FMEA에서 시간을 고려한 기대손실모형에 기초한 위험 평가

권혁무† · 홍성훈* · 이민구** · Agung Sutrisno***
부경대학교 시스템경영공학과 · *전북대학교 산업정보시스템공학과 · **충남대학교 정보통계학과
***Department of Mechanical Engineering, Sam Ratulangi University

(2011. 8. 26. 접수 / 2011. 12. 1. 채택)

Risk Evaluation Based on the Time Dependent Expected Loss Model in FMEA

Hyuck Moo Kwon† · Sung-Hoon Hong* · Min Koo Lee · Agung Sutrisno*****
Department of Systems Management and Engineering, Pukyong National University
*Department of Industrial & Information Systems Engineering, Chonbuk National University
**Department of Information and Statistics, Chungnam National University
***Department of Mechanical Engineering, Sam Ratulangi University, Indonesia

(Received August 26, 2011 / Accepted December 1, 2011)

Abstract : In FMEA, the risk priority number(RPN) is used for risk evaluation on each failure mode. It is obtained by multiplying three components, i.e., severity, occurrence, and detectability of the corresponding failure mode. Each of the three components are usually determined on the basis of the past experience and technical knowledge. But this approach is not strictly objective in evaluating risk of a given failure mode and thus provide somewhat less scientific measure of risk. Assuming a homogeneous Poisson process for occurrence of the failures and causes, we propose a more scientific approach to evaluation of risk in FMEA. To quantify severity of each failure mode, the mission period is taken into consideration for the system. If the system faces no failure during its mission period, there are no losses. If any failure occurs during its mission period, the losses corresponding to the failure mode incurs. A longer remaining mission period is assumed to incur a larger loss. Detectability of each failure mode is then incorporated into the model assuming an exponential probability law for detection time of each failure cause. Based on the proposed model, an illustrative example and numerical analyses are provided.

Key Words : FMEA, RPN, homogeneous poisson process, time dependent loss model

1. Introduction

Correct evaluation of failure risk is an important part toward the efficiency of a firm's resource allocation. In industrial practices, firms utilized FMEA(failure mode and effect analysis) as a means to estimate the risk of system failure and to provide an appropriate way of reducing its impact on the end customer. In FMEA, risk of failure is measured based on the metric called the Risk Priority Number(RPN). It is a metric obtained by multiplying the ratings of severity, occurrence, and detectability of each failure. But the rating on each of the three components is usually based on the past experiences and intuition of the FMEA team. Thus, the conventional FMEA based on MIL STD1629 A provides only rough evaluation of relative risk priority for each failure mode. For detailed explanation of the conventional FMEA, the big 3 motor companies' FMEA reference manual4) can be referenced.

Up to now, many authors including Eubanks et al.5), Blivbamd et al.3), Bertolini et al.2), Jeegadeshan et al.6) and Oolkalkar et.al8) tried to find out improved methods that complement the conventional FMEA. Narayanagounder and Karuppusami7) and Sawhney et al.9) presented summary to previous efforts to improve the RPN prioritization method. Agung and Kwon1) suggested an

---

† To whom correspondence should be addressed.
iehmkwon@pknu.ac.kr

expected loss model for improving risk prioritization in the conventional FMEA. The previous studies seem to neglect the role of time in modeling the likelihood of failure occurrences in failure cause and its corresponding failure effect relationship. Besides, the scales to estimate the criticality of failure effects are still determined qualitatively. Senol[10)] determined the component of failure occurrence on the basis of Poisson distribution. However, the approach used in estimating the probability of failure occurrences is neglecting the influence of failure cause occurrence. Also, the Poisson-based FMEA proposed is still referring to 1-10 scale rating of failure detectability and severity.

In this paper, a model for quantifying the failure risk is presented assuming that a failure can occur only after at least one of its causes has occurred in advance. Constant occurrence rates are assumed for every failure and its causes. The failure risk is evaluated by the loss that results from the failure. The loss due to each failure mode is assumed to depend on the remaining mission period of the system. To evaluate the risk of each failure mode, the expected value of its corresponding loss is obtained. In the proposed model, the expected loss of each failure mode includes the two components of FMEA, i.e. severity and occurrence, intrinsically. We then incorporate the last component of detectability into our model as the probability that every cause of a failure mode is detected before the corresponding failure actually occurs. This new risk evaluation model will facilitate FMEA team to quantify the risk of each failure mode and prioritize the failure modes.

This paper is organized as follows. In Section 2, the three components of the conventional FMEA are expressed as mathematical equations assuming the Homogeneous Poisson Process(HPP) for failure and cause occurrence. In Section 3, the time dependent expected loss model is constructed for constant, linear and quadratic loss functions. In Section 4, an illustrative example is provided and the proposed model is compared with the conventional FMEA based on the given example. Section 5 concludes with possible future extensions for further studies.

2. Quantifying Risk Components in FMEA

FMEA evaluates the risk of each failure mode on the basis of three components; severity, occurrence and detectability. In the conventional FMEA, each component is estimated using 1-10 scales, which largely depends on the past experience and intuition of the FMEA team. There are guidelines to assign an appropriate number to each component related with a specific failure. But these guidelines are not precise enough and provide only a rough estimation of relative risk priority for each failure mode. When sufficient information is available from the past experience and scientific knowledge, a more systematic approach may be applicable. In this section, we suggest a scientific approach to quantification of each component under some reasonable assumptions.

2.1. Severity

The severity of each failure mode may reasonably be evaluated by its resultant loss. The loss will be incurred if the system or process fails during its mission time duration(0,T). If the system does not fail during(0,T), no loss is confronted. The loss function may be reasonably assumed to be a non-decreasing function of the length of the remaining mission time period. Here, three different types of loss function are considered for each failure; constant, linear and quadratic.

When the system suffers a fixed amount of loss if any kind of failure occurs, a constant loss function will be appropriate. If we denote the loss due to failure k connected to its i^{th} cause by Lki, the constant loss function will be given as

$$L_{ki} = \begin{cases} \alpha_k, & 0 < W_{ki} < T \\ 0, & otherwise \end{cases} \tag{1}$$

where α_k is a constant reflecting the severity of the failure k and W_{ki} is the time elapsed until failure k occurs due to its i^{th} cause from the beginning time point of the system or process operation. When a failure has several effects, α_k should reflect the total aggregated effects that result from failure k.

Next, when the loss amount is proportional to the length of the remaining mission time period, the linear loss function will be appropriate. In this situation, the loss function is given by

$$L_{ki} = \begin{cases} \alpha_k(T - W_{ki}), & 0 < W_{ki} < T \\ 0, & otherwise \end{cases} \tag{2}$$

And finally, when the unfulfilled mission period severely affects the loss due to the failure, the quadratic loss function will be appropriate. The quadratic loss function is defined by

$$L_{ki} = \begin{cases} \alpha_k(T - W_{ki})^2, & 0 < W_{ki} < T \\ 0, & otherwise \end{cases} \tag{3}$$

2.2. Occurrence

The occurrence of a failure may be described by the time elapsed from the beginning of system operation to the system failure. Any failure must have some cause or causes which lead to the failure itself. Occurrence of every failure comes after occurrence of one or more of its causes. Thus, the failure time is composed of two components; the cause occurrence time and its corresponding failure occurrence time.

Now let X_{ki} be the time elapsed until the i^{th} cause, $i = 1, 2, ..., n_k$ of failure mode k, $k = 1, 2, ..., l$ occurs. Assume that each cause occurs at a constant rate λ_{ki} over time. Then the probability density function of X_{ki} will be

$$f_{X_{ki}}(x) = \lambda_{ki} e^{-\lambda_{ki} x}, \quad 0 < x. \tag{4}$$

Next, let Y_{ki} be the time elapsed until failure k occurs from the occurrence time point of its i_{th} cause. Assume the occurrence rate of failure k due to its i_{th} cause is constant over time, say μ_{ki}. Then the probability density function of Y_{ki} will be

$$f_{Y_{ki}}(y) = \mu_{ki} e^{-\mu_{ki} y}, \quad 0 < y. \tag{5}$$

Let W_{ki} be the time elapsed until failure k occurs due to its i_{th} cause from the beginning time point of operation of the system or process. Then

$$W_{ki} = X_{ki} + Y_{ki}. \tag{6}$$

The probability density function of can be shown to be

$$f_{W_{ki}}(w) = \frac{\lambda_{ki}\mu_{ki}}{\lambda_{ki} - \mu_{ki}} \left\{ e^{-\mu_{ki} w} - e^{-\lambda_{ki} w} \right\}, \ 0 < w \tag{7}$$

See Appendix for detailed derivation. The distribution function can be easily obtained as

$$F_{W_{ki}}(w) = 1 - \frac{1}{\lambda_{ki} - \mu_{ki}} \left\{ \lambda_{ki} e^{-\mu_{ki} w} - \mu_{ki} e^{-\lambda_{ki} w} \right\}, \ 0 < w. \tag{8}$$

2.3. Detectability

Once a failure occurs, it takes much time and cost for remedy. But the cause is detected before the failure actually occurs, it usually does not take much time and cost for correction. When the cause is detected before the corresponding failure occurs, immediate corrective action is assumed to be taken without any loss. The detectability is determined by the probability distributions of detection time and failure occurrence time after the corresponding cause occurrence.

Let U_{ki} be the time elapsed until the i_{th} cause of failure mode k is detected from its occurrence time point. Here, U_{ki} is assumed to have an exponential distribution with its probability density function

$$f_{U_{ki}}(u) = \tau_{ki} e^{-\tau_{ki} u}, \ 0 < u \tag{9}$$

Assuming U_{ki} and Y_{ki} to be independent, the probability that the i_{th} cause of the failure mode k is detected before the corresponding failure occurs can easily be obtained as

$$P[U_{ki} < Y_{ki}] = \frac{\tau_{ki}}{\mu_{ki} + \tau_{ki}}. \tag{10}$$

In FMEA, the detectability component actually is undetectability which is the probability that failure occurrence is not prevented. Thus, the detectability D_{ki}, i.e. the probability of occurrence of failure k connected to its i_{th} cause is obtained by

$$D_{ki} = P[U_{ki} > Y_{ki}] = \frac{\mu_{ki}}{\mu_{ki} + \tau_{ki}}. \tag{11}$$

3. The Time Dependent Expected Loss Model

The overall risk for a failure mode can be evaluated by the expected value of its resultant total loss. To get an accurate expected value of loss for failure k, we must use the exact failure time distribution of failure mode k connected to its i_{th} cause which is somewhat different from (7) and (8). It actually is far more complicated than these formulas and cannot be expressed in a simple form. To be strict, W_{ki} depends on X_{ki}, Y_{ki} and U_{ki}, $i = 1, 2, \ldots, n_k$.

Notice that even after the i_{th} cause of failure mode k is detected and corrected, it may reoccur after X_{ki} units of time. Thus, W_{ki} includes Y_{ki} and several random variables which are identical with X_{ki} and U_{ki}. This situation is illustrated in Fig. 1, assuming that every failure cause is detected up to the $(v\text{-}1)^{th}$ occurrence and corrected before actual failure occurs. At the v^{th} occurrence of failure cause, actual failure occurs before its cause is detected.

In this article, however, we are going to avoid too much complexity in obtaining the expected loss. We simply consider detectability separately. We first calculate the expected loss $E[L_{ki}]$ based on the distribution of W_{ki} given by formulas (7) and (8). Then the risk measure of failure k due to its i_{th} cause is obtained by multiplying $E[L_{ki}]$ with the detectability D_{ki} of formula (11).

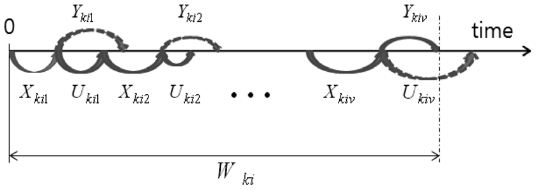

Fig. 1. The actual components of W_{ki}.

For constant, linear, quadratic loss function, the expected loss can be obtained by

$$E[L_{ki}] = \alpha_k F_{W_{ki}}(T), \tag{12}$$

$$E[L_{ki}] = \alpha_k \int_0^T F_{W_{ki}}(w)\,dw, \tag{13}$$

$$E[L_{ki}] = \alpha_k \int_0^T (T-w)^2 f_{W_{ki}}(w)\,dw, \tag{14}$$

respectively. We can evaluate the risk of failure k due to its i_{th} cause by multiplying $E[L_{ki}]$ with the detectability of formula (11) for every $i = 1, 2, \ldots, n_k$, $k = 1, 2, \ldots, l$ and the risk priorities of all the failure causes can be determined.

To facilitate the evaluation procedure, a modified FMEA sheet of fig. 2 can be used. Note that some quantities in the sheet are not easy to get directly from the given information, requiring some additional calculations. And even though several failure effects may be present for one failure mode, only one loss function is applicable for each failure mode. This means that the loss function should sum up all the effects corresponding to the given failure mode. We should choose the value of α_k taking this fact into account.

Fig. 2. The modified FMEA sheet for risk evaluation.

Failure mode	Severity		Occurrence			Expected loss	Detectability		Risk measure
	Effect	Loss	Cause		Failure				
	Item	Loss function	item	λ	μ	$E[L]$	τ	D	$E[L]\times D$
…	…	…	…	…	…	…	…	…	…
F_k	E_{k1}	α_k $\alpha_k(T-W_k)$ $\alpha_k(T-W_k)^2$	C_{k1}	λ_{k1}	μ_{k1}	$E[L_{k1}]$	τ_{k1}	D_{k1}	$E[L_{k1}]D_{k1}$
	E_{k2}		C_{k2}	λ_{k2}	μ_{k2}	$E[L_{k2}]$	τ_{k2}	D_{k2}	$E[L_{k2}]D_{k2}$
	…		…	…	…	…	…	…	…
	E_{kn_k}		C_{kn_k}	λ_{kn_k}	μ_{kn_k}	$E[L_{kn_k}]$	τ_{kn_k}	D_{kn_k}	$E[L_{kn_k}]D_{kn_k}$
…	…	…	…	…	…	…	…	…	…

Table 1. Design FMEA sheet for the front door of an automobile(excerpted from TS 16949 FMEA manual)

Potential Effects	S	Potential causes	O	Current Design Controls	D	RPN
Deteriorated life of door leading to: Unsatisfactory appe-arance. Impaired function of interior door hard-ware	7	Upper edge of protective wax application specified for inner door panels is too low	6	Vehicle general durability test	7	294
		Insufficient wax thickness specified	4	Vehicle general durability test	7	196
		Inappropriate wax formulation specified	2	Physical and chem. Lab test	2	28
		Entrapped air prevents wax from entering comer/edge access	5	Design aid investigation with non-functioning spray head	8	280
		Wax application plugs door drain holes	3	Lab test using "worst case" wax application and hole size	1	21
		Insufficient room between panels for spray head access	4	Drawing evaluation of spray head access	4	112

4. Numerical Analyses

In this section, we provide an illustrative example to explain how to use our model in the field. We also compare the results of three types of loss functions with that of the conventional FMEA. The example is taken from the FMEA reference manual of TS 16949 (2008) and slightly modified to fit our model.

Example. Let's consider the front door of an automobile whose main functions are i) ingress to and egress from vehicle, ii) occupant protection from weather, noise, and side impact, iii) support anchorage for door hardware including mirror, hinges, latch and window regulator, iv) provide proper surface for appearance items, and v) paint and soft trim. Table 1 describes potential effects, causes, and current controls with RPN ratings for the potential failure mode "Corroded interior lower door panels." To apply our model to this example, we assign numerical values for α, λ, μ and τ considering those numbers given in Table 1. The mission period is assumed ten years. Table 2 shows the modified example fit for our model. Based on the parameter values given in Table 2, we evaluate the expected losses corresponding to each failure cause and summarize the results in Table 3.

Next, the risk for each failure cause is evaluated by multiplying the expected loss with its corresponding detectability. Here, for easy handling of number, the

Table 2. Modified example for the front door of an automobile

Potential Effects	α	Potential causes	λ	μ	τ
Deteriorated life of door leading to: Unsatisfactory appe-arance. Impaired function of interior door hard-ware	7	Upper edge of protective wax application specified for inner door panels is too low	1/80	1/8	1/7
		Insufficient wax thickness specified	1/2000	1/200	1/7
		Inappropriate wax formulation specified	1/150000	1/15000	1/2
		Entrapped air prevents wax from entering comer/edge access	1/400	1/40	1/8
		Wax application plugs door drain holes	1/15000	1/1500	1
		Insufficient room between panels for spray head access	1/2000	1/200	1/4

Table 3. The expected losses due to each failure cause for three types of loss function

Potential Effects	α	Potential causes	Types of loss function		
			Constant	Linear	Quadratic
Deteriorated life of door leading to: Unsatisfactory appe-arance. Impaired function of interior door hard-ware	7	Upper edge of protective wax application specified for inner door panels is too low	0.35897	1.3265	7.0573
		Insufficient wax thickness specified	0.000859	0.002878	0.014424
		Inappropriate wax formulation specified	$1.551*10^{-7}$	$5.184*10^{-7}$	$2.592*10^{-6}$
		Entrapped air prevents wax from entering comer/edge access	0.019990	0.068146	0.345345
		Wax application plugs door drain holes	$1.552*10^{-5}$	$5.176*10^{-5}$	$2.589*10^{-4}$
		Insufficient room between panels for spray head access	0.000859	0.002878	0.014424

Table 4. The evaluated risk and priority of each failure cause

Potential Effects	α	Potential causes	Detection	Types of loss function			Risk priority
				Constant	Linear	Quadratic	
Deteriorated life of door leading to: Unsatisfactory appearance. Impaired function of interior door hardware	7	Upper edge of protective wax application specified for inner door panels is too low	7/15	167520	619027	3293407	1
		Insufficient wax thickness specified	7/207	29	97	488	3
		Inappropriate wax formulation specified	7/7501	0.000145	0.000484	0.00242	6
		Entrapped air prevents wax from entering comer/edge access	1/6	3332	11358	57558	2
		Wax application plugs door drain holes	1/1501	0.01034	0.03448	0.1725	5
		Insufficient room between panels for spray head access	1/51	16.8	56.4	282.8	4

result is again multiplied with 1,000,000. Then the risk priority of each failure cause is determined. Table 4 shows the evaluated risks and priorities.

Notice that there is no difference in the order of the risk priority between the conventional FMEA and our proposed model. But the evaluated sizes of corresponding risks are much different. For example, in Table 1, the maximum and minimum RPN's are 294 and 21, respectively. In Table 4, the maximum and minimum expected losses for quadratic loss function are 3293407 and 0.00242, respectively. It seems that the proposed model might yield the order of risk priorities not much different from that of the conventional FMEA. But it provides more realistic information on the degree of the corresponding risk. One may think that the potential cause "Inappropriate wax formulation specified" cannot be neglected in Table 1. If he refers to Table 4, however, he may consider that cause negligible without slightest hesitation. Moreover, Table 4 shows that how much effort should be focused on the failure cause with the highest risk priority.

5. Conclusion

Integration of the aspect of failure time occurrence in estimating the RPN in FMEA is one key strategic approach toward proactive error free operation. Our literature survey indicated that previous techniques to estimate the RPN are neglecting the role of time horizon as key dimension to estimate the risk and still relying on qualitative basis to determine the severity of failure effects. In this study, we have presented a new model to estimate the RPN of FMEA accomplishment by considering the role of time span and loss incurred by failing to carry out the mission of an item.

Assuming exponential probability distribution for the occurrence time of failure and cause, we proposed an expected loss model for evaluating the failure risks. Compared with the conventional FMEA, the proposed model can provide more specific information on the magnitude of risk for each failure and its corresponding causes. The conventional FMEA may help to determine the priority of preventive action for each failure. But it does not tell us on the suitable degree of efforts or the right amount of expenses required. When there is sufficient information on the failure mechanism, the proposed model can provide a way of solution for such problems. If we have no information on the failure occurrence, the maximum likelihood estimation methodology can be used to obtain reasonable estimates for the occurrence rates of each failure and its causes. Besides, the proposed model enables the FMEA practitioner to monitor the trend of expected loss in a more manageable way.

Since this study is still in its initial stage, extensions for further studies are encouraged. Many systems are usually monitored periodically for occurrence of failure or failure causes. Failure occurrence may follow a non-homogeneous Poisson process(NHPP). We are not going to mention all the possible situations not considered in our model. These unexplored situations provide possible areas of extension.

Acknowledgement: This work was supported by the Quality Innovation and Infrastructure Development program through the Korea Institute for Advancement of Technology funded by the Ministry of Knowledge Economy, Republic of Korea.

References

1) Agung, S. and Kwon, H., “An Expected Loss Model for Risk Prioritization in Service FMEA”, California Journal of Operations Management, Vol. 8, No. 2, 27~38, 2010.
2) Bertolini, M., Braglia., M. and Carmignani, G., “An FMECA-based Approach to Process Analysis”, International Journal of Process Management and Benchmarking, Vol. 1, No. 20, 127~145, 2006.
3) Blivband, Z., Grabov, P. and Nakar, O., “Expanded FMEA(EFMEA)”, Annual Reliability and Maintainability Symposium, 31~36, 2004.
4) Chrysler LLC, Ford motor Company, and General Motors Corporation, FMEA Reference Manual, 4thed, 2008.
5) Eubanks, C. F., Kmenta, S. and Ishii, K., “Advanced FMEA Using behavior Modeling”, ASME Design Engineering Technical Conference, Sacramento, California, 1~10, 1997.
6) Jeegadeshan, C., Arunachalam, V. P., Devadasan, S. R. and Srinivasan, P. S. S., “Design and Development of Modified Service FMEA Model”, International Journal of Services and Operations Management, Vol. 3, No. 1, 111~126, 2007.
7) Narayanagounder, S. and Karuppusami, G., “A New Approach for Prioritizing Failure Mode in Design FMEA Using Anova”, World Academy of Science, Engineering, and technology, Vol. 49, 524~531, 2009.
8) Oolkalkar, A. D., Joshi, A. G. and Oolkalkar, D. S., “Quality Improvement in Haemodialysis Process Using FMEA”, International Journal of Quality and Reliability Management, Vol. 26, No. 8, 817~830, 2009.
9) Sawhney, R., Subburaman, K., Sonntag, C., Rao, P. R. V. and Capizzi, P., “A Modified FMEA Approach to Enhance Reliability In Lean Systems”, International Journal of Quality and Reliability Management, Vol. 24, No. 5, 525~546, 2010.
10) Senol, S., “Poisson Approach to Determine the Occurrence Degree in Failure Mode and Reliability Analysis”, The Quality Management Journal, Vol. 14, No. 2, 12~24, 2007.

Appendix : Proof of (7)

1) Since X_{ki} and Y_{ki} are mutually independent, their joint probability density function is given by

$$f_{X_{ki},\,Y_{ki}}(x,y)=\lambda_{ki}\mu_{ki}e^{-\lambda_{ki}x-\mu_{ki}y},\,0<x,\;0<y. \qquad \text{(A1)}$$

2) If we change variable as $W_{ki} = X_{ki} + Y_{ki}$ and $X_{ki} = Y_{ki}$, the joint probability density function of X_{ki} and W_{ki} will be

$$f_{X_{ki},\,W_{ki}}(x,w)=\lambda_{ki}\mu_{ki}e^{-(\lambda_{ki}-\mu_{ki})x-\mu_{ki}w},\,0<x<w. \text{ (A2)}$$

3) The probability density function of W_{ki} is obtained by integrating (A2) with x over its range, that is,

$$\begin{aligned} f_{W_{ki}}(w) &= \int_0^w \lambda_{ki}\mu_{ki}e^{-(\lambda_{ki}-\mu_{ki})x-\mu_{ki}w}dx \\ &= \frac{\lambda_{ki}\mu_{ki}}{\lambda_{ki}-\mu_{ki}}\{e^{-\mu_{ki}w}-e^{-\lambda_{ki}w}\},\;0<w. \end{aligned}$$

4) Thus, we get Equation (7).