

GPS Integrity Monitoring Method Using Auxiliary Nonlinear Filters with Log Likelihood Ratio Test Approach

Jongsun Ahn*, Rosihan*, Dae Hee Won*, Young Jae Lee*,
Gi Wook Nam**, Moon-Beom Heo** and Sangkyung Sung[†]

Abstract – Reliability is an essential factor in a navigation system. Therefore, an integrity monitoring system is considered one of the most important parts in an avionic navigation system. A fault due to systematic malfunctioning definitely requires integrity reinforcement through systematic analysis. In this paper, we propose a method to detect faults of the GPS signal by using a distributed nonlinear filter based probability test. In order to detect faults, consistency is examined through a likelihood ratio between the main and auxiliary particle filters (PFs). Specifically, the main PF which includes all the measurements and the auxiliary PFs which only do partial measurements are used in the process of consistency testing. Through GPS measurement and the application of the autonomous integrity monitoring system, the current study illustrates the performance of the proposed fault detection algorithm

Keywords: Reliability, Integrity monitoring, Fault, Particle filter, Likelihood ratio, GPS

1. Introduction

Numerous fault detection (FD) studies have been conducted with real time sensor measurement. Generally, FD methods are approached in a statistical manner. For example, a stochastic approach through Kalman filter is used when the given system model is linear and the noise and disturbance possess Gaussian characteristics; thus, one can obtain the most ideal state estimates [1]. In case the nonlinear system possessing the Gaussian type of noise characteristics, finding an optimal solution becomes complicated. To solve this problem, an extended Kalman filter is used, which gives suboptimal solution through local linearization [2]. Recently, much attention has been given to the nonlinear and non-Gaussian estimation problem that is based on Monte Carlo methods such as particle filtering. The strong merit of the particle filter (PF) is that a nonlinear system model, system noise, and measurement noise can be stochastically managed via point mass representation of probability distribution [3, 16].

With the help of nonlinear filtering mechanization, the main interest of this paper is to ensure the reliability of the GPS signal in a navigation system and thus enhance the probability of sending timely fault warning to users. Integrity means notifying people about systematic fallacies within a given time. Generally, the GPS control center

takes 15 minutes to notify users about the pseudorange fault measurement [4]. Thus, much swifter failure detection to notify in-operation users and equipment is required [7–10]. In an aviation navigation system that uses GPS, determining user mobility depends on the relative geographic movement between the user and the satellite. More accurate location estimation needs a processing algorithm to manage time-varying anomalous measurements. This is vital for extracting reliable location information utilizing GPS. Thus, to provide valid information, FD and removal techniques are critical to the GPS based system. Although many procedures for FD have been introduced, residual testing through statistical approach is a fundamental method [13–15]. In the residual test scheme, picking out the sporadic errors is easy, but detecting the gradually increasing error in the measurement is not. FD may be conducted by examining the probabilistic property of the estimated state itself instead of the residuals [5], yet a precise reference system unaffected by failures is required in implementation [17].

In this present paper, a new GPS integrity monitoring method is presented by applying the particle filter based state estimation and subsequent likelihood ratio test. Specifically, the presented fault detection and isolation (FDI) scheme takes advantage of the robust performance of the PF with respect to nonlinear and non-Gaussian system and highly sensitive log likelihood ratio (LLR) change between the main and N auxiliary filters, where N matches with the number of available satellites for GPS data acquisition. Particularly, the FD performance is notable in the case of a ramp type fault through numerical simulation. This paper is organized as follows: Section 2 briefly

[†] Corresponding Author: Department of Aerospace Information Engineering, Konkuk University, Korea. (sksung@konkuk.ac.kr)

* Department of Aerospace Information Engineering, Konkuk University, Korea.

** Department of Satellite Navigation, Space Application and Future Technology Center in Korea Aerospace Research Institute (KARI).

introduces the general principle and structure of FD, which presents a preliminary discussion for the succeeding equation development. In Section 3, a detailed GPS integrity monitoring algorithm that employs complementary PF and ratio test is illustrated. Numerical simulation using real GPS measurement data is presented in Section 4, followed by the conclusion in Section 5.

2. Principle and Structure of FDI Algorithm

2.1 Multi-model FD scheme for GPS monitoring

The FD problem consists of making the decision in the presence or absence of faults in the monitored system, whereas the problem of fault isolation (FI) consists of deciding the present faulty mode among a number of possible modes. In the current study, both FD and FI systems for GPS integrity monitoring are considered. The proposed FDI system structure originates from a simple idea that multiple failures hardly happen in the GPS monitoring system. Thus, the main failure occurring in the FDI system is assumed to be related to a subset of the system measurements. Fig. 1 shows the exemplary structure of the proposed failure detection system using the PF approach.

In Fig. 1, the Main PF, using the GPS measurements and with $y_{main} := [y_{sub1} \ y_{sub2}]^T$ as input, yields an optimal state estimate \hat{x}_{main} when the system is working in normal state. The auxiliary filters Sub1 PF and Sub2 PF, using y_{sub1} and y_{sub2} as inputs, respectively, generate the state estimates \hat{x}_{sub1} and \hat{x}_{sub2} as failure detection references. Under the single failure condition, wherever a failure occurs, either Sub1 PF or Sub2 PF is unaffected. Consequently, failure detection can be conducted by checking the consistency

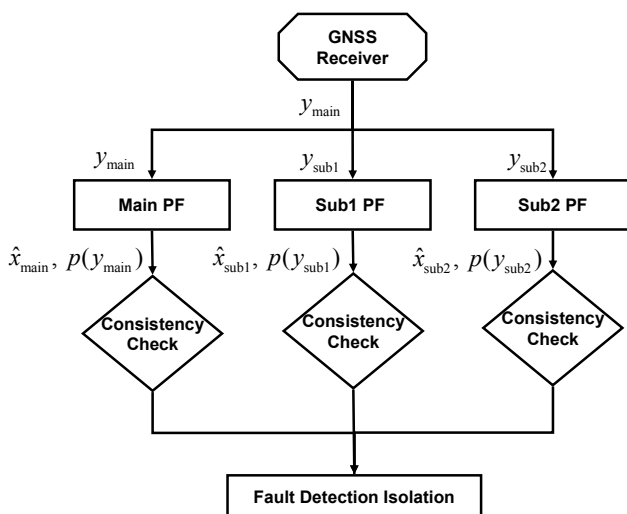


Fig. 1. Failure detection system structure based on the PF approach.

between the estimates, \hat{x}_{main} and \hat{x}_{sub1} and between \hat{x}_{main} and \hat{x}_{sub2} . For consistency check to detect failures, the LLR test is applied. As observed from Fig. 1, a reference system unaffected by failures is required for the FD. The proposed scheme also adopts a LLR test for the consistency check to detect failures. Meanwhile, the GPS signal consists of a clock signal and a navigation message that is modulated by amplitude. Each satellite sends the clock signal via two different bands, L_1 and L_2 [2]. The GPS receiver typically receives the signal corrupted by noise and other sources of error.

By considering error sources, the raw measurements of the code and carrier phase pseudoranges are presented as follows [3, 6]:

$$\rho^i(k) = R^i(k) + c\Delta\delta^i + T^i(k) + I^i(k) + E^i(k) + \varepsilon^i(k) \quad (1)$$

where ρ refers to the code pseudoranges (m), i is the satellite number, R is the distance between the receiver and the satellite position (m), c is the speed of light (m/s), $\Delta\delta$ is the combined clock offsets of the receiver and the satellite clock with respect to GPS time (s), E is the effect of ephemeris error (m), I is the ionospheric delay (m), T is the tropospheric delay, and ε is the code observation noise (m). The distance R^i is computed by the root of error squared sum between the position of the receiver and the i^{th} satellite in the coordinate system, respectively. Consequently, the typical measurement equation of GPS consists of the nonlinear system model and the non-Gaussian noise distribution, and thus, requires a robust nonlinear filter for reliable estimation of state innovation.

2.2 Particle filter and likelihood ratio method

Considering the nonlinear system model, particle filtering is chosen for probability density function (pdf) approximation. The PF is an algorithm that provides iterative Monte Carlo approximations for a given sequence of state variable distribution. The algorithm approximates the required pdf by swarms of points in the state space. The predictive pdf $p(x_k | Z_{k-1})$, where Z_k is denoted as the set of measurements up to time, is approximated by a set of M particles $\{x_k(i) : i = 1, \dots, M\}$. By allocating weights on particles, the state variable's distribution can be approximated by a discrete distribution of each particle. The probability density is proportional to the weight of each particle, which is recursively computed. The particles are random samples; thus with increased particle numbers, they can provide an effective approximation to the target PDF.

A summarized PF algorithm is presented following Gordon's approach [12], which is generally characterized by the sampling importance re-sampling (SIR) filter. For further equation development, let us first consider the dynamic state space model below [3, 16]:

$$\begin{aligned} x_{k+1} &= f(x_k, w_k) \\ z_k &= h(x_k, v_k) \end{aligned} \quad (2)$$

where x_k is the state vector, z_k is the output measurement vector, f and h are the nonlinear state transition and measurement function, respectively, w_k is the process noise vector independent of the current state, and v_k is the measurement noise vector independent of the states and the system noise. Under a general assumption for the SIR filter construction, i.e., f and h are obtained in a functional form and the initial PDF of states are available with the measurements given, the PF can perform state estimation recursively by regenerating random samples from the distribution of w_k and from the predicted distribution of the prior state. Obviously, the likelihood function $p(x_k | Z_k)$ needs to be known in a functional form. Filtering algorithm can be summarized through the following steps [6, 11, 12]. First, assume that there is a set of random samples (i.e., particles of filter) $\{x_{k-1}(i) : i = 1, 2, \dots, M\}$ from the pdf $p(x_{k-1} | Z_{k-1})$. In a prediction step, by sampling M values $\{w_{k-1}(i) : i = 1, 2, \dots, M\}$ from the pdf of the system noise w_{k-1} , new swarms of points $\{x_{k|k-1}(i) : i = 1, 2, \dots, M\}$ are generated which approximate the predicted pdf $p(x_{k|k-1} | Z_{k-1})$, where $x_{k|k-1}(i) = f_{k-1}(x_{k-1}(i), w_{k-1}(i))$. In the updated step, for each $x_{k|k-1}(i)$, a weight $w_k(i)$ for $i = 1, 2, \dots, M$ can be assigned after measurement z_k is received. The weights are given by $w_k(i) = p(z_k | x_{k|k-1}(i)) \cdot [\sum_{j=1}^M p(z_k | x_{k|k-1}(j))]^{-1}$. This equation defines a discrete distribution over $\{x_{k|k-1}(i) : i = 1, 2, \dots, M\}$, which assigns the probability mass $w_k(i)$ to the element $x_{k|k-1}(i)$ and results in the posterior pdf $p(x_k | Z_k)$ being represented in terms of weighted samples. Finally, the resampling step is followed.

By re-sampling independently M times from the above discrete distribution, the resulting particles $\{x_k(i) : i = 1, 2, \dots, M\}$ which satisfy $\Pr\{x_k(i) = x_{k|k-1}(j)\} = w_k(j)$ form an appropriate set of samples from the posterior pdf $p(x_k | Z_k)$ with equal weight to each element. Finally, the prediction, update, and re-sampling step form a single iteration and are recursively applied.

In detecting faults, the LLR method is employed due to its proven performance in FD systems using the PF [6, 9]. The system model used for the FDI problem is formulated in a multi-model environment. Consider the nonlinear stochastic system expanded from (2),

$$\begin{aligned} x_k &= f(x_{k-1}, \alpha, w_{k-1}) \\ z_k &= h(x_k, \alpha, v_k) \end{aligned} \quad (3)$$

where $f(\cdot)$ and $h(\cdot)$ are nonlinear functions parameterized by a vector, $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_D]^T$. D denotes the total number of parameters to be monitored and the faults are modeled as unknown changes in the parameter

α . FD ascertains a model shift or detects a jump from the normal (i.e., fault free) model indexed by $d = 1$ to the faulty model indexed by $d = 1, \dots, D$. FI determines the index that indicates possible faulty models. In computing the index, a likelihood-based function of LLR is used. LLR is a function of random variable z , which is defined as

$$s(z) = \ln \frac{p_{\alpha_1}(z)}{p_{\alpha_0}(z)} \quad (4)$$

where $p_{\alpha_i}(z)$ ($i = 0, 1$) is a pdf parameterized by α_i . If the observations z_k ($k = 1, 2, \dots$) with a pdf $p_\alpha(z)$ are independent of each other, the cumulative LLR for the observations from z_j to z_k can be expressed as

$$S_j^k = \sum_{i=j}^k s(z_i) \quad (5)$$

The typical behavior of this cumulative LLR S_j^k shows, on average, a negative drift before change and a positive drift after change as k increases, which can be effectively used to detect any faults. By using (5), the detection problem, given the observation up to k , comprises testing two hypotheses H_0 and H_1 that reflect the different parameterizations of the pdf before and after the onset of a fault, respectively. The change detector is achieved by checking if $\max_j S_j^k$ exceeds a predetermined threshold or not.

In solving an FDI problem in a multi-model environment as described in (3), an efficient method is to use a set of adaptive PFs that monitor only a strict subset of the parameter vector. The FDI decision made by the LLR test uses the complete pdf information obtained from every PF. The system model of the nominal and the d -th step auxiliary PF for this problem are illustrated as follows:

Nominal PF D_0 :

$$\begin{aligned} x_k^{(0)} &= f(x_{k-1}^{(0)}, \alpha^{(0)}, w_{k-1}), \\ z_k &= h(x_k^{(0)}, \alpha^{(0)}, v_k) \end{aligned} \quad (6)$$

Auxiliary PF D_d :

$$\begin{aligned} \begin{bmatrix} x_k^{(d)} \\ \bar{\alpha}_{d,k} \end{bmatrix} &= \begin{bmatrix} f(x_{k-1}^{(d)}, \bar{\alpha}_d^{(0)}, \bar{\alpha}_{d,k-1}, w_{k-1}) \\ \bar{\alpha}_{d,k-1} + w'_d \end{bmatrix}, \\ z_k &= h(x_k^{(d)}, \bar{\alpha}_d^{(0)}, \bar{\alpha}_{d,k}, v_k) \end{aligned} \quad (7)$$

where $\bar{\alpha}$ denotes a subset of parameter vector α . Each auxiliary PF estimates only a subset of parameters and assumes that the remaining parameters are known and constant. However, a change in a single subset affects the model equation, hence the overall parameter estimates. Further steps are needed to identify which subset of

parameters has really changed.

The joint likelihood of the observations based on each auxiliary PF is computed using the complete sample-based pdf information obtained from PFs. The joint LLR can be activated in a parallel-D LLR test by combining H_d ($d=1,2,\dots,D$) and H_0 , which is computed as

$$S_j^k(d) = \sum_{r=j}^k \ln \frac{p(z_r | H_d, Z_{r-1})}{p(z_r | H_0, Z_{r-1})} \quad (8)$$

With the PF, the innovation likelihood in the numerator and denominator term (8) can be estimated using the pdf information of the predicted state $x_{r|r-1}^d$, which is represented by a swarm of particles [1]. The joint likelihood of the observation is computed using the complete sample-based pdf information provided by PF as follows:

$$p(z_r | H_d, Z_{r-1}) = \int p(z_r | H_d, x_r) p(x_r | H_d, Z_{r-1}) dx_r \approx \frac{1}{M} \sum_{i=1}^M p(z_r | x_{r|r-1}^d(i)). \quad (9)$$

Finally, the decision function for FD is given by

$$\beta_k = \max_{k-U+1 \leq j \leq k} \max_{1 \leq d \leq D} S_j^k(d) \begin{matrix} > & H_1 \\ < & H_0 \end{matrix} \tau \quad (10)$$

where U is the sliding window size of recent observations and τ is the threshold value. The FI is achieved by determining the index g of the faulty subset of parameters, which is given by

$$g = \arg \max_{1 \leq d \leq D} S_{t_a}^k(d) \quad \text{for } k > t_a \quad (11)$$

where t_a is the time where the fault alarm is set.

3. FDI Application to GPS Integrity Monitoring

The structure of the proposed GPS integrity monitoring method is illustrated in Fig. 2. In considering a normal GPS positioning condition, four measurements are assumed for the integrity monitoring in the figure. In general, if the number of measurements $y_{main} = [z_1, z_2, \dots, z_n]^T = Z_n$ is n , then $n+1$ units of PFs are required, where one is the main filter and other n filters are auxiliaries.

In Fig. 2, there are four measurements and five PFs for integrity monitoring. The Main PF processes all the four measurements to generate the best state estimate \hat{x}_{main} and its pdf $p(y_{main})$, while the auxiliary PFs process three measurements out of four to provide the state estimates

\hat{x}_{subN} (the subscript N denotes 1, 2, 3, and 4) and the pdf $p(y_{subN})$ for consistency testing using the LLR test.

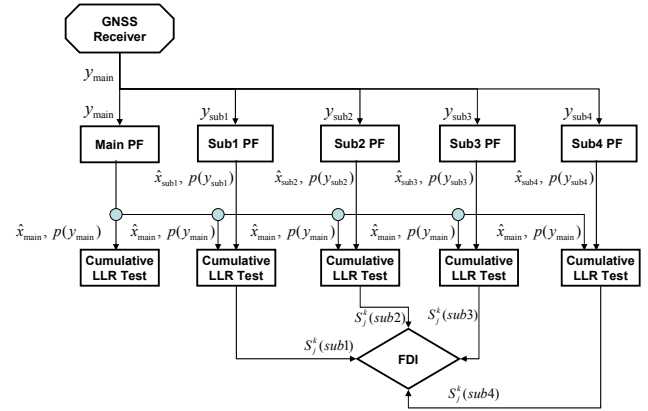


Fig. 2. GPS integrity monitoring using PF-based LLR test approach.

The pdf $p(y_{main})$ and $p(y_{subN})$ can be estimated by utilizing the complete pdf information of the predicted state particles computed during the particle filtering process. Let the measurement vectors for each PF be as follows:

$$y_{main}(k) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}, y_{sub1}(k) = \begin{bmatrix} z_2 \\ z_3 \\ z_4 \end{bmatrix}, y_{sub2}(k) = \begin{bmatrix} z_1 \\ z_3 \\ z_4 \end{bmatrix}, \quad (12)$$

$$y_{sub3}(k) = \begin{bmatrix} z_1 \\ z_2 \\ z_4 \end{bmatrix}, y_{sub4}(k) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

When one of the tracked satellite signals is faulty, the corresponding auxiliary PF that has no faulty data remains uncontaminated. Therefore, at least one of the consistency tests is expected to produce a correct reference for an alarm. The LLR test in this case is defined as the comparison between the pdf resulting from each auxiliary PF and the main PF. Following the equations from (8) to (11), the formulation for LLR is illustrated by

$$s(y_{subN}) = \ln \frac{p(y_{subN})}{p(y_{main})} \quad (13)$$

Moreover, the cumulative LLR for the observations from z_j to z_k can be expressed as

$$S_j^k(y_{subN}) = \sum_{r=j}^k \ln \frac{p_{subN}(z_r | Z_{r-1})}{p_{main}(z_r | Z_{r-1})} \quad (14)$$

The decision function is given by

$$\beta_k = \max_{k-U+1 \leq j \leq k} \max_{1 \leq N \leq Q} S_j^k(y_{subN}) \underset{H_0}{\overset{H_1}{>}} \tau \quad (15)$$

where U is the window size of the most recent past observations and τ is the threshold. If the decision function β_k exceeds the threshold τ , then failure has occurred and an alarm will be issued. The FI is achieved by determining the index g of the faulty subset of measurements given by [6]

$$g = \arg \max_{1 \leq N \leq Q} S_{t_a}^k(y_{subN}) \quad \text{for } k > t_a \quad (16)$$

The complete FDI algorithm for GPS integrity monitoring based on the PF and LLR test is summarized as follows:

- Generate the initial set of M samples/particles $\{x_0^{main}(i) : i = 1, 2, \dots, M\}$ for the main PF from the prior pdf $p(x_0)$. The samples $\{x_0^{subN}(i) : i = 1, 2, \dots, M\}$ in each of the auxiliary PFs are then initialized as

$$x_0^{subN}(i) = x_0^{main}(i) \quad \text{for } i = 1, 2, \dots, M \quad (17)$$

The main PF processes complete measurements (n measurements) while the auxiliaries process the subset of measurements ($n-l$ measurements). The text must include a citation of each figure and table.

- Repeat the following steps for each time instant k .
 - ① State prediction. Generate a new swarm of particles $\{x_{k|k-1}(i) : i = 1, 2, \dots, M\}$ for each main and auxiliary PF which approximates the predicted pdf $p(x_k | Z_{k-1})$

$$\text{Main PF : } x_{k|k-1}^{main}(i) = f(x_{k-1}^{main}(i), w_{k-1}(i)) \quad (18)$$

$$\text{Auxiliary PF : } x_{k|k-1}^{subN}(i) = f(x_{k-1}^{subN}(i), w_{k-1}(i)) \quad (19)$$

- ② Likelihood evaluation. Upon receiving the measurement z_k , the likelihood of the predictive state samples from the main PF is obtained as

$$\tilde{w}_k^{main}(i) = p(z_k | x_{k|k-1}^{main}(i)) \quad (20)$$

Thus, the likelihood in (11) is approximated using a swarm of particles as follows:

$$\text{Main PF : } p^{main}(z_k | Z_{k-1}) \approx \frac{1}{M} \sum_{i=1}^M \tilde{w}_k^{main}(i) \quad (21)$$

$$\text{Auxiliary PF : } p^{subN}(z_k | Z_{k-1}) \approx \frac{1}{M} \sum_{i=1}^M \tilde{w}_k^{subN}(i) \quad (22)$$

- ③ LLR calculation. LLR defined in (11) is calculated using the results obtained in step ②.

$$S_j^k(y_{subN}) = \sum_{r=j}^k \ln \frac{p_{subN}(z_r | Z_{r-1})}{p_{main}(z_r | Z_{r-1})} \quad (23)$$

- ④ Decision function evaluation. Decision function defined by (15) for FD is evaluated as

$$\beta_k = \max_{1 \leq N \leq Q} \zeta_k^u(y_{subN}) \quad (24)$$

where the maximum of the window-limited cumulative LLR between the auxiliary PF and the main PF is given by

$$\zeta_k^u(y_{subN}) = \max_{k-U+1 \leq j \leq k} S_j^k(y_{subN}) \quad (25)$$

- ⑤ Fault detection. FD is evaluated as follows:
If $\beta_k > \tau \rightarrow$ fault alarm is noted, then go to step ⑥.
If $\beta_k < \tau \rightarrow$ no fault, go to step ⑦.

- ⑥ Fault isolation. Following a fault alarm, FI is achieved by comparing the $subN$ cumulative LLRs $S_{t_a}^k(y_{subN})$ for $k > t_a$, and the index g of the faulty subset of measurements is obtained by (16).

- ⑦ State update. The importance weight $w_k^{main}(i)$ of the predictive state samples $\{x_{k|k-1}^{main}(i) : i = 1, 2, \dots, M\}$ from the main PF and $w_k^{subN}(i)$ of the predictive state samples $\{x_{k|k-1}^{subN}(i) : i = 1, 2, \dots, M\}$ from the auxiliary PFs are calculated as follows:

$$w_k^{main}(i) = \frac{\tilde{w}_k^{main}(i)}{\sum_{j=1}^M \tilde{w}_k^{main}(j)}, w_k^{subN}(i) = \frac{\tilde{w}_k^{subN}(i)}{\sum_{j=1}^M \tilde{w}_k^{subN}(j)} \quad (26)$$

Then the filtered samples $\{x_k^{main}(i) : i = 1, 2, \dots, M\}$ for the main PF and $\{x_k^{subN}(i) : i = 1, 2, \dots, M\}$ for the auxiliaries are obtained by re-sampling $\{x_{k|k-1}^{main}(i) : i = 1, 2, \dots, M\}$ and $\{x_{k|k-1}^{subN}(i) : i = 1, 2, \dots, M\}$, respectively.

4. Numerical Simulation Results

In this section, numerical simulation is done to illustrate the operation and evaluation of the FDI performance using the proposed method for GPS integrity monitoring. The simulation uses the GPS observation data measured on a static standalone positioning condition. The GPS measurement data were obtained on October 8, 2008 in an

engineering building in Konkuk University using a NovAtel GPS receiver. The GPS data were collected from six satellites with 2 Hz during 1000 epoch. By processing the measured data, the position information of satellites and the pseudoranges are extracted. A simple GPS measurement model in (1) is used, while the system model is as follows:

$$x_k = F_{k-1}x_{k-1} + w_{k-1} \quad (27)$$

where $x_k = [r_x \ r_y \ r_z \ \Delta\delta]^T$ is the three-dimensional position and receiver clock offset, respectively; and F_{k-1} is a transition matrix, which in a static case is the identity matrix. The proposed FDI system for GPS integrity monitoring is tested under nominal and failure conditions for comparison. Under nominal conditions, the decision functions always stay below the relevant detection threshold for all visible satellites in the GPS dataset collected. In the case of failure, pseudorange contamination is intentionally induced by modifying stored GPS data collected under nominal conditions, which consequently results in position anomalies. The corrupted measurements are injected back into the FDI system. In implementing PFs, the number of particles for the nominal and auxiliary PFs are chosen as $N=1,000$. The window size used for calculating the decision function β_k in (25) is chosen as $U=50$. Since there are six satellites, a total of seven PFs are used, with one as a main PF, i.e., *Main PF*, and the others as auxiliaries, i.e., *Sub 1 PF*, *Sub 2 PF*, *Sub 3 PF*, *Sub 4 PF*, *Sub 5 PF*, and *Sub 6 PF*. The measurement vectors for each PF are arranged as follows:

$$\begin{aligned}
 y_{main}(k) &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}; y_{sub1}(k) = \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}; y_{sub2}(k) = \begin{bmatrix} y_1 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}; y_{sub3}(k) = \begin{bmatrix} y_1 \\ y_2 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}; \\
 y_{sub4}(k) &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_5 \\ y_6 \end{bmatrix}; y_{sub5}(k) = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_6 \end{bmatrix}; y_{sub6}(k) = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}.
 \end{aligned} \quad (28)$$

A. Test for failure-free case

Fig. 3 presents simulation results when the proposed FDI algorithm for GPS integrity monitoring is applied to a failure-free case. The threshold τ for the FD is chosen as $\tau=100$. In the figure, the decision function β_k is maintained around 120 and the cumulative LLR for each auxiliary PF is bounded by the threshold with slightly varied offset values, which confirms nominal operation in the GPS positioning.

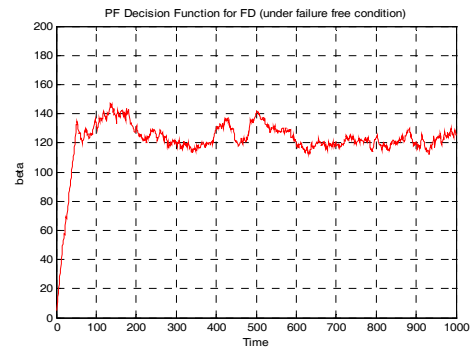


Fig. 3(a). Decision function for FI under failure-free condition

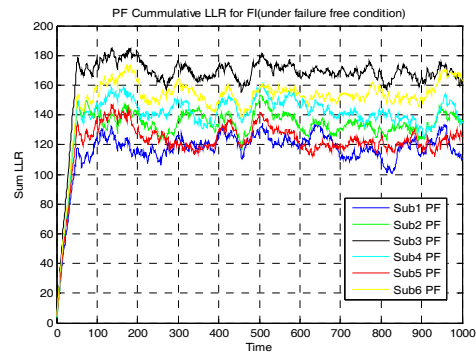


Fig. 3(b). Cumulative LLR for FI under failure-free condition.

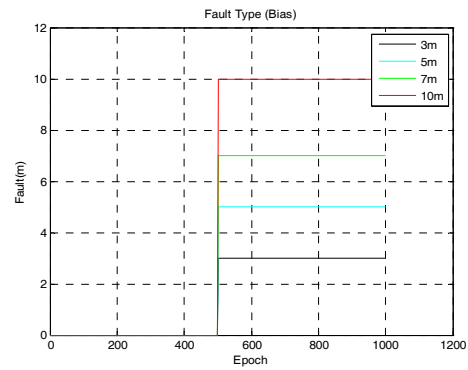


Fig. 4(a). Applied error signal having bias fault.

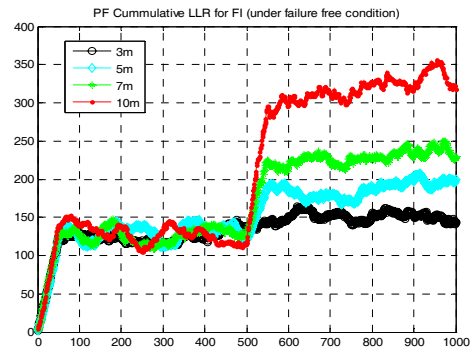


Fig. 4(b). Decision function for FD.

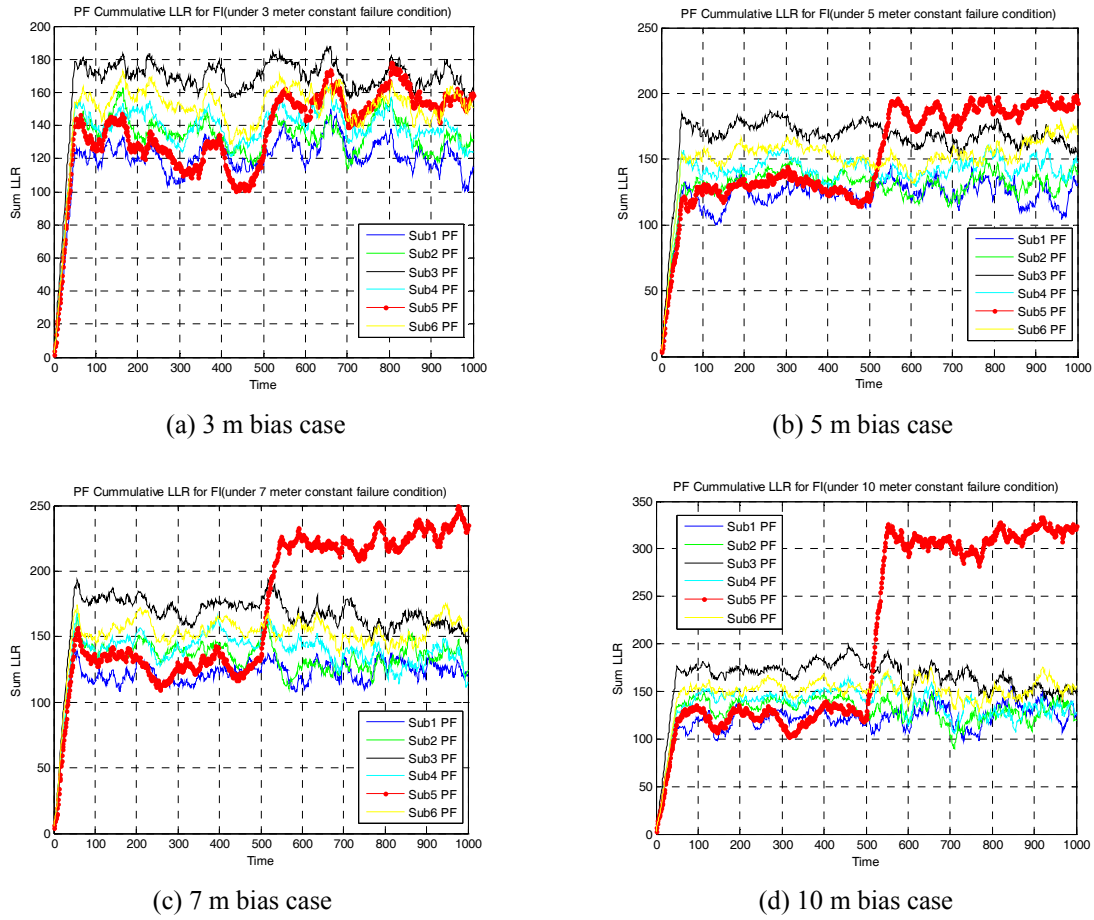


Fig. 5. Cumulative LLF for FI under step failure condition.

B. Test for failure case

To demonstrate FDI performance, errors are intentionally inserted into the nominal GPS data measured. To illustrate FDI performance of the proposed method, two fault types are considered, i.e., bias error and ramp error. In the simulation, pseudorange measurements of satellite 5th (PRN 18) are contaminated, without loss of generality, by adding constant bias errors or time-varying ramp errors at time instant $k=501$. Figs. 4 and 5 show the results of applying the proposed FDI when the artificial failure occurred in the bias error form. In Fig. 4(a), bias errors are applied at epoch 500 with four different error magnitudes. Fig. 4(b) shows that the decision function β_k is kept around 180 prior to the occurrence of a fault, but jumps significantly after the onset of the fault. Using the value of decision function and threshold τ set to 230, results show that the bias error larger than 5 m can be explicitly detected within several tens of epoch time. On the other hand, fault occurrence with another bias error less than 3 m is hardly detected because the threshold to distinguish the fault from the ordinary case cannot be obtained. Accordingly, in Fig. 5, FI is conducted by checking each cumulative LLR graph when a fault is detected. By plotting $S_{i_a}^k(\cdot)$ in (23) for each auxiliary PF, the auxiliary PF that is free from the

pseudorange contamination of bias error is determined numerically. The bias error of 3 m cannot be isolated while other cases that have larger errors are readily identified. Fig. 5 indicates that the LLR graph with the greatest magnitude among the six candidates is provided by the auxiliary *Sub 5 PF*. This means that the fault occurred in satellite 5th(PRN 18) because the LLR of the auxiliary *Sub 5 PF* $S_{i_a}^k(\cdot)$ does not include GPS measurement data from satellite 5th(PRN 18).

Previous results showed that the proposed FDI method provides limited performance on FI in the bias error case. Figs. 6 and 7 show the results of the proposed FDI when the failure occurs in the ramp error form. Similar to the bias error case illustrated in Fig. 6(a), ramp errors start at 500 epoch with four different gradient values, 0.03, 0.07, 0.05, and 0.1 m/s, respectively. As seen in Fig. 6(b), the decision function β_k grows significantly according to the fault occurrence, and thus anomalies are detected by setting the threshold of decision function to 230. In Fig. 7, FI is conducted by checking each cumulative LLR graph when a fault is detected. By plotting $S_{i_a}^k(\cdot)$ in (23) for each auxiliary PF, *Sub 5 PF* thus has the largest cumulative LLR and the fault occurred in the data from satellite 5th (PRN 18). The advantage of the proposed FDI method is its

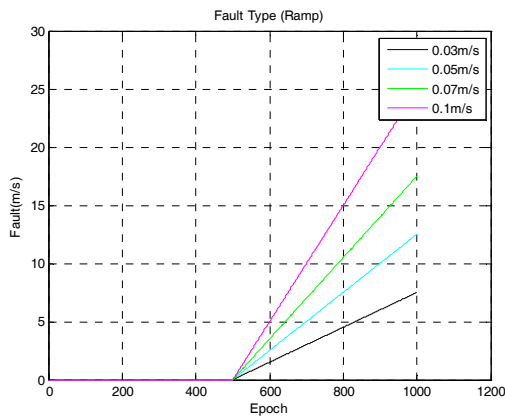


Fig. 6(a). Applied error signal having ramp fault.

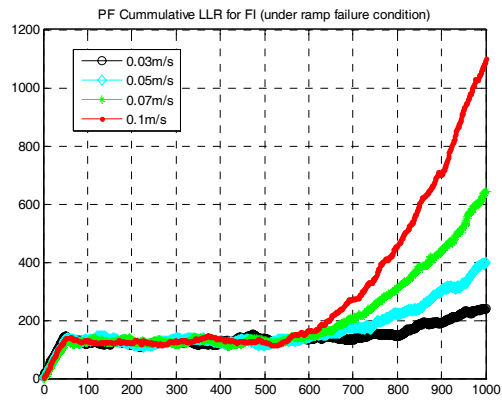
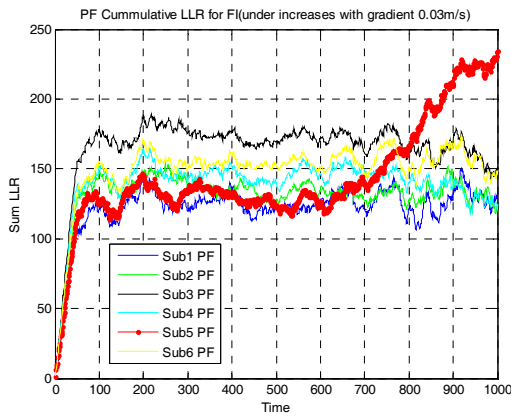
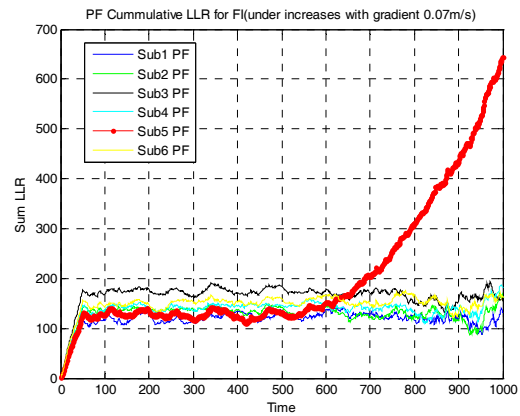


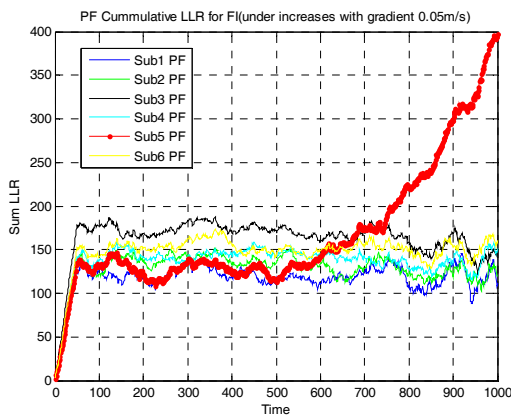
Fig. 6(b). Applied error signal having ramp fault.



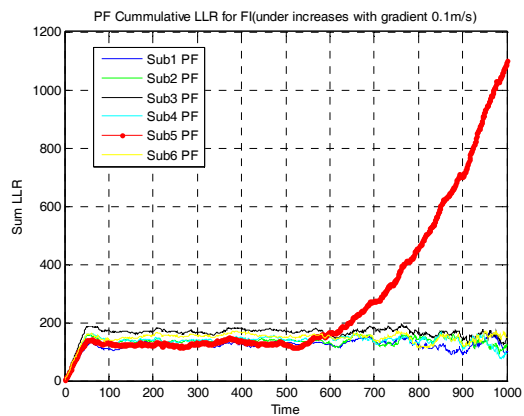
(a) 0.03 m/s gradient fault



(b) 0.05 m/s gradient fault



(c) 0.07 m/s gradient fault



(d) 0.1 m/s gradient fault

Fig. 7. Cumulative LLF for FI under ramp failure condition.

applicability to the ramp type error with small gradient value.

Table 1 summarizes the FD performance of the fault types considered. In the case of bias fault, the proposed FD algorithm failed to detect the bias error of 3 m while errors larger than 5 m have been successfully detected within 20

epochs. On the other hand, the simulation demonstrated outstanding performance when the fault type is given by a ramp type signal. The detection time decreases as the magnitude of the bias error and the gradient of ramp error increases. In conclusion, the proposed algorithm demonstrates a fair capability in addressing bias error.

Moreover, this algorithm illustrated a strong FDI performance in case of ramp type fault. Therefore, the possible combination of various faults consisting of bias, noise, and ramp error, which can be induced through many stages in the GPS signal acquisition and processing part, can be effectively detected and isolated by the FDI performance of the proposed method to address ramp errors.

Table 1. Fault type and detection time

Bias fault		Ramp fault	
Applied bias	Detection time (epoch)	Applied ramp	Detection time (epoch)
3 m	-	0.03 m/s	715
5 m	520	0.05 m/s	647
7 m	515	0.07 m/s	597
10 m	508	0.10 m/s	590

5. Conclusion

This paper proposes a new FDI method in GPS integrity monitoring by using the particle filter and likelihood ratio test of the main and auxiliary filtering structure. The proposed method makes it possible to detect ramp fault in partial measurements, such as the pseudorange that comes from the GPS satellite measurement. The basic concept is to use the particle filtering system in parallel architecture. The likelihood function is established and examined by integrating state estimate from both the main and auxiliary PFs. Furthermore, the LLR test is used in fault detecting, which compares the consistency of the measurement between the central and auxiliary PFs. The evaluation of FDI is conducted through simulation using the real GPS measurement data. The measured data from GPS are deliberately contaminated with two different error types, the bias and ramp terms. Based on the simulation result, the proposed approach successfully demonstrated that it can detect the GPS measurement fault, particularly in showing its outstanding performance when error is gradually increasing.

Acknowledgements

This research was supported by the "Multiple GNSS RAIM for Aviation Application" project through the Korean Research Council of Fundamental Science & Technology (KRF).

References

- [1] Greg Welch and Gray Bishop, "An Introduction to the Kalman Filter," UNC-Chapel Hill, TR 95-041, 2006.
- [2] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Prentice-hall, Englewood Cliffs, NJ, 1979.
- [3] M. Sanjeev Arulampalam, Simmon Maskell, Neil Gordon, and Tim Clapp, "A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking," *IEEE Transaction on signal processing*, vol. 50, no. 2, Feb. 2002.
- [4] B. Hofmann-Wellenhof, H. Lichtenegger and E. Wasle, *GNSS GPS, GLONASS, Galileo and More*. Springer, New York, 2007.
- [5] DA, R.: "Failure detection of dynamical systems with the state chi-square test," *Jr. Guid. Control Dyn.*, 1994, vol. 17, no. 2, pp. 271–277.
- [6] J. J. Spilker, "GPS signal Structure and Performance Characteristics. Navigation," *Journal of the Institute of Navigation*, vol. 25 no. 2, pp. 121–146.
- [7] B. Azimi-Sadjadi and P. S. Krishnaprasad, "Change detection for nonlinear systems: A particle filtering approach," in *Proceedings of Amer. Control Conf.*, Anchorage, AK, 2002.
- [8] X. X. Jin, "Algorithm for Carrier-Adjusted DGPS Positioning and Some Numerical Results," *Journal of Geodesy*, pp. 411–423, 1997.
- [9] P. Li and V. Kadiramanathan, "Particle filtering based likelihood ratio approach to fault diagnosis in nonlinear stochastic systems," *IEEE Trans. Syst., Man, Cybern. C*, vol. 31, pp. 337–343, Aug. 2001.
- [10] A. Doucet, S. Godsill, and C. Ardrieu, "On sequential Monte-Carlo sampling methods for Bayesian filtering," *Statist. Comput.*, vol. 10, pp. 197–208, 2000.
- [11] Rosihan, Arif Indryatmoko, Sebum Chun, Dae Hee Won, Young Jae Lee, Taesam Kang, Jeongrae Kim, Hyan-sig Jun. "Particle Filtering Approach to Fault Detection and Isolation for GPS Integrity Monitoring," in *Proceedings of ION GNSS 19th International Technical Meeting*, Sep. 2006, Fort Worth, TX
- [12] N. J. Gordon, D. J. Salmond, and A. F. M. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation," *Proc. Inst. Elect. Eng. F*, vol. 140, no. 2, pp. 107–113, 1993.
- [13] B. W. Parkinson, and P. Axelrad, "Autonomous GPS integrity monitoring using the pseudorange residual," *Navig., J. Inst. Navig.*, vol. 35, no. 2, pp. 255–274, 1988.
- [14] J. C. Juang, and C. W. Jang, "A failure detection approach applying to GPS autonomous integrity monitoring," *IEE Proc., Radar Sonar Navig.*, vol. 145, no. 6, pp. 342–346, 1998.
- [15] Y. C. Lee, "Analysis or range and position comparison methods as a means to provide GPS integrity in the user receiver," in *Proceedings of the Annual Meeting of the Institute of Navigation*, Seattle, WA, Jun. 1986, pp. 1–4.
- [16] D. H. Won, S. Chun, S. Sung, Y. J. Lee, J. Cho, J. Joo,

J. Park, "INS/vSLAM System Using Distributed Particle Filter," *International Journal of Control, Automation, and Systems*, vol. 8, no. 6, 1232–1240, 2010.

- [17] S. Feng, W. Y. Ochieng, T. Moore, and C. Hide, "Carrier Phase Based Integrity Monitoring for High Accuracy Positioning," *GPS Solutions*, vol. 13, no. 1, pp.13–22.



Jongsun Ahn He obtained his B.S and M.S. degrees in Aerospace Information Engineering from Konkuk University, Korea, in 2007 and 2009, respectively. Currently, he is a Ph.D. candidate in the Department of Aerospace Information Engineering at Konkuk University. His research interests include Global Navigation Satellite Augmentation System (GNSS) and GPS signal fault detection.



Rosihan Arif He earned his B.S degree from Bandung Institute of Technology, Indonesia in 2004, and his M.S. degree in Aerospace Information Engineering from Konkuk University, Korea, in 2007. His research interests include fault detection and isolation, optimal filtering, neural network design, and application to GPS signal processing.



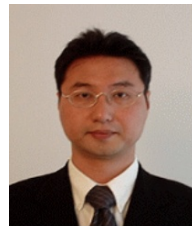
Dae Hee Won He received his B.S and M.S. degrees in Aerospace Information Engineering from Konkuk University, Korea, in 2006 and 2008, respectively. He is a Ph.D. candidate in the Department of Aerospace Information Engineering at Konkuk University. His research interests include GPS/INS/Vision system integration, GPS RTK, and nonlinear estimation.



Young Jae Lee He is a Professor in the Department of Aerospace Information Engineering at Konkuk University, Korea. He received his Ph.D. degree in Aerospace Engineering from the University of Texas at Austin in 1990. His research interests include integrity monitoring of GNSS signal, GBAS, RTK, attitude determination, orbit determination, and GNSS-related engineering problems.



Gi Wook Nam He received his Ph.D degree in Aerospace Engineering from Cranfield University in the U.K. He is a principal researcher in the Department of Satellite Navigation, Space Application and Future Technology Center in Korea Aerospace Research Institute (KARI). His research interests are safety analysis of aerospace systems and Global Navigation Satellite System (GNSS) Augmentation System.



Moon-Beom Heo He received M.S. and Ph.D degrees in Mechanical and Aerospace Engineering Aerospace Information Engineering from Illinois Institute of Technology, U.S. in 1997 and 2004, respectively. He is the head of the Navigation Department, Space Application and Future Technology Center in Korea Aerospace Research Institute (KARI). His research interests include GNSS-based navigation system including Ground Based Augmentation System (GBAS).



Sankyung Sung He received his B.S. and Ph.D. degrees in Electrical Engineering from Seoul National University, Seoul, Korea, in 1996 and 2003, respectively. From March 1996 to February 2003, he worked for the Automatic Control Research Center in Seoul National University. Currently, he is an Associate Professor of the Department of Aerospace Information Engineering, Konkuk University. His research interests include avionic system hardware and IT fusion technology, inertial sensors, integrated navigation, and application to unmanned systems.