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On Fuzzy r-Minimal Semicompactness on Fuzzy r-Minimal Spaces

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Abstract

The concepts of fuzzy r-minimal semicompact, almost fuzzy r-minimal semicompact, and nearly fuzzy r-minimal semicompact on fuzzy r-minimal spaces are introduced. We also investigate the relationships between fuzzy r-M-semicontinuous mappings and several types of fuzzy r-minimal semicompactness.

Key Words : fuzzy r-minimal semiopen, fuzzy r-M-continuous, fuzzy r-M-semicontinuous, fuzzy r-M^*-semiopen, fuzzy r-minimal semicompact, almost fuzzy r-minimal semicompact, nearly fuzzy r-minimal semicompact

1. Introduction

The concept of fuzzy set was introduced by Zadeh [7]. Chang [1] defined fuzzy topological spaces using fuzzy sets. In [2], Chattopadhyay, Hazra and Samanta introduced smooth topological spaces which are a generalization of fuzzy topological spaces. In [5], we introduced the concept of fuzzy r-minimal space which is an extension of the smooth topological space. The concepts of fuzzy r-minimal open sets and fuzzy r-M-continuous mappings were also introduced and studied. In [3], we introduced the concepts of fuzzy r-minimal semiopen sets and fuzzy r-M-semicontinuous mappings, which are generalizations of fuzzy r-minimal open sets and fuzzy r-M-continuous mappings, respectively. Yoo et al. [6] introduced the concepts of fuzzy r-minimal compactness on fuzzy r-minimal spaces. The purpose of this paper is to extend the concepts of fuzzy r-minimal compactness defined in [6]. So in this paper, we introduce and study the concepts of fuzzy r-minimal semicompactness, almost fuzzy r-minimal semicompactness, nearly fuzzy r-minimal semicompactness on fuzzy r-minimal spaces. We also introduce the concept of fuzzy r-M^*-semiopen mapping, and investigate the relationships among fuzzy r-M-semicontinuous mappings, fuzzy r-M^*-semiopen mappings and several types of fuzzy r-minimal semicompactness.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a *fuzzy set* [7] of X. By $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$, we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{\mathbf{1}} - A$. All other notations are standard notations of fuzzy set theory.

An *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

Let $f : X \to Y$ be a function and $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} \sup_{z\in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise }, \end{cases}$$

for $y \in Y$ and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x)), x \in X$.

A *smooth topology* [2,4] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:
(1) $\mathcal{T}(\tilde{\mathbf{0}}) = \mathcal{T}(\tilde{\mathbf{1}}) = 1$.
(2) $\mathcal{T}(A_1 \cap A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$.
(3) $\mathcal{T}(\cup A_i) \geq \wedge\mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space*.

Let X be a nonempty set and $r \in (0,1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a *fuzzy r-minimal structure* [5] if the family

$$\mathcal{M}_r = \{A \in I^X \mid \mathcal{M}(A) \geq r\}$$

contains $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$.

Then the $(X, \mathcal{M})$ is called a *fuzzy r-minimal space* [5] (simply, fuzzy r-FMS). Every member of $\mathcal{M}_r$ is called a *fuzzy r-minimal open* set. A fuzzy set A is called a *fuzzy r-minimal closed* set if the complement of A is a fuzzy r-minimal open set.

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Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure of A, denoted by $mC(A, r)$, is defined as

$$mC(A, r) = \cap\{B \in I^X : \tilde{\mathbf{1}} - B \in \mathcal{M}_r \text{ and } A \subseteq B\}.$$

The fuzzy r-minimal interior of A, denoted by $mI(A, r)$, is defined as

$$mI(A, r) = \cup\{B \in I^X : B \in \mathcal{M}_r \text{ and } B \subseteq A\}.$$

Theorem 2.1 ([5]). Let $(X, \mathcal{M})$ be an r-FMS and $A, B \in I^X$.

(1) $mI(A, r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A, r) = A$.

(2) $A \subseteq mC(A, r)$ and if A is a fuzzy r-minimal closed set, then $mC(A, r) = A$.

(3) If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.

(4) $mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.

(5) $mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.

(6) $\tilde{\mathbf{1}} - mC(A, r) = mI(\tilde{\mathbf{1}} - A, r)$ and $\tilde{\mathbf{1}} - mI(A, r) = mC(\tilde{\mathbf{1}} - A, r)$.

Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is called a *fuzzy r-minimal semiopen set* [3] in X if

$$A \subseteq mC(mI(A, r), r).$$

A fuzzy set A is called a *fuzzy r-minimal semiclosed* set if the complement of A is fuzzy r-minimal semiopen.

We showed that any union of fuzzy r-minimal semiopen sets is fuzzy r-minimal semiopen [3].

For $A \in I^X$, $msC(A, r)$ and $msI(A, r)$, respectively, are defined as the following:

$$msC(A, r) = \cap\{F \in I^X : A \subseteq F, \; F \text{ is fuzzy } r\text{-minimal semiclosed}\};$$

$$msI(A, r) = \cup\{U \in I^X : U \subseteq A, \; U \text{ is fuzzy } r\text{-minimal semiopen }\}.$$

Theorem 2.2 ([3]). Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then

(1) $msI(A, r) \subseteq A \subseteq msC(A, r)$.

(2) If $A \subseteq B$, then $msI(A, r) \subseteq msI(B, r)$ and $msC(A, r) \subseteq msC(B, r)$.

(3) A is r-minimal semiopen iff $msI(A, r) = A$.

(4) F is r-minimal semiclosed iff $msC(F, r) = F$.

(5) $msI(msI(A, r), r) = msI(A, r)$ and $msC(msC(A, r), r) = msC(A, r)$.

(6) $msC(\tilde{\mathbf{1}} - A, r) = \tilde{\mathbf{1}} - msI(A, r)$ and $msI(\tilde{\mathbf{1}} - A, r) = \tilde{\mathbf{1}} - msC(A, r)$.

3. Main Results

We recall the concepts of several types of fuzzy r-minimal compactness introduced in [6]. Let $(X, \mathcal{M})$ be an r-FMS and $\mathcal{A} = \{A_i \in I^X : i \in J\}$. $\mathcal{A}$ is called a *fuzzy r-minimal cover* if $\cup\{A_i : i \in J\} = \tilde{\mathbf{1}}$. It is a *fuzzy r-minimal open cover* if each A_i is a fuzzy r-minimal open set. A subcover of a fuzzy r-minimal cover $\mathcal{A}$ is a subfamily of it which also is a fuzzy r-minimal cover. A fuzzy set A in X is said to be

(1) *fuzzy r-minimal compact* if every fuzzy r-minimal open cover $\mathcal{A} = \{A_i \in \mathcal{M}_r : i \in J\}$ of A has a finite subcover;

(2) *almost fuzzy r-minimal compact* (resp., *nearly fuzzy r-minimal compact*) if for every fuzzy r-minimal open cover $\mathcal{A} = \{A_i \in I^X : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} mC(A_k, r)$ (resp., $A \subseteq \cup_{k \in J_0} mI(mC(A_k, r), r)$).

Definition 3.1. Let $(X, \mathcal{M})$ be an r-FMS and $\mathcal{A} = \{A_i \in I^X : i \in J\}$ a fuzzy r-minimal cover. It is a *fuzzy r-minimal semiopen cover* if each A_i is fuzzy r-minimal semiopen.

Definition 3.2. Let $(X, \mathcal{M})$ be an r-FMS. A fuzzy set A in X is said to be

(1)*fuzzy r-minimal semicompact* if every fuzzy r-minimal semiopen cover $\mathcal{A} = \{A_i \in \mathcal{M}_r : i \in J\}$ of A has a finite subcover;

(2) *almost fuzzy r-minimal semicompact* if for every fuzzy r-minimal semiopen cover $\mathcal{A} = \{A_i \in I^X : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} msC(A_k, r)$;

(3) *nearly fuzzy r-minimal semicompact* if for every fuzzy r-minimal semiopen cover $\mathcal{A} = \{A_i : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} msI(msC(A_k, r), r)$.

From the definitions of several types of fuzzy r-minimal semicompactness, the following implications are easily obtained but the converses may not be true as shown in the next example.

fuzzy r-minimal semicompact $\Rightarrow$ nearly fuzzy r-minimal semicompact $\Rightarrow$ almost fuzzy r-minimal semicompact

Example 3.3. Let $X = I$ and $n \in N - \{1\}$. Let A_1, A_n, B, C and D be fuzzy sets defined as follows

$$A_n(x) = \begin{cases} 0.8, & \text{if } x = 0, \\ nx, & \text{if } 0 < x \leq \frac{1}{n}, \\ 1, & \text{if } \frac{1}{n} < x \leq 1; \end{cases}$$

$$A_1(x) = \begin{cases} 1, & \text{if } x = 0, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

$$B(x) = \begin{cases} 0.1, & \text{if } x = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$C(x) = \frac{1}{2}, \text{if } x \in I.$$

$$D(x) = -0.4x + 0.9, \text{if } x \in I.$$

(1) Consider a fuzzy r-minimal structure $\mathcal{M} : I^X \to I$ on X as follows

$$\mathcal{M}(A) = \begin{cases} \frac{4}{5}, & \text{if } A = \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ \frac{n}{n+1}, & \text{if } A = A_n, \\ \frac{2}{3}, & \text{if } A = A_1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $\mathcal{A} = \{A_n : n \in N\}$ be a fuzzy $\frac{1}{2}$-minimal semiopen cover of X. Since the semi-closure of any fuzzy r-semiopen set is $\tilde{\mathbf{1}}$, X is nearly fuzzy $\frac{1}{2}$-minimal semicompact. But there does not exist a finite subcover of $\mathcal{A}$, and so X is not fuzzy $\frac{1}{2}$-minimal semicompact.

(2) Consider a fuzzy r-minimal structure $\mathcal{M} : I^X \to I$ on X as follows

$$\mathcal{M}(\sigma) = \begin{cases} \frac{4}{5}, & \text{if } \sigma = B, C, D, \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ \frac{n}{n+1}, & \text{if } \sigma = A_n, \\ \frac{2}{3}, & \text{if } \sigma = A_1, \tilde{\mathbf{1}} - D, \\ 0, & \text{otherwise.} \end{cases}$$

Obviously X is almost fuzzy $\frac{1}{2}$-minimal semicompact. Let $\mathcal{A} = \{A_n : n \in N\}$ be a fuzzy $\frac{1}{2}$-minimal semiopen cover of X. Note that: $msI(msC(A_n)) = D$ for $n \in N$. Consequently, X is not nearly fuzzy $\frac{1}{2}$-minimal semicompact.

Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be r-FMS's. Then a mapping $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is said to be *fuzzy r-M-semicontinuous* [3] if for each point x_α and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal semiopen set U containing x_α such that $f(U) \subseteq V$.

Theorem 3.4 ([3]). Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the following statements are equivalent:

(1) f is fuzzy r-M-semicontinuous.

(2) $f^{-1}(V)$ is a fuzzy r-minimal semiopen set for each fuzzy r-minimal open set V in Y.

(3) $f^{-1}(B)$ is a fuzzy r-minimal semiclosed set for each fuzzy r-minimal closed set B in Y.

(4) $f(msC(A, r)) \subseteq mC(f(A), r)$ for $A \in I^X$.

(5) $msC(f^{-1}(B), r) \subseteq f^{-1}(mC(B, r))$ for $B \in I^Y$.

(6) $f^{-1}(mI(B, r)) \subseteq msI(f^{-1}(B), r)$ for $B \in I^Y$.

Theorem 3.5. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a fuzzy r-M-semicontinuous mapping on two r-FMS's. If A is a fuzzy r-minimal semicompact set, then $f(A)$ is fuzzy r-minimal compact.

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then since f is a fuzzy r-M-semicontinuous mapping, $\{f^{-1}(B_i) : i \in J\}$ is a fuzzy r-minimal semiopen cover of A in X. Since A is fuzzy r-minimal semicompact, there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} f^{-1}(B_k)$. It implies $f(A) \subseteq \cup_{k \in J_0} B_k$ for the finite subset J_0 of J, and hence $f(A)$ is fuzzy r-minimal compact. □

Theorem 3.6. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a fuzzy r-M-semicontinuous mapping on two r-FMS's. If A is an almost fuzzy r-minimal semicompact set, then $f(A)$ is almost fuzzy r-minimal compact.

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is a fuzzy r-minimal semiopen cover of A in X. Since A is almost fuzzy r-minimal semicompact, there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} msC(f^{-1}(B_k), r)$. From Theorem 3.4 (5), it follows

$$\begin{aligned} \cup_{k \in J_0} msC(f^{-1}(B_k, r)) &\subseteq \cup_{k \in J_0} f^{-1}(mC(B_k, r)) \\ &= f^{-1}(\cup_{k \in J_0} mC(B_k, r)). \end{aligned}$$

So $f(A) \subseteq \cup_{k \in J_0} mC(B_k, r)$ and $f(A)$ is almost fuzzy r-minimal compact. □

Definition 3.7. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a fuzzy mapping on two r-FMS's. Then f is said to be *fuzzy r-M^*-semiopen* if for each fuzzy r-minimal semiopen set U in X, $f(U)$ is fuzzy r-minimal open.

Lemma 3.8. Let $f : X \to Y$ be a function on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. If f is fuzzy r-M^*-semiopen, then $f(A) = mI(f(A), r)$ for every fuzzy r-minimal semiopen set A in X.

Proof. Obvious. □

Theorem 3.9. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a fuzzy mapping on two r-FMS's.

(1) f is fuzzy r-M^*-semiopen.

(2) $f(msI(A, r)) \subseteq mI(f(A), r)$ for $A \in I^X$.

(3) $msI(f^{-1}(B), r) \subseteq f^{-1}(mI(B), r)$ for $B \in I^Y$.

Then (1) ⇒ (2) ⇔ (3).

Proof. (1) ⇒ (2) For $A \in I^X$,
$f(msI(A), r)$
$= f(\cup\{B \in I^X : B \subseteq A, B$ is fuzzy r-semiopen$\})$
$= \cup\{f(B) \in I^Y : f(B) \subseteq f(A),$
$f(B)$ is fuzzy r-minimal open$\}$
$\subseteq \cup\{U \in I^Y : U \subseteq f(A), U$ is fuzzy r-minimal open$\}$
$= mI(f(A), r)$.
Hence $f(msI(A), r) \subseteq mI(f(A), r)$.

(2) ⇒ (3) For $B \in I^Y$, from (3) it follows that

$$f(msI(f^{-1}(B), r)) \subseteq mI(f(f^{-1}(B)), r) \subseteq mI(B, r).$$

This implies (3).

Similarly, we get the implication (3) ⇒ (2). □

Example 3.10. Let $X = I$ and let A, B be fuzzy sets defined as follows:

$$A(x) = \frac{1}{2}x, \;\; x \in I;$$

$$B(x) = -\frac{1}{2}(x-1), \;\; x \in I.$$

Define

$$\mathcal{M}(\sigma) = \begin{cases} \frac{7}{8}, & \text{if } \sigma = A, B, \\ \frac{2}{3}, & \text{if } \sigma = A \cup B, \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ 0, & \text{otherwise.} \end{cases}$$

Consider

$$\mathcal{N}(\sigma) = \begin{cases} \frac{2}{3}, & \text{if } \sigma = \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ \frac{1}{2}, & \text{if } \sigma = A, B, \\ 0, & \text{otherwise.} \end{cases}$$

Note that $A \cup B$ is fuzzy r-minimal semiopen. Let $f : (X, \mathcal{N}) \to (X, \mathcal{N})$ be the identity mapping. Then the mapping f satisfies the condition (2) of Theorem 3.9 but it is not fuzzy $\frac{1}{2}$-M^*-semiopen.

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy family $\mathcal{M}$ is said to have the property $(\mathcal{U})$ [5] if for $A_i \in \mathcal{M}$ $(i \in J)$,

$$\mathcal{M}(\cup A_i) \geq \wedge \mathcal{M}(A_i).$$

Theorem 3.11 ([5])**.** Let $(X, \mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$ and $A \in I^X$. Then
(1) A is fuzzy r-minimal open if and only if $mI(A, r) = A$.
(2) A is fuzzy r-minimal closed if and only if $mC(A, r) = A$.

Corollary 3.12. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a fuzzy mapping on two r-FMS's. If $\mathcal{M}_Y$ has the property $(\mathcal{U})$, then the following are equivalent:
(1) f is fuzzy r-M^*-semiopen.
(2) $f(msI(A, r)) \subseteq mI(f(A), r)$ for $A \in I^X$.
(3) $msI(f^{-1}(B), r) \subseteq f^{-1}(mI(B, r))$ for $B \in I^Y$.

Definition 3.13. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two r-FMS's. Then a function $f : X \to Y$ is called *fuzzy r-M^*-semiclosed* if for every fuzzy r-minimal semiclosed set A in X, $f(A)$ is a fuzzy r-minimal closed set in Y.

Lemma 3.14. Let $f : X \to Y$ be a function on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. If f is fuzzy r-M^*-semiclosed, then $f(A) = mC(f(A), r)$ for every fuzzy r-minimal semiclosed set A in X.

Proof. Obvious. □

Theorem 3.15. Let $f : X \to Y$ be a function on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$.
(1) f is fuzzy r-M^*-semiclosed.
(2) $mC(f(A), r) \subseteq f(msC(A, r))$ for $A \in I^X$.
(3) $f^{-1}(mC(B, r)) \subseteq msC(f^{-1}(B), r)$ for $B \in I^Y$.
Then $(1) \Rightarrow (2) \Leftrightarrow (3)$.

Proof. It is similar to Theorem 3.9. □

From Theorem 3.11, obviously the next corollary is obtained:

Corollary 3.16. Let $f : X \to Y$ be a function on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. If $\mathcal{N}$ has the property $(\mathcal{U})$, the following are equivalent:
(1) f is fuzzy r-M^*-semiclosed.
(2) $mC(f(A), r) \subseteq f(msC(A, r))$ for $A \in I^X$.
(3) $f^{-1}(mC(B, r)) \subseteq msC(f^{-1}(B), r)$ for $B \in I^Y$.

Theorem 3.17. Let a mapping $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be fuzzy r-M-semicontinuous and fuzzy r-M^*-semiopen on two r-FMS's. If A is a nearly fuzzy r-minimal semicompact set, then $f(A)$ is nearly fuzzy r-minimal compact.

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is a fuzzy r-minimal semiopen cover of A in X. Since X is nearly fuzzy r-minimal semicompact, there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} msI(msC(f^{-1}(B_k), r), r)$. From Theorem 3.4 and Theorem 3.9, it follows

$$\begin{aligned} f(A) &\subseteq \cup_{k \in J_0} f(msI(msC(f^{-1}(B_k), r), r)) \\ &\subseteq \cup_{k \in J_0} mI(f(msC(f^{-1}(B_k), r)), r) \\ &\subseteq \cup_{k \in J_0} mI(f(f^{-1}(mC(B_k, r))), r) \\ &\subseteq \cup_{k \in J_0} mI(mC(B_k, r), r). \end{aligned}$$

Hence $f(A)$ is a nearly fuzzy r-minimal compact set. □

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