Control and Synchronization of New Hyperchaotic System using Active Backstepping Design

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Abstract

In this paper, an active backstepping design is proposed to achieve control and synchronization of a new hyperchaotic system. The proposed method is a systematic design approach and exists in a recursive procedure that interlaces the choice of a Lyapunov function with the design of the active control. The proposed controller enables stabilization of chaotic motion to the origin as well as synchronization of the two identical new hyperchaotic systems. Numerical simulations illustrate the validity of the proposed control technique.

Key Words : hyperchaotic system, chaos control, chaos synchronization, active control, backstepping control.

1. Introduction

Since the pioneering work by Ott, Grebogi and Yorke (OGY) [1] and Pecora and Carroll [2], chaos control and synchronization, as an important topic in the nonlinear science, has been widely investigated in a variety of fields, such as engineerings, physics, mathematics, life sciences, biomedical communities, heart beat regulations, etc.

It is well known that for regular chaotic systems, there is just one positive Lyapunov exponent. Messages masked by regular chaotic systems are not always safe [3]. It was suggested that this problem can be overcome by using higher-dimensional hyperchaotic systems, which have increased randomness and unpredictability [4]. The hyperchaotic attractor is characterized as a chaotic attractor with more than one positive Lyapunov exponent, and indicates that the dynamics of the system are expanded in more than one direction. Due to its higher unpredictability than regular chaotic systems, hyperchaos may be more useful in some relevant applications. Therefore, how to realize control and synchronization of hyperchaotic systems is an interesting and challenging work.

Until now, enormous progresses have been made in understanding various methods [5–17] to achieve control and synchronization of chaotic systems. Fortunately, the existing method to control and synchronize chaotic systems can be generalized to control and synchronize hyperchaotic systems [18–20]. Among the aforementioned method, the active control [9-13] and the backstepping design [14-17] have been widely recognized as two powerful design methods for control and synchronization of chaotic systems. Chaos synchronization

using the active control was proposed by Bai and Lonngren for the Lorenz system [9]. The Active control technique can be used widely to control various nonlinear systems including chaotic systems since it has the flexiblity to design a control law. The backstepping design method can guarantee the global stability, and the tracking and transient performance for a broad class of strict-feedback nonlinear systems [21].The technique is a systematic design approach and consists in a recursive procedure that skillfully interlaces the choice of a Lyapunov function with the control. Consequently, the main aim of this paper is in an attempt to use the combination of the two control approaches, i.e., active backstepping method, to control and synchronize hyperchaotic system.

In this paper, the active backstepping control scheme is proposed to control and synchronize a new hyperchaotic system that is recently presented by Qi at al. [22]. The new hyperchaotic system has two large positive and one small negative Lyapunov exponents over the large range of parameters. Spectral analysis shows that the system in the hyperchaotic mode has an extremely broad frequency bandwidth of high magnitudes, verifying its unusual random nature and indicating its great potential for some pertinent engineering applications [23-24].

The rest of the paper is organized as follow. In Section 2, a brief description of the new hyperchaotic system is introduced. We present the control scheme of hyperchaotic system in Section 3. Section 4 deals with the synchronization behavior of two identical new hyperchaotic systems and finally some concluding remarks are made in Section 5.

2. System Description

The new 4D hyperchaotic system was performed by Qi et al. [22]. This chaotic system, named as Qi system in this paper, is described by the following nonlinear differential equations.

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\
\dot{x}_3 &= -cx_3 - gx_4 + x_1 x_2 \\
\dot{x}_4 &= -fx_4 + hx_3 + x_1 x_2
\end{aligned} \qquad (1)$$

where x_1, x_2, x_3 and x_4 are state variables and a, b, c, f, g and h are all positive real constant parameters. When $a = 50$, $b = 24$, $c = 13$, $f = 8$, $g = 33$ and $h = 30$, the system (1) is hyperchaotic, and the hyperchaotic attractors are shown in Fig. 1.

(a) $x_1 - x_2 - x_3$ (b) $x_1 - x_2 - x_4$
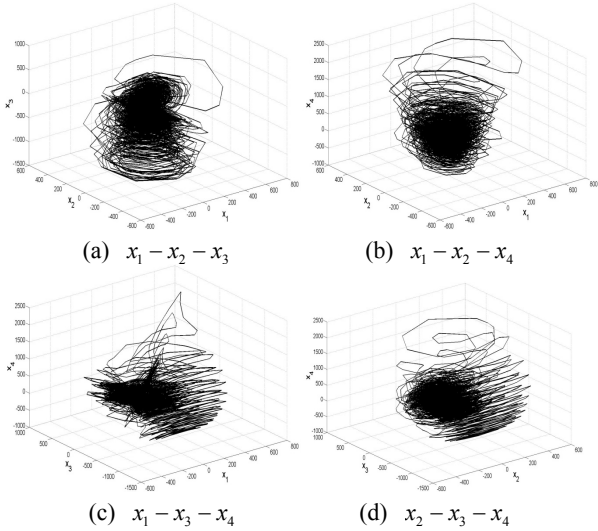
(c) $x_1 - x_3 - x_4$ (d) $x_2 - x_3 - x_4$

Fig 1. 3-D Phase portrait of the hyperchaotic system (1)

3. Control of the New Hyperchaotic System

3.1. Active Backstepping Control

In the following, we will use the backstepping design method to design an active controller for the hyperchaotic system presented by the equation (1) to the origin. According to the active control theory, the controlled hyperchaotic system can be written in the following form

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 + u_1 \\
\dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 + u_2 \\
\dot{x}_3 &= -cx_3 - gx_4 + x_1 x_2 + u_3 \\
\dot{x}_4 &= -fx_4 + hx_3 + x_1 x_2 + u_4
\end{aligned} \qquad (2)$$

where $u = [u_1, u_2, u_3, u_4]^T$ is the active control function.

In practical applications, the controller to be designed must be simple, efficient and easy to implement. Thus, let $u_1 = 0$ and $u_4 = 0$, then the controlled dynamics can be written as

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 + u_2 \\
\dot{x}_3 &= -cx_3 - gx_4 + x_1 x_2 + u_3 \\
\dot{x}_4 &= -fx_4 + hx_3 + x_1 x_2
\end{aligned} \qquad (3)$$

Now our objective is to find u_2 and u_3 that make the state vectors $x_i = [x_1, x_2, x_3, x_4]^T$ converge to zero as time t goes to infinity. In order to achieve such goal, the backstepping design method is adopted.

The backstepping design procedure is recursive. At the ith-step, the ith-order subsystem is stabilized with respect to a Lyapunov function V_i by the design of a virtual control α_i and the control input function u_i. Now we begin to design the active controller based on the backstepping design method as follows.

Step 1. Let $z_1 = x_1$, then we can obtain its derivative as follows

$$\begin{aligned}
\dot{z}_1 &= \dot{x}_1 \\
&= -az_1 + (a + x_3)x_2
\end{aligned} \qquad (4)$$

where $x_2 = \alpha_1(z_1)$ is regarded as a virtual control input.

For the design of α_1 to stabilize z_1-subsystem (4), we choose Lyapunov function V_1 as

$$V_1 = \frac{z_1^2}{2} \qquad (5)$$

The derivative of V_1 is obtained as

$$\begin{aligned}
\dot{V}_1 &= z_1 \dot{z}_1 \\
&= -az_1^2 + (a + x_3)z_1 \alpha_1
\end{aligned} \qquad (6)$$

If we choose $\alpha_1 = 0$, then $\dot{V}_1$ is negative definite. This implies that the z_1-subsystem (4) is asymptotically stable.

Since the virtual control function α_1 is estimative, the error between x_2 and α_1 is

$$z_2 = x_2 - \alpha_1 \qquad (7)$$

Then, we can obtain the following (z_1, z_2)-subsystem

$$\begin{aligned}
\dot{z}_1 &= -az_1 + az_2 + z_2 x_3 \\
\dot{z}_2 &= bz_1 + bz_2 - z_1 x_3 + u_2
\end{aligned} \qquad (8)$$

where $x_3 = \alpha_2(z_1, z_2)$ is regarded as a virtual controller.

Step 2. In this step, we stabilize the (z_1, z_2)-subsystem (8). We can choose a Lyapunov function V_2 as follows

$$V_2 = V_1 + \frac{z_2^2}{2} \qquad (9)$$

Its derivative is given by

$$\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 \\
&= -az_1^2 + z_2 \{(a + b)z_1 + bz_2 + u_2\}
\end{aligned} \qquad (10)$$

If we choose $\alpha_2 = 0$ and $u_2 = -(a + b)z_1 - 2bz_2$, then $\dot{V}_2 = -az_1^2 - bz_2^2 < 0$ makes (z_1, z_2)-subsystem (8) asymptotically stable. Similarly, assume that $z_3 = x_3 - \alpha_2$, then we can derive the following (z_1, z_2, z_3)-subsystem

$$\dot{z}_1 = -az_1 + az_2 + z_2 z_3$$
$$\dot{z}_2 = -az_1 - bz_2 - z_1 z_3 \qquad (11)$$
$$\dot{z}_3 = -cz_3 - gx_4 + z_1 z_2 + u_3$$

Step 3. In order to stabilize the (z_1, z_2, z_3) -subsystem (11), we can choose a Lyapunov function V_3 as follows

$$V_3 = V_2 + \frac{z_3^2}{2}. \qquad (12)$$

Its derivative of V_3 is

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + z_3 \dot{z}_3 \\ &= -az_1^2 - bz_2^2 + z_3(-cz_3 - gx_4 + z_1 z_2 + u_3)\end{aligned} \qquad (13)$$

If we choose $u_3 = gx_4 - z_1 z_2$,

then $\dot{V}_3 = -az_1^2 - bz_2^2 - cz_3^2 < 0$ makes the (z_1, z_2, z_3) - subsystem (11) asymptotically stable

Since $\dot{V}_3$ is negative definite, it follows that the equilibrium $(0,0,0)$ of the subsystem (11) is global asymptotically stable. Furthermore, since $z_1 = x_1$, $z_2 = x_2 - \alpha_1 = x_2$ and $z_3 = x_3 - \alpha_2 = x_3$, x_1, x_2 and x_3 go to zeros asymptotically as well. According to $x_1 \to 0, x_2 \to 0, x_3 \to 0$ and the fourth equation of system (3), we get that (x_1, x_2, x_3, x_4) in the controlled system (3) tend to $(0,0,0,0)$ as $t \to \infty$. In other words, the controlled system (3) is asymptotically stable with the proposed control inputs.

3.2. Numerical Results

For the purpose of numerical simulation, we set $a = 50$, $b = 24$, $c = 13$, $f = 8$, $g = 33$ and $h = 30$, as in Fig. 1, to ensure hyperchaotic behaviors. The initial conditions for the hyperchaotic system (3) set to be $x_1(0) = 0.1, x_2(0) = 0.2$, $x_3(0) = 0.3, x_4(0) = 0.4$. Fig. 2 shows the time response of states with the proposed control functions. The controller is added to the hyperchaotic system (3) at $t = 20$. As expected, it shows that the hyperchotic system can be stabilized to the origin point $(0,0,0,0)$.

4. Synchronization of New Hyperchaotic System

4.1. Active Backstepping Synchronization

In this section, we will use the backstepping method to design an active controller to synchronize two identical Qi hyperchaotic systems.

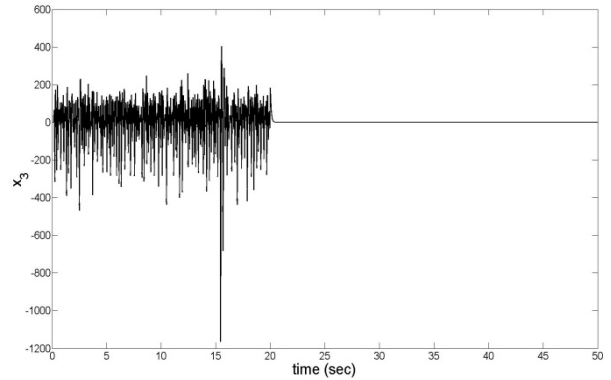In order to observe the synchronization behavior in the Qi hyperchaotic system, we assume the drive system as

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= b(x_1 + x_2) - x_1 x_3 \\ \dot{x}_3 &= -cx_3 - gx_4 + x_1 x_2 \\ \dot{x}_4 &= -fx_4 + hx_3 + x_1 x_2\end{aligned} \qquad (14)$$
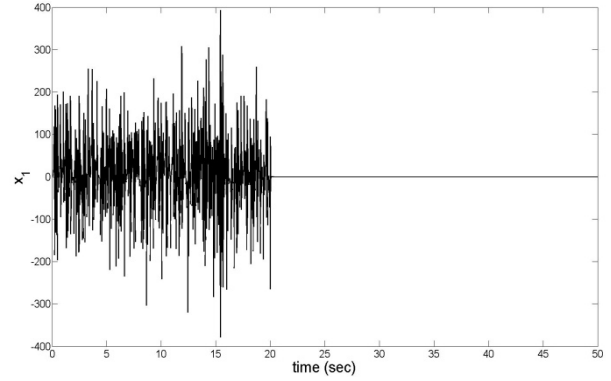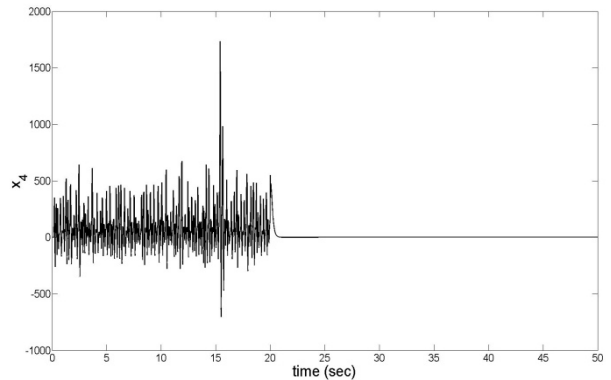

(a) The time response of states x_1
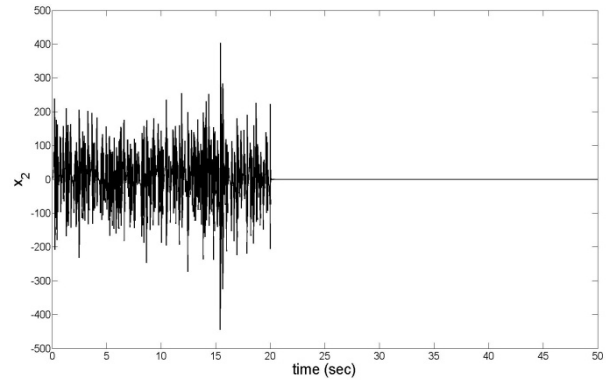
(b) The time response of states x_2

(c) The time response of states x_3

(d) The time response of states x_4

Fig. 2. The time response of states for the system (3) with the proposed controller.

and the response system is

$$\dot{y}_1 = a(y_2 - y_1) + y_2 y_3 + u_1$$
$$\dot{y}_2 = b(y_1 + y_2) - y_1 y_3 + u_2$$
$$\dot{y}_3 = -cy_3 - gy_4 + y_1 y_2 + u_3$$
$$\dot{y}_4 = -fy_4 + hy_3 + y_1 y_2 + u_4$$
(15)

where $u = [u_1, u_2, u_3, u_4]^T$ is the active control function. Likewise in section 2, let $u_1 = 0$ and $u_4 = 0$, then the response system dynamics can be written as

$$\dot{y}_1 = a(y_2 - y_1) + y_2 y_3$$
$$\dot{y}_2 = b(y_1 + y_2) - y_1 y_3 + u_2$$
$$\dot{y}_3 = -cy_3 - gy_4 + y_1 y_2 + u_3$$
$$\dot{y}_4 = -fy_4 + hy_3 + y_1 y_2$$
(16)

Here, we aim at determining the controllers u_2 and u_3 which are required for the controlled response system (16) to synchronize with the drive system (14). For this purpose, let the error states between the state variables of the response system (16) and the drive system (14) be

$$e_1 = y_1 - x_1, e_2 = y_2 - x_2, e_3 = y_3 - x_3, e_4 = y_4 - x_4 \quad (17)$$

Subtracting (14) from (16), we obtain the following error dynamics

$$\dot{e}_1 = a(e_2 - e_1) + e_2 e_3 + x_3 e_2 + x_2 e_3$$
$$\dot{e}_2 = b(e_1 + e_2) - e_1 e_3 - x_3 e_1 - x_1 e_3 + u_2$$
$$\dot{e}_3 = -ce_3 - ge_4 + e_1 e_2 + x_2 e_1 + x_1 e_2 + u_3$$
$$\dot{e}_4 = -fe_4 + he_3 + e_1 e_2 + x_2 e_1 + x_1 e_2$$
(18)

The next step is to find u_2 and u_3 that make the error vectors $e = [e_1, e_2, e_3, e_4]^T$ converge to zero as time t goes to infinity. This implies that the trajectory of the response system (16) asymptotically approaches the trajectory of the drive system (14).

Again, we design the active controller based on the backstepping method outlined in subsection 3.1

Step 1. Let $z_1 = e_1$, then we can obtain its derivative

$$\dot{z}_1 = \dot{e}_1$$
$$= -az_1 + (a + x_3 + e_3)e_2 + x_2 e_3$$
(19)

where $e_2 = \alpha_1(z_1)$ is regarded as a virtual control input.

For the design of α_1 to stabilize z_1-subsystem (19), we choose Lyapunov function V_1 as

$$V_1 = \frac{z_1^2}{2}.$$
(20)

The derivative of V_1 is as following

$$\dot{V}_1 = z_1 \dot{z}_1$$
$$= -az_1^2 + x_2 z_1 e_3 + (a + x_3 + e_3) z_1 \alpha_1$$
(21)

If we choose $\alpha_1 = 0$, then $\dot{V}_1 = -az_1^2 + x_2 z_1 e_3$. The second term $x_2 z_1 e_3$ in $\dot{V}_1$ will be cancelled at the next step.

Since the virtual control input α_1 is estimative the error between e_2 and α_1 is

$$z_2 = e_2 - \alpha_1.$$
(22)

Then, we can obtain the following (z_1, z_2)-subsystem;

$$\dot{z}_1 = -az_1 + (a + x_3)z_2 + x_2 e_3 + z_2 e_3$$
$$\dot{z}_2 = (b - x_3)z_1 + bz_2 - (x_1 + z_1)e_3 + u_2$$
(23)

where $e_3 = \alpha_2(z_1, z_2)$ is regarded as a virtual controller.

Step 2. In this step, we will stabilize the (z_1, z_2)-subsystem (23). We can choose a Lyapunov function V_2 as follows

$$V_2 = V_1 + \frac{z_2^2}{2}.$$
(24)

Its derivative is given by

$$\dot{V}_2 = \dot{V}_1 + z_2 \dot{z}_2$$
$$= -az_1^2 + (a + x_3 + \alpha_2)z_1 z_2 + x_2 z_1 \alpha_2$$
$$+ z_2 \{(b - x_3)z_1 + bz_2 - (x_1 + z_1)\alpha_2 + u_2\}$$
$$= -az_1^2 + (x_2 z_1 - x_1 z_2)\alpha_2$$
$$+ z_2 \{(a + b)z_1 + bz_2 + u_2\}$$
(25)

If we choose $\alpha_2 = 0$ and $u_2 = -(a + b)z_1 - 2bz_2$, then $\dot{V}_2 = -az_1^2 - bz_2^2 < 0$ makes (z_1, z_2)-subsystem (23) asymptotically stable. Similarly, assume that $z_3 = e_3 - \alpha_2$, then we can derive the following (z_1, z_2, z_3)-subsystem

$$\dot{z}_1 = -az_1 + (a + x_3)z_2 + x_2 z_3 + z_2 z_3$$
$$\dot{z}_2 = -(a + x_3)z_1 - bz_2 - x_1 z_3 - z_1 z_3$$
$$\dot{z}_3 = x_2 z_1 + x_1 z_2 - cz_3 - ge_3 + z_1 z_2 + u_3$$
(26)

Step 3. In order to stabilize the (z_1, z_2, z_3)-subsystem (26), we can choose a Lyapunov function V_3 as follows

$$V_3 = V_2 + \frac{z_3^2}{2}.$$
(27)

Its derivative of V_3 is

$$\dot{V}_3 = \dot{V}_2 + z_3 \dot{z}_3$$
$$= -az_1^2 - bz_2^2$$
$$+ z_3(2x_2 z_1 - cz_3 - ge_4 + z_1 z_2 + u_3)$$
(28)

If we choose $u_3 = -2x_2 z_1 + ge_4 - z_1 z_2$,
then $\dot{V}_3 = -az_1^2 - bz_2^2 - cz_3^2 < 0$ makes the (z_1, z_2, z_3)-subsystem (26) asymptotically stable.

Likewise, since $\dot{V}_3$ is negative definite, it follows that in the (z_1, z_2, z_3) coordinates the equilibrium $(0, 0, 0)$ of the

subsystem (26) is global asymptotically stable. In view of $z_1 = e_1$, $z_2 = e_2 - \alpha_1 = e_2$ and $z_3 = e_3 - \alpha_2 = e_3$, this implies that e_1, e_2 and e_3 go to zero asymptotically. According to $e_1 \to 0, e_2 \to 0, e_3 \to 0$ and the fourth equation of system (18), we get that (e_1, e_2, e_3, e_4) of the controlled system (18) go to $(0,0,0,0)$ as $t \to \infty$. In other words, the trajectory of the controlled response system (16) asymptotically approaches the trajectory of the drive system (14) with the proposed control inputs.

4.2. Numerical Results

Similarly, we set $a = 50$, $b = 24$, $c = 13$, $f = 8$, $g = 33$ and $h = 30$ to ensure hyperchaotic behavior. The initial conditions for the drive hyperchaotic system (14) and the response hyperchaotic system (16) are

$x_1(0) = 0.1, x_2(0) = 0.2, x_3(0) = 0.3, x_4(0) = 0.4$ and

$y_1(0) = 1.0, y_2(0) = 2.0, y_3(0) = 3.0, y_4(0) = 4.0$.

Thus, the initial values of the error states are
$e_1(0) = 0.9, e_2(0) = 1.8\ e_3(0) = 2.7, e_4(0) = 3.6$.

Fig. 3 shows the time response of states determined by the drive system and the response system with the proposed control function. The trajectories of synchronization errors for the drive system and the response system are shown in Fig. 4. The controller turns on at $t = 3$. As expected, it shows that all state variables are synchronized and the synchronization errors converge to zero.

(a) The time response of states x_1 and y_1

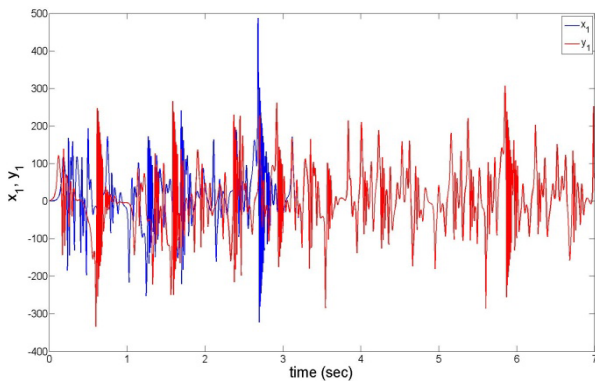(b) The time response of states x_2 and y_2
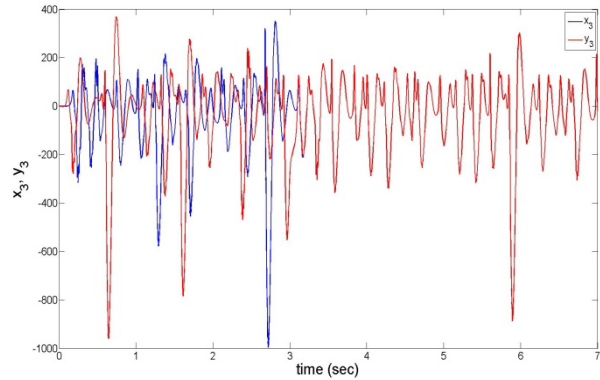
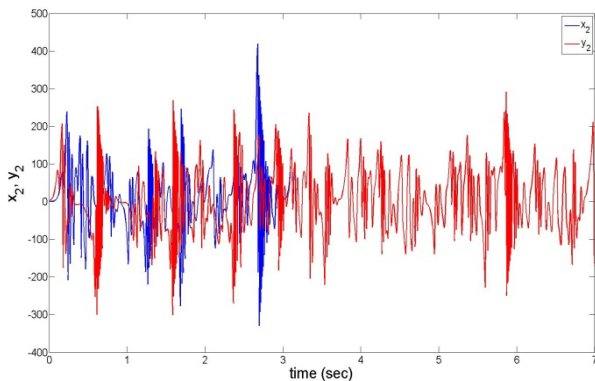(c) The time response of states x_3 and y_3
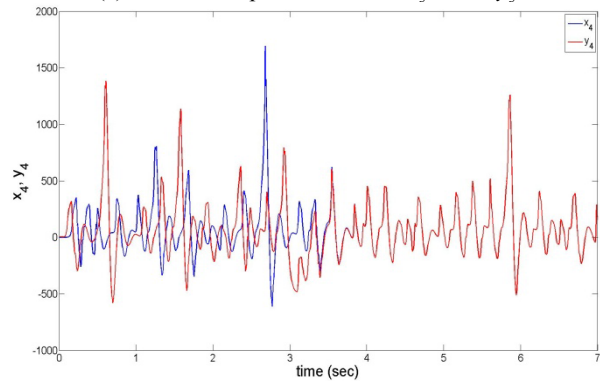
(d) The time response of states x_4 and y_4

Fig. 3. The time response of states for the drive system (14) and the response system (16) with the proposed controller.
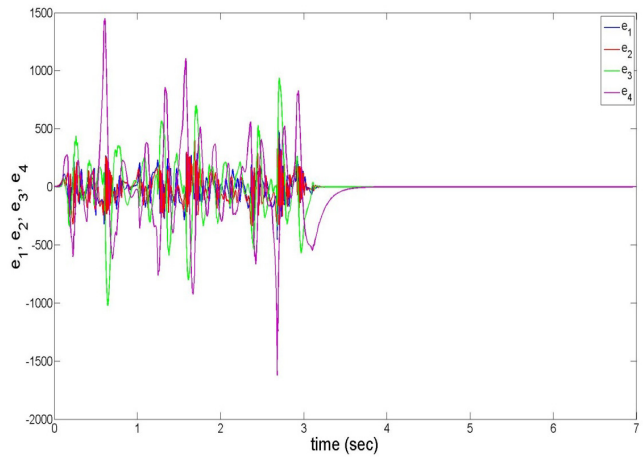

Fig. 4. The time response of synchronization error states

5. Conclusions

This paper has examined control and synchronization of the new hyperchaotic systems performed by Qi et al. [22] using the active backstepping control method. The proposed scheme is a systematic design approach and consists in a recursive procedure that interlaces the choice of a Lyapunov function with the design of active control. The proposed control approach is able to stabilize the chaotic motion to the origin. In addition, it

synchronizes the two identical Qi hyperchaotic systems with the systematic way. Numerical simulations were also carried out to illustrate the effectiveness of the approach. We verified that the proposed method has following advantages. First, it does not need to calculate the Lyapunov exponents and eigenvalues of the Jacobian matrix. Hence, it is simple and convenient. Second, This approach is applicalbe to control high dimensional hyperchaotic systems by adopting the active control technique, which has the flexiblity to design a control law.

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