Interference Tolerant Based CR System with Imperfect Channel State Information at the CR-Transmitter

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Abstract

In interference tolerance based spectrum sharing systems, primary receivers (PRs) are protected by a predefined peak or average interference power constraint. To implement such systems, cognitive radio (CR) transmitters are required to adjust their transmit power so that the interference power received at the PR receivers is kept below the threshold value. Hence, a CR-transmitter requires knowledge of its channel and the primary receiver in order to allocate the transmit power. In practice, it is impossible or very difficult for a CR transmitter to have perfect knowledge of this channel state information (CSI). In this paper, we investigate the impact of imperfect knowledge of this CSI on the performances of both a primary and cognitive radio network. For fixed transmit power, average interference power (AIP) constraint can be maintained through knowledge of the channel distribution information. To maintain the peak interference power (PIP) constraint, on the other hand, the CR-transmitter requires the instantaneous CSI of its channel with the primary receiver. First, we show that, compared to the PIP constraint with perfect CSI, the AIP constraint is advantageous for primary users but not for CR users. Then, we consider a PIP constraint with imperfect CSI at the CR-transmitter. We show that inaccuracy in CSI reduces the interference at the PR-receivers that is caused by the CR-transmitter. Consequently the proposed schemes improve the capacity of the primary links. Contrarily, the capacities of the CR links significantly degrade due to the inaccuracy in CSI.

Key words : Cognitive Radio, Interference Temperature, Channel Capacity, Fading Channel, Imperfect CSI.

I. Introduction

The main feature of CR systems is to take the advantage of unoccupied or partially-occupied frequency bands to improve spectrum utilization. In CR networks, the CR users may coexist with the primary users either on a non-interference basis or an interference tolerant basis. In the case of non-interference systems, the CR users are allowed to operate in the unused frequency bands, commonly known as spectrum holes or white spaces [1]. Contrarily, the interference tolerant systems allow CR users to access the frequency band of the primary users provided that the interference power level at the primary receivers is kept below some certain threshold [1], [2]. This threshold is an interference temperature. In this interference temperature, Quality of Service (QoS) of the primary user can be guaranteed. So a CR system based on interference temperature permits primary users and CR users to use the interference temperature to simultaneously transmit signals. The fundamental limits of interference temperature in a single channel CR-system are studied in [1]. The capacities, along with the power allocation problems of such systems for fading channels,

are investigated in [2]. The closed form expressions of the capacities are derived in [3]. So far, most of the work on interference temperature limited spectrum-sharing systems considers that the CR transmitter has perfect knowledge of the channel state information (CSI) of the CR-transmitter relative to the primary receiver channel. Such perfect knowledge of the CSI ensures that the interference power received at the primary receivers due to the transmission of the CR-transmitter strictly satisfies the pre defined constraint. In this paper, we consider that the CSI known at the CR-transmitter is not perfect. The impact of imperfect CSI on both the primary and cognitive radio links is investigated. The analysis and results show that imperfect CSI actually increases the capacity of the primary-link but significantly reduces the capacity of the CR-link.

II. System and Channel Model

We consider a spectrum sharing scenario in which CR users are coexisting with primary users and are sharing the same frequency band. For the purpose of exposition, we consider a single CR-link consisting of a CR trans-

mitter (CR-tx) and a CR-receiver (CR-rx), and a single primary-link consisting of a primary-transmitter (PR-tx) and a primary receiver (PR-rx). The primary users are willing to share their spectrum provided that interference power received at the primary receivers is below some certain threshold. The primary transmitters are assumed to be located far away from the CR-receiver. So, the interference power received at the CR-receiver due to the transmission of the primary transmitters is negligible. We consider a flat Rayleigh fading channel with additive white Gaussian noise (AWGN). Perfect CSI is assumed to be available at the receivers so that coherent detection is possible. The instantaneous CSI between the PR-transmitter and PR-receiver, CR-transmitter and CR-receiver, and CR-transmitter and PR-receiver can be represented by f, g and h, respectively. In Rayleigh fading, the channel coefficients f, g, and h are zero mean circularly symmetric complex Gaussian random variables with variances σ_f^2, σ_g^2, and σ_h^2, respectively. The interference constraint can be imposed on the PIP or AIP. The PIP and AIP constraints at the PR-receiver can be defined as $E\{P_{CR}|h|^2\} \leq I_{av}$ and $P_{CR}|h|^2 \leq I_{pk}$, respectively. Here, I_{av} and I_{pk} are the predefined AIP and PIP constraints, P_{CR} is the transmit power of the CR-tx and E represents the statistical expectation operator. Considering that $E\{|h|^2\}= \sigma_h^2$, the fixed power allocation to the CR-tx under the AIP constraint can be written as $P_{CR} \leq I_{gv}/\sigma_h^2$. Hence, to implement and AIP limited CR system, the CR transmitter only requires an average channel power gain of consistent with the CR-tx to PR-rx link.

The power allocation to the CR-tx considering the PIP constraint can be given as, $P_{CR} \leq I_{pk}/|h|^2$. Hence, to satisfy the peak interference constraint, the CR-transmitter must know the amplitude of the CSI between the CR transmitter and PR-receiver, |h|. The CR-tx and PR-rx are two nodes from two different networks. Consequently, measuring the CSI (h) of such cross network links requires some sort of cooperation among the PR and CR system. Simply, the CR-transmitter can achieve the information about h through feedback. The feedback may be carried out directly by the primary receivers [1] or through a band manager [4]. Alternatively, the PR-rx can send a pilot signal so that the CR-tx can measure the CSI from the pilot symbols. If the PR system uses some handshake signaling e.g., ready-to-send (RTS) and clear-to-send (CTS), then the pilot symbols can be included in the CTS. Due to the different radio specifications of the two different networks, it is not easy to measure the perfect value of |h| at the CR-transmitter in any one of the methods explained above. The accuracy of the feedback or channel measurement is

vulnerable to noise, estimation error, and feedback delay, etc. Considering this reality, we assume that the CR-transmitter knows a noisy version of the channel state amplitude $|\hat{h}|$ where $\hat{h}=h+e$. Similar to [5] and [6], the additional error term in the estimated channel coefficient, e, and it is modeled as a zero mean complex Gaussian random variable with a variance σ_e^2. Now, the transmit power allocated to the CR-tx with the imperfect knowledge of CSI can be written as $P_{CR} \leq I_{pk}/|\hat{h}|^2$.

III. Performance of the PR-link

In this section, we investigate the ergodic capacity of the primary link under both the AIP and PIP constraints. In the case of PIP constraint, we consider that the CR-transmitter knows an erroneous version of the CSI $\hat{h}$. The error in $\hat{h}$ leads a power allocation to the CR transmitter which may cause degradation in the performance of the PR-link. The instantaneous interference power, considering the PIP constraint, received at the PR-rx can be given as

$$I^{PIP} = P_{CR}|h|^2 = I_{pk}\frac{|h|^2}{|h+e|^2} \qquad (1)$$

Eq. (1) clearly shows that at zero error (e=0), the instantaneous received interference power is exactly equal to the PIP constraint. For any nonzero error component, the received interference power is always less than the PIP constraint. Moreover, the interference power received at the PR-receiver decreases as the error component of the CSI increases. Hence the erroneous CSI at the CR-transmitter always satisfies the PIP constraint at the primary-receiver. The ergodic capacity of the PR-link considering PIP constraint is given as

$$C_{PR}^{PIP,er} \leq E\left[\log\left(1+\frac{P_{PR}|f|^2}{N_0+I^{PIP}}\right)\right] \qquad (2)$$

where P_{PR} is the transmit power of the PR-tx and N_0 is the noise variance. The expectation operation of (2) should be performed over the probability density function (PDF) of the random variables f, h and e.

In the case of the AIP constraint we assume that the CR-tx knows its distance from the PR-rx perfectly. Hence, the average channel power gain (σ_h^2) is also perfectly known. Under the AIP constraint, the instantaneous interference power received at the PR-rx is

$$I^{AIP} = P_{CR}|h|^2 \leq I_{av}\frac{|h|^2}{\sigma_h^2} \qquad (3)$$

Obviously, the instantaneous interference power is a function of the random variable $|h|^2$, which is normalized with its mean value. Therefore, the long term aver-

age interference always satisfies the AIP constraint. The ergodic capacity of the PR-link considering AIP can be given as

$$C_{PR}^{AIP,er} \leq E\left[\log\left(1 + \frac{P_{PR}|f|^2}{N_0 + I^{AIP}}\right)\right] \tag{4}$$

Here, the expectation should be taken over the PDF of f and h.

Ⅳ. Performance of the CR-link

The ergodic capacities of CR links with AIP and PIP constraints for different power allocations have been investigated in [1], [2]. In this paper we will investigate the impact of the different levels of knowledge on CR-tx to PR-rx links at the CR-transmitter. The ergodic capacity of the CR-link considering the PIP and AIP constraints can be given as

$$C_{CR}^{PIP,er} \leq E\left[\log\left(1 + \frac{I_{pk}}{N_0}\frac{|g|^2}{|\hat{h}|^2}\right)\right] \quad \text{and} \tag{5}$$

$$C_{PR}^{AIP,er} \leq E\left[\log\left(1 + \frac{I_{av}}{N_0\sigma_h^2}|g|^2\right)\right]. \tag{6}$$

Eq. (5) shows that the received SNR part of the capacity is inversely proportional to the error term, e. Hence any nonzero error will cause degradation in the ergodic capacity of the CR-link.

Along with the ergodic capacity, outage probability is also an important measure of performance. The outage probability of the CR link can be defined as the probability that the instantaneous achievable rate at the CR-receiver is less than that at a certain threshold, R. The outage probability of the CR-link with PIP and AIP constraints can be expressed as

$$OUT_{CR}^{PIP} = Pr\left[\log\left(1 + \frac{I_{pk}}{N_0}\frac{|g|^2}{|\hat{h}|^2}\right) < R\right]$$
$$= Pr\left[\frac{|g|^2}{|\hat{h}|^2} \leq \beta_1\right] = F(\beta_1) \quad \text{and} \tag{7}$$

$$OUT_{CR}^{AIP} = Pr\left[\log\left(1 + \frac{I_{av}}{N_0\sigma_h^2}|g|^2\right) < R\right]$$
$$= Pr[|g|^2 \leq \beta_2\sigma_h^2] = 1 - \exp\left(-\beta_2\frac{\sigma_h^2}{\sigma_g^2}\right) \tag{8}$$

where $\beta_1 = (2^R - 1)N_0/I_{pk}$, $\beta_2 = (2^R - 1)N_0/I_{av}$ and $F(\cdot)$ is the CDF of the random variable $|g|^2/|\hat{h}|^2$ which can be given as

$$F(z) = \frac{z(\sigma_h^2 + \sigma_e^2)}{z(\sigma_h^2 + \sigma_e^2) + \sigma_h^2} \tag{9}$$

Eq. (9) is derived considering that $|g|^2$ and $|h|^2$ are exponentially distributed random variables and it follow the basic theory of deriving the PDF of two random variables.

Ⅴ. Simulation Results

In this section, we provide some numerical and simulation results to show the effect of inaccurate CSI at the CR-transmitter on the performance of the primary and CR links. For all cases, we consider $N_0 = 1$ and $\sigma_f^2 = \sigma_g^2 = \sigma_h^2 = 1$. In this paper, we consider both PIP and AIP constraint and compare their performances. For fair comparison among the PIP and AIP constraints, we assume $I_{pk} = I_{av} = Q$. The ergodic capacity of the primary link of Eqs. (2) and (4) against P_{PR}/N_0 is plotted in Fig. 1. First we compare the capacity achieved under the AIP and PIP constraints with perfect CSI at the CR-tx ($\sigma_e^2 = 0$. It has been shown in [7] that the AIP constraint is more advantageous than the PIP constraint in terms of the performance of the PR-link. Our results also show that the PR-link achieves higher ergodic capacity under the AIP constraint than the PIP constraint over the whole range of interference power constraint, Q.

In Fig. 1 we also investigate the effect of the error component in the CSI of the CR-tx to PR-rx channel, $\hat{h}$. One could expect the erroneous CSI at CR-transmitter to cause a incorrect power allocation which would degrade the performance of the PR-link. Surprisingly, our results show that inaccurate knowledge of the CSI of the CR-tx to PR-rx channel ($\sigma_e^2 > 0$) actually improves the ergodic capacity of the PR-link. The capacity of the PR-link linearly increases as the variance of the error, σ_e^2, increases. This is due to the fact that the interference power received at the primary-receiver under the PIP constraint decreases with e as shown in Eq. (1). This capacity im-
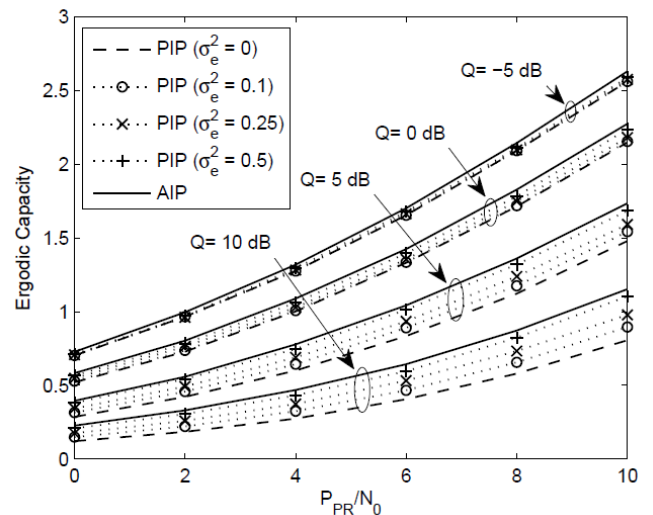

Fig. 1. Ergodic capacity of the primary link.

provement in the PR-link is very low at the lower PIP constraint but increases as the PIP constraint increases. Therefore, an inaccurate CSI of the CR-tx to PR-rx link at the CR-tx improves the performance of the PR-link rather than causes any performance degradation.

Fig. 2 shows the ergodic capacity of the CR-link of Eqs. (5) and (6) against the interference power constraint, Q. Unlike the PR-link, the CR-link achieves higher ergodic capacity under the PIP constraint compared to the AIP constraint over the entire range of Q. This extra capacity gain associated with the PIP constraint comes from its implementation complexity which requires accurate knowledge of instantaneous CSI of the CR-tx to PR-rx link, h, at the CR-tx. Fig. 2 shows that the ergodic capacity achieved by the PIP constraint is highly dependent on the accuracy of the knowledge of h. As the variance of the error component, σ_e^2, increases, the ergodic capacity of the CR-link decreases. Eq. (5) shows that the SNR term in the ergodic capacity is inversely proportional to the error component e which justifies the degradation in the capacity of the CR-link. Importantly, at σ_e^2=0.5 (which is half the actual channel variance) the capacity of the CR-link with the PIP constraint is well above the capacity with the AIP constraint. Therefore, the PIP constraint with inaccurate knowledge of the instantaneous CSI of the CR-tx to PR-rx link is superior to the AIP constraint even though the level of inaccuracy is very high.

Fig. 3 shows the outage probability of the CR link of Eqs. (7) and (8). We consider σ_g^2=2, σ_h^2=1, and R=0.5 for this figure. Fig. 3 depicts that, at the low interference constraint region, the outage probability of the CR-link under the AIP constraint is higher than the outage probability under the PIP constraint. As the interfer-
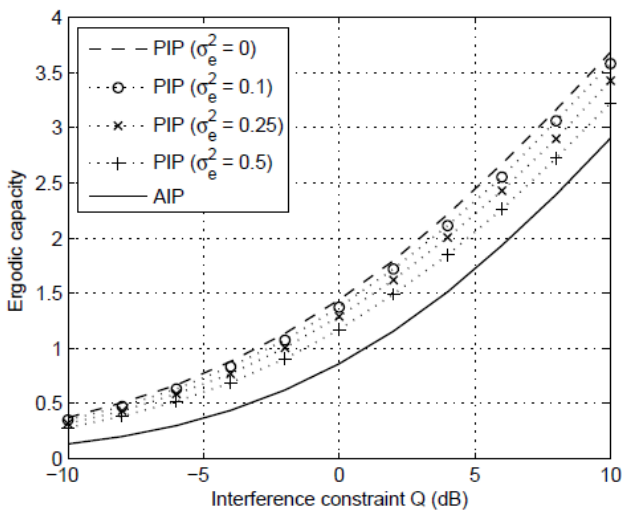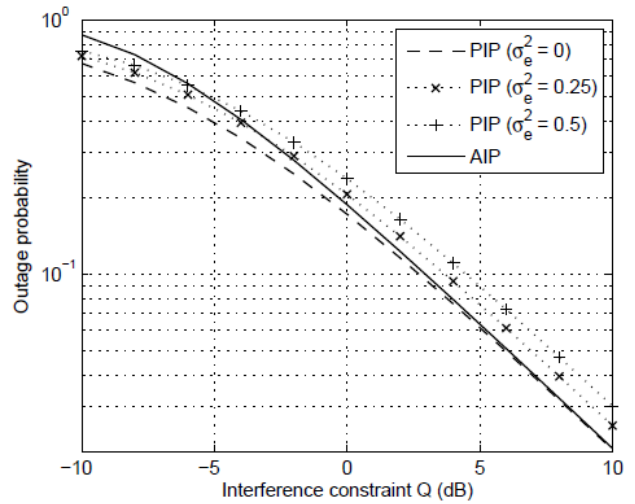

Fig. 3. Outage probability of the cognitive radio link.

ence constraint increases, the outage probability of the AIP constraint approaches the outage probability of the PIP constraint. At a high interference constraint the outage probability of both the AIP and PIP constraints is the same. We show in Fig. 2 how the AIP constraint achieves less ergodic capacity compared to the PIP constraint over the entire range of the interference constraint. Fig. 3 shows that the outage behavior under the PIP and AIP constraints is different from the ergodic capacity. This observation is very interesting, since the implementation complexity of the AIP constraint is considered to be lower than that of the PIP constraint but it achieves the same performance in the low outage probability region. Fig. 3 also confirms that the outage performance of the CR-link, under the PIP constraint, degrades as the error variance increases over the entire range of the interference constraint.

VI. Conclusion

The implementation issues of the interference tolerant basis cognitive radio network have been investigated. The simplest implementation of one such CR network is done by allocating a fixed power to the CR-transmitter, to satisfy the AIP constraint that requires only the channel distribution information of the CR-tx to PR-rx link at the CR transmitter. Such implementation achieves a higher capacity for the PR-links but does not fully leverage the capacity of the CR-link. Under the PIP constraint based implementation, on the other hand, the CR network requires perfect and instantaneous CSI of the CR-tx to PR-rx link which is difficult to achieve at the CR-transmitter. The effects of erroneous CSI on the performance of the PR and CR links are investigated. We

Fig. 2. Ergodic capacity of the cognitive radio link.

show that the imperfect CSI at the CR-transmitter reduces the instantaneous interference power received at the PR-receiver. But, this imperfect CSI causes a reduction in the ergodic and outage capacities of the CR-link.

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