

Low Complexity Multiuser Scheduling in Time-Varying MIMO Broadcast Channels

Seung Hwan Lee¹ · Junho Lee²

Abstract

The sum-rate maximization rule can find an optimal user set that maximizes the sum capacity in multiple input multiple output (MIMO) broadcast channels (BCs), but the search space for finding the optimal user set becomes prohibitively large as the number of users increases. The proposed algorithm selects a user set of the largest effective channel norms based on statistical channel state information (CSI) for reducing the computational complexity, and uses Tomlinson-Harashima precoding (THP) for minimizing the interference between selected users in time-varying MIMO BCs.

Key words : Multiuser Scheduling, Multiple Input Multiple Output (MIMO), Statistical Channel State Information (CSI), Scheduling Delay, Channel Norm.

I . Introduction

Statistical channel state information (SCSI)-assisted multiuser scheduling [1] can improve the sum capacity by minimizing the effect of the delay-induced channel estimation error caused by the Doppler spread in time-varying MIMO BCs, where a base station supports data traffic to multiple users. However, the computational complexity involved in finding an optimal user set using the sum-rate maximization rule makes real-time implementation very difficult if the number of users is large.

Due to its simplicity, linear zero-forcing beamforming (ZFBF) has attracted more attention than nonlinear THP for developing practical multiuser scheduling algorithms in MIMO broadcast channels [2], [3]. However, ZFBF performs worse than THP in terms of the sum capacity: ZFBF only uses orthogonal channel directions, while THP can cancel the interference from other users using the dirty-paper coding (DPC) principle. This motivates the combination of a norm-based user selection and THP.

This paper proposes a low complexity multiuser scheduling algorithm considering MIMO precoding techniques together in a cross-layer fashion in time-varying MIMO BCs. The proposed algorithm consists of a statistical CSI-assisted norm-based selection rule and THP. The algorithm simply finds users with the largest time-varying channel norms up to the number of transmit antennas, in decreasing order, considering their Doppler spread regardless of the orthogonality between users. This re-

duces the sum capacity and also the computational complexity in the user selection procedure. In this case, THP with a CSI-assisted norm-based selection rule can minimize the performance degradation compared to ZFBF because any possible interference due to non-orthogonal users can be removed by the pre-subtraction of interference at the transmitter in the time-varying MIMO BCs.

This paper is organized as follows. In section II, the system model is described, including a channel model and the sum capacity with MIMO precoding techniques. The proposed statistical CSI-assisted norm-based rule is introduced in section III. Then, the numerical results are given in section IV and, finally, we conclude this paper in section V.

II . System Model

We use boldface to denote matrices and vectors. For any general matrix \mathbf{A} , \mathbf{A}^T and \mathbf{A}^H denote the transpose and the conjugate transpose, respectively; \mathbf{I} denotes the identity matrix. For any general set B , $|B|$ denotes the cardinality of the set.

2-1 Channel Model

Consider a MIMO BC with M_T transmit antennas at a base station and K ($K \geq M_T$) users, each with a single receive antenna. Let $\mathbf{h}_k(t) \in \mathbb{C}^{M_T \times 1}$ denote the channel at time t between the transmit antenna array and the re-

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ceive antenna for user k , whose elements are complex Gaussian random variables with a zero mean and unit variance. The MIMO BC at time t can then be represented as in [4]:

$$y_k(t) = \mathbf{h}_k^T(t)\mathbf{s}(t) + z_k(t), \quad k = 1, \dots, K, \quad (1)$$

where $\mathbf{s}(t) \in \mathbb{C}^{M_T \times 1}$ is the transmit signal with a covariance matrix $\mathbf{R}_{ss} = E[\mathbf{s}(t)\mathbf{s}(t)^H] = \sigma_s^2 \mathbf{I}$ and a power constraint $\text{Tr}(\mathbf{R}_{ss}) \leq P$, the scalar $y_k(t)$ is the received signal for user k , and $z_k(t)$ is complex additive white Gaussian noise (AWGN) with a zero mean and unit variance for user k at time instant t . The temporal correlation of user k with delay τ can be given by $\rho_k(\tau) = J_0(2\pi f_k^d \tau)$, where f_k^d is the Doppler spread and J_0 is the zeroth order Bessel function of the first kind.

It is assumed that one downlink frame of the MIMO BC with time period T_f consists of N_S ($N_S > 1$) time slots with time period T_s . The instantaneous CSI, which is updated once per frame, is assumed to be perfect, so that the mismatch of the channel estimate due to scheduling delay exists in the statistical CSI.

If a user set S_0 ($|S_0| \leq M_T$) forms a channel matrix $\mathbf{H}(S_0)$, an equivalent time-varying channel matrix reflecting scheduling delay within the frame at slot index n is given by [1]:

$$\mathbf{H}(n) = \mathbf{P}(n)\mathbf{H}_0 + \sqrt{\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)}\mathbf{H}_m, \quad n = 1, \dots, N_S, \quad (2)$$

where $\mathbf{P}(n) = \text{diag}\{\rho_1(nT_s), \dots, \rho_{M_T}(nT_s)\}$ denotes the autocorrelation matrix of user set S_0 , $\sqrt{\mathbf{I} - \mathbf{P}(n)\mathbf{P}^H(n)}$ represents the amplitude increase in the channel estimation error due to scheduling delay, \mathbf{H}_0 is the perfectly estimated instantaneous CSI at the beginning of each frame, and \mathbf{H}_m is an uncorrelated estimation error matrix that has the same statistical characteristics as the estimated channel \mathbf{H}_0 .

2-2 Sum Capacity with MIMO Precoding Techniques

For THP, a channel matrix $\mathbf{H}(S_0)$ is decomposed into a unitary beamforming matrix \mathbf{F} and lower triangular matrix \mathbf{B} by taking the QR decomposition. Throughout this paper, an equal power allocation over spatial channels and the same number of elements in a user set as the number of transmit antennas, $|S_0| = M_T$, are assumed for simplicity. If the modulo loss of THP can be ignored, the sum capacities of THP and ZFBF reduce respectively to

$$C_{THP} = \sum_{k=1}^{M_T} \log_2 \left(1 + \frac{b_{kk}^2 P}{M_T} \right), \quad \text{and} \quad (3)$$

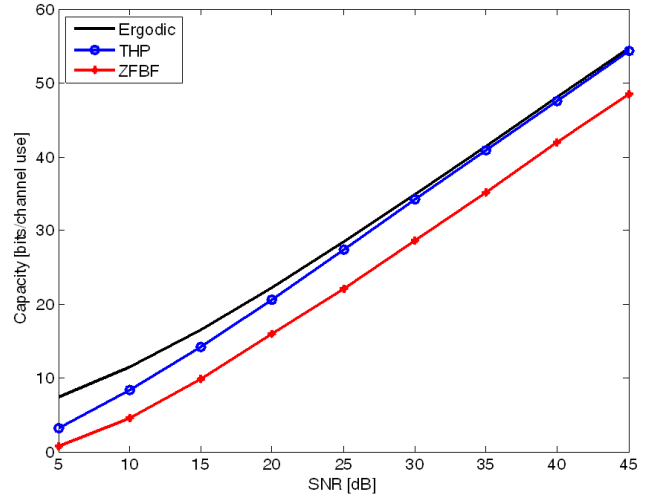


Fig. 1. Comparison of the sum capacity of THP and ZFBF with the ergodic MIMO capacity, $M_T=4$ and $K=4$.

$$C_{ZFBF} = \sum_{k=1}^{M_T} \log_2 \left(1 + \frac{\gamma_k P}{M_T} \right), \quad (4)$$

where b_{kk} is the diagonal element of matrix \mathbf{B} and γ_k is the effective channel gain, which is given by [2]:

$$\gamma_k = \frac{1}{\left[(\mathbf{H}(S_0)\mathbf{H}(S_0)^H)^{-1} \right]_{k,k}}. \quad (5)$$

Fig. 1 compares the sum capacity of MIMO precoding techniques such as THP and ZFBF with the ergodic MIMO capacity in an *i.i.d.* MIMO channel. We notice that as SNR increases, the modulo loss of THP can be ignored. This means that the sum capacity of THP approaches the ideal ergodic capacity with a high SNR assumption. Linear ZFBF performs more poorly than non-linear THP. This is because THP can cancel the interference from other users using the DPC principle. However, ZFBF only uses orthogonal channel directions, which reduces the effective channel gains. This justifies THP as an appropriate MIMO precoding technique for practical implementation from the viewpoint of the sum capacity.

Accordingly, the sum capacities of THP and ZFBF at slot index n in the time-varying MIMO BC are respectively given by [4]:

$$C_{THP}(nT_s) = \sum_{k=1}^{M_T} \log_2 \left(1 + \frac{b_{kk}^2 \rho_k^2(nT_s) P}{M_T \{(1 - \rho_k^2(nT_s))^{P+1}\}} \right) \quad (6)$$

$$C_{ZFBF}(nT_s) = \sum_{k=1}^{M_T} \log_2 \left(1 + \frac{\rho_k^2(nT_s) P}{M_T \{(1 - \rho_k^2(nT_s)) \gamma_k^{-1} P + 1\}} \right) \quad (7)$$

III. Statistical CSI-assisted Norm-based Rule

When the number of users K is larger than the num-

number of transmit antennas M_T in the MIMO BC, a multi-user selection algorithm finds a user set S , which satisfies certain performance criteria by considering all possible choices of user set S . If the performance criterion is to maximize the sum-rate, the sum capacity can be obtained by the sum-rate maximization rule as

$$C_{MAX} = \max_{S_0} R(S_0), \quad (8)$$

where sum rate, $R(S_0)$, is determined by the type of MIMO precoding technique used.

Fig. 2 shows the sum capacity of THP plotted against the frame period using the sum-rate maximization rule in the time-varying MIMO BC. When the frame period is sufficiently small compared to the coherence time of the channel, the performance improvement considering SCSi is negligible. However, as the frame period increases, the performance advantage associated with using SCSi becomes evident. The SCSi-assisted multiuser scheduling algorithm performs better than the multiuser scheduling without SCSi considerations when $T_f > 0.2$ ms. When $T_f = 2$ ms, the throughput gain due to the SCSi-assisted algorithm is about 1.5 [bits/channel use]. Note that the amount of performance improvement and complexity increase of the proposed algorithm depends on the channel conditions and the number of slots in the frame structure.

The computational burden of the exhaustive search for this rule is prohibitively large, even for practical values of K , *i.e.* $K > 30$. In order to reduce the computational complexity, the channel norm $\|\mathbf{h}_k\|^2$ is used to select a user set because it is closely related to the chan-

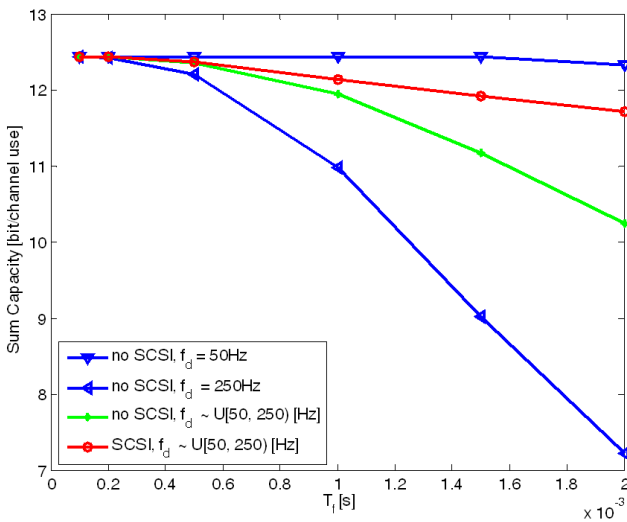


Fig. 2. The sum capacity of THP plotted against the frame period with different Doppler spread configurations, $f_c = 2$ GHz, SNR = 15 dB, $M_T = 2$, and $K = 32$.

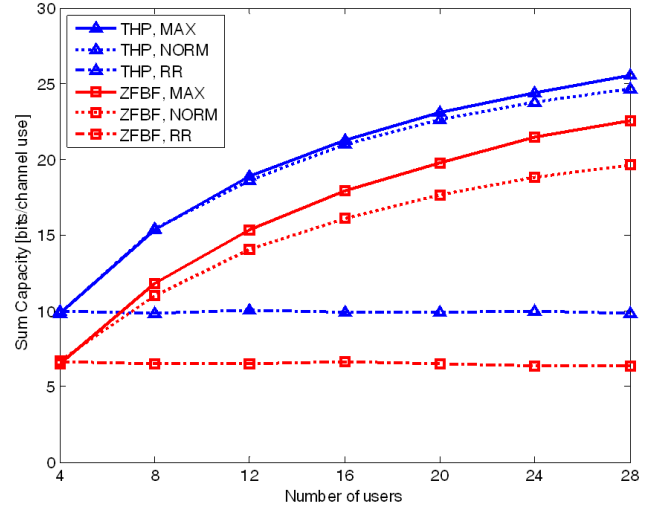


Fig. 3. The sum capacity of THP and ZFBF with different user selection criteria, $M_T = 4$ and SNR = 12 dB.

nel capacity. However, the user selection rule using the channel norm cannot maximize the sum capacity because it only considers individual channel energy and ignores interference between users, so that it only finds a suboptimal user set.

We notice that user selection algorithms affect the sum-rate capacity. The sum-rate maximization rule (MAX) shows the best performance regardless of the type of MIMO precoding techniques in terms of the sum capacity because it optimizes the user set choice for that parameter. However, it requires more computation than other selection rules. Because the norm-based rule (NORM) does not consider interference among users, its performance is worse than the sum-rate maximization rule. The round-robin rule (RR) shows the poorest performance, which is unchanged regardless of the number of users. This is because RR rule does not exploit multiuser diversity when it selects a user set for transmission. The amount of degradation of ZFBF when using the norm-based rule is larger than that of THP. This implies that THP is more robust to the type of user selection criterion used than ZFBF.

The proposed statistical CSI-assisted norm-based multiuser scheduling algorithm operating on a slot-by-slot basis can be described as follows:

Step 1: Calculation of Effective Norm. Denote the perfect channel estimate of user k at the beginning of each frame as \mathbf{h}_k^0 , obtain the effective channel norms of all users using statistical CSI for every time slot index n . The effective channel norms reflect the channel estimation error due to scheduling delay as follows:

$$\|\mathbf{h}_k^e(nT_s)\|^2 = \frac{\rho_k^2(nT_s)}{1 - \rho_k^2(nT_s)} \|\mathbf{h}_k^0\|^2. \quad (9)$$

Step 2: Sorting. Place the effective channel norms obtained in step 1 in decreasing order and extract a sorting index.

Step 3: Selection and MIMO precoding. Use the sorting index from step 2, to find a user set S of size M_T from a total user set U of size K , and perform MIMO precoding on the selected user set.

The computational complexity of the proposed algorithm will be compared with the sum-rate maximization rule in the next section.

IV. Numerical Results

Fig. 4 compares the sum capacity for different combinations of MIMO precoding techniques and user selection criteria. In this plot, it is assumed that the number of transmit antennas is $M_T=4$, the number of users $K=20$, and the number of slots in a frame $N_S=5$. The relative Doppler spread $f_d T_f$ of each user has a uniform distribution over $[0.1, 0.5]$.

We notice that the sum capacity using statistical CSI (SCSI) can be improved regardless of the multiuser selection rules or MIMO precoding techniques used. This is because the statistical CSI-assisted multiuser selection rule performed on a slot-by-slot basis ensures that any user showing a rapid decrease in temporal correlation is assigned to the first few slots in the frame. Between MIMO precoding techniques, ZFBF always performs worse than THP in all cases. The proposed effective norm-based selection rule (NORM) performs worse than the sum-rate maximization rule (MAX) because it selects a sub-optimal user set that only maximizes the sum of in di-

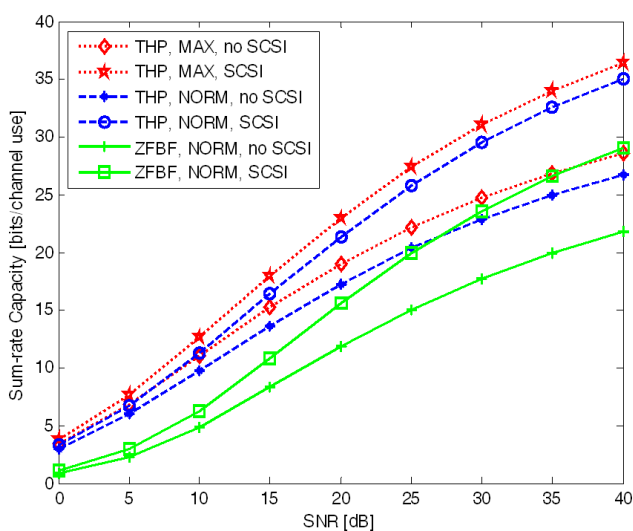


Fig. 4. Sum-rate for different MIMO precoding techniques and multiuser selection criteria with $M_T=4$, $K=20$, and $N_S=5$.

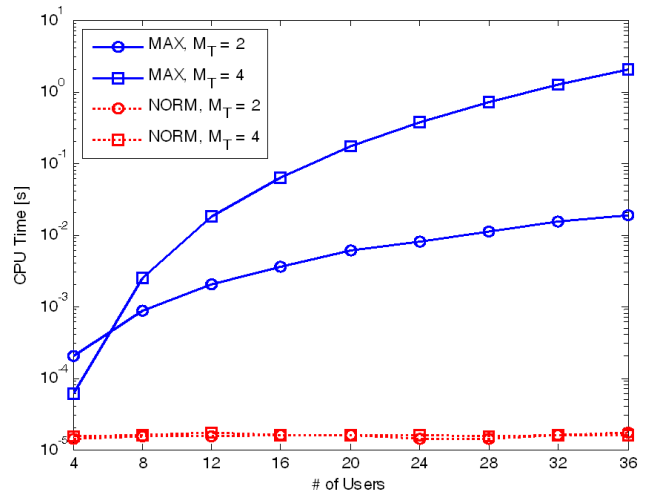


Fig. 5. CPU time relative to the number of users: estimating the computational complexities of selection rules.

vidual channel energy of the selected users.

However, THP can minimize the performance degradation due to the norm-based selection by adapting interference pre-subtraction at the transmitter. Therefore, the sum-rate with the proposed selection rule using THP closely follows that of an optimally selected case using the sum-rate maximization rule and THP.

Fig. 5 compares the complexities between the proposed selection rule and the sum-rate maximization rule in terms of CPU times, generated by the command 'cpu-time' in MATLAB (Pentium IV 3.0 GHz PC), which is commonly used for measuring the run-time of various scheduling algorithms [5]. It is obvious that the proposed selection rule requires much less CPU time than the sum-rate maximization rule, regardless of the number of transmit antennas M_T . The computational complexity of the sum-rate maximization rule increases rapidly as the number of transmit antennas and users increases, which makes real-time implementation very difficult in realistic multiuser MIMO BC scenarios. On the contrary, the CPU time of the proposed selection rule is not affected by the number of transmit antennas because it always computes the effective channel norm of all users, irrespective of the number of transmit antennas. Indeed, an increase in the computational complexity of the norm-based selection is negligible as the number of users grows. This makes practical implementation feasible at the expense of reduced sum capacity compared to the sum-rate maximization rule.

V. Conclusions

In this paper, a low complexity multiuser scheduling algorithm consisting of a statistical CSI-assisted norm-

based selection rule and THP was proposed. Numerical results showed that the proposed algorithm can reduce the computational complexity associated with finding a user set by using an effective channel norm, and can minimize the performance degradation by using THP in time-varying MIMO broadcast channels. This provides a trade-off between performance and complexity from the viewpoint of practical implementation.

However, further work should be performed to consider fairness as well as complexity in selecting users for time-varying MIMO broadcast channels.

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