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# Two Dimensional Chloride Ion Diffusion in Reinforced Concrete Structures for Railway

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**Abstract**

Chloride ion diffusion at the corner of rectangular-shaped concrete structures is presented. At the corner of rectangular-shaped concrete, chloride ion diffusion is in two-dimensional process. Chloride ions accumulate from two orthogonal directions, so that corrosion-free life of concrete structures is significantly reduced. A numerical procedure based on finite element method is used to solve the two-dimensional diffusion process. Orthotropic property of diffusion coefficient of concrete is considered and chloride ion profile obtained from numerical analysis is used to produce transformed diffusion coefficient. Comparisons of experimental data are also carried out to show the reliability of proposed numerical analysis. As a result of two-dimensional chloride diffusion, corrosion-free life of concrete structure for railway is estimated using probability of corrosion initiation. In addition, monographs that produces transformed diffusion coefficient and corrosion-free life of concrete structure are made for maintenance purpose.

***Key words :*** *Chloride-induced corrosion of reinforcement, 2D Chloride diffusion in concrete, Corrosion-free life of reinforced concrete structure*

## 1. Introduction

Deterioration of plain and reinforced concrete structures due to physical and chemical attack has been major concerns in civil engineers. The idea that reinforced concrete structures are durable maintenance-free construction material has been threaten in recent years, and it has been shown that such deterioration mechanisms like carbonation, alkali-silica reaction, chloride ion diffusion, and reinforcing steel corrosion can cause the performance of reinforced concrete structures to be much lower than expected.

One of the main causes for deterioration of concrete structures attributes to chloride-induced corrosion of reinforcing steel. The major sources of the chloride ions are the deicing salts applied to bridge and roadway during winter traffic maintenance and the sea water. Chloride ions penetrate the concrete and cause reinforcing steel to corrode. The corrosion products, occupying more volume than the original steel, produce tensile stresses that cause cracking and spalling of the concrete. In alkaline environments ($pH > 11.5$), reinforcing steel is normally passivated by the formation of ferric oxide, which prevents further oxidation (Tutti, 1980). Concrete presents just such an environment. However, chloride ions present in concentrations in excess of threshold level destroy passivation and promote the development of corrosion cells.

Chloride diffusion in concrete is complex phenomenon involving various influence factors such as chloride diffusion coefficient of concrete and surface chloride ion concentration, etc. The basic parameters that should be considered when studying chloride diffusion in concrete and related risk of corrosion of reinforcing steel are mainly the diffusion characteristics of concrete.

Many researchers have attempted to model chloride diffusion in concrete, but the scope of theses works were generally limited to particular problems such as one-dimensional diffusion process and other broad simplifications. In most cases, one-dimensional diffusion modeling is sufficient. However, chloride diffusion at the corner of rectangular-shaped concrete structure, one-dimensional modeling of chloride ion diffusion will certainly underestimate the amount of chloride ions in concrete. For the

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accurate prediction of corrosion-free life, two-dimensional modeling is necessary.

The complexity of the mathematical formulation of two-dimensional chloride diffusion process in concrete makes it necessary to apply numerical method. This paper considers numerical procedure to model two-dimensional chloride ion diffusion at the corner of rectangular-shaped concrete. For the sake of simplicity, the result of two-dimensional diffusion process is expressed by transformed diffusion coefficient. Transformed diffusion coefficient represents the effect of two-dimensional diffusion process in one spatial dimension. It is acquired by least square fit analysis to the chloride ion profile obtained from numerical analysis of two-dimensional diffusion process. Using transformed diffusion coefficient, the effect of two-dimensional diffusion process can be expressed by remaining service life to the onset of corrosion. The ability to predict the probable extent of steel corrosion due to two-dimensional chloride diffusion process and corrosion-free life of structure would enhance the bridge custodian's ability to schedule the maintenance and to plan.

## 2. Mathematical Modeling

The one-dimensional chloride ion diffusion in a porous concrete media can be viewed to follow Fick's second law of diffusion as

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) \tag{1}$$

where, $C(x, t)$ is the chloride concentration at a point $x$ at time $t$; $D$ is the chloride diffusion coefficient of concrete; and $x$ is the spatial coordinate. Eq (1) can be solved by assuming constant diffusion coefficient and the following conditions.

$$C(x,0) = 0,\ x>0 \tag{2}$$

$$C(0,t) = C_o,\ t\geq 0 \tag{3}$$

$$C(\infty,t) = 0,\ t\geq 0 \tag{4}$$

The solution to above boundary value problem is then

$$C(x,t) = C_o\left[erfc\left(\frac{x}{2\sqrt{D_{ac}t}}\right)\right] \tag{5}$$

where, $C(x, t)$ is the chloride ion concentration at a depth $x$ from the surface, $C_o$ is the surface chloride ion concentration, and $D_{ac}$ is the apparent diffusion coefficient. In practice, Eq (5) is used to calculate apparent diffusion coefficient by least square fit analysis with chloride ion profiles obtained either in cores taken from real structures or in specimens tested in the laboratory.

Chloride diffusion at the corner of rectangular-shaped concrete structure such as diffusion at the corner of rectangular-shaped bridge pier, diffusion process is in two-dimensional. If it can be assumed that chloride diffusion takes place in parallel planes and these planes are taken parallel to the $x$-$y$ plane, the governing differential equation is given as

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D_x\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y\frac{\partial C}{\partial y}\right) \tag{6}$$

where, $D_x$ and $D_y$ are the diffusion coefficients in $x$ and $y$ directions, respectively. Unlike the one-dimensional diffusion process, there are very few analytical solutions of Eq (6) and these solutions must have certain boundary condition. H.S. Carslaw (1986) provides product solution of Eq (6) with unit initial chloride ion concentration and zero surface chloride ion concentration.

$$C(x,y,t) = erf\left[\frac{x}{2\sqrt{D_x t}}\right]erf\left[\frac{y}{2\sqrt{D_y t}}\right] \tag{7}$$

In general, the analytical solution of Eq (6) with various boundary conditions can not be simply obtained. Instead, Eq (6) must be numerically integrated.

## 3. Finite Element Implementation

To formulate finite element scheme of two-dimensional diffusion equation, standard residual procedure (Galerkin°Øs method) is introduced. Applying standard residual procedure on Eq (6) gives

$$\int_\Omega w_i\left[\frac{\partial C}{\partial t} - \frac{\partial}{\partial x}\left(D_x\frac{\partial C}{\partial x}\right) - \frac{\partial}{\partial y}\left(D_y\frac{\partial C}{\partial y}\right)\right]dxdy = 0 \tag{8}$$

where W is the problem domain and $w_i$ is the weighting function. Using interpolation function as weighting function ($w_i = N_i$) and Green's formulae to eliminate the second spatial derivatives of $C$

$$\int_\Omega\left[\frac{\partial N_i}{\partial x}D_x\frac{\partial C}{\partial x} + \frac{\partial N_i}{\partial y}D_y\frac{\partial C}{\partial y} - N_i\frac{\partial C}{\partial t}\right]dxdy$$

$$-\oint_\Gamma N_i\left[D_x\frac{\partial C}{\partial x}n_x + D_y\frac{\partial C}{\partial y}n_y\right]d\Gamma = 0 \tag{9}$$

where $\Gamma$ is the boundary of domain and $n_x$ is the cosine between outward normal and $x$ axis. Introducing the operator $\nabla$ and the orthogonal diffusion property matrix **D**, Eq (9) is rewritten as

$$\int_\Omega\left(\nabla^T N_i\mathbf{D}\nabla C + N_i\frac{\partial C}{\partial t}\right)dxdy - \oint_\Gamma N_i(\nabla^T C\mathbf{D}\mathbf{n})d\Gamma = 0 \tag{10}$$

where $\nabla$ and $\mathbf{D}$ are defined as

$$\nabla = \left\{ \frac{\partial}{\partial x} \ \frac{\partial}{\partial y} \right\}^T \ and\ \mathbf{D} = \begin{bmatrix} D_x & 0 \\ 0 & D_y \end{bmatrix}$$

Applying finite element approximation with the interpolation function $N_i$, Eq (10) becomes

$$C = \mathbf{N}(x,y) \cdot \mathbf{c}(t) = \mathbf{N} \cdot \mathbf{c}$$

and

$$\int_\Omega (\mathbf{B}^T\mathbf{D}\mathbf{B})dxdy \cdot \mathbf{c} + \int_\Omega (\mathbf{N}^T\mathbf{N})dxdy \cdot \frac{\partial \mathbf{c}}{\partial t}$$

$$-\oint_\Gamma N_i(\nabla^T C\mathbf{D}\mathbf{n})d\Gamma = 0 \qquad (11)$$

The latter equation in Eq (11) can be rewritten in the following form

$$\mathbf{V}\dot{\mathbf{c}} + \mathbf{K}\mathbf{c} = \mathbf{f} \qquad (12)$$

where, $\mathbf{V} = \int_\Omega (\mathbf{N}^T\mathbf{N})dxdy; \mathbf{K} = \int_\Omega (\mathbf{B}^T\mathbf{D}\mathbf{B})dxdy$ and $\mathbf{f}$ is the vector of external forces given by the integration of the boundary condition.

The ordinary differential equation of Eq (12) can be numerically integrated in time domain by single time-step algorithms, applying generalized trapezoidal method. Introducing discrete time step as $\Delta t = t_{n+1} - t_n$, Eq (12) is rewritten as

$$\mathbf{V}\dot{\mathbf{c}}_{n+1} + \mathbf{K}\mathbf{c}_{n+1} = \mathbf{f}_{n+1} \qquad (13)$$

$$\mathbf{c}_{n+1} = \mathbf{c}_n + \Delta t \dot{\mathbf{c}}_{n+\alpha} \qquad (14)$$

$$\dot{\mathbf{c}}_{n+\alpha} = (1-\alpha)\dot{\mathbf{c}}_n + \alpha\dot{\mathbf{c}}_{n+1} \qquad (15)$$

Taking Eq (15), Eq (14) becomes:

$$\mathbf{c}_{n+1} = [\mathbf{c}_n + \Delta t(1-\alpha)\dot{\mathbf{c}}_n] + \alpha\Delta t\dot{\mathbf{c}}_{n+1}$$

$$= \mathbf{c}_{tr} + \alpha\Delta t\dot{\mathbf{c}}_{n+1} \qquad (16)$$

where, $\mathbf{c}_{tr}$ is the predictor value.

Taking Eq(16) into Eq (13) and solve for $\dot{\mathbf{c}}_{n+1}$ yields

$$\dot{\mathbf{c}}_{n+1} = \frac{\mathbf{f}_{n+1} - \mathbf{K}\mathbf{c}_{tr}}{\mathbf{V} - \alpha\Delta t\mathbf{K}} \qquad (17)$$

Substituting Eq (17) into Eq (16) and solve for $\mathbf{c}_{n+1}$ gives

$$\mathbf{c}_{n+1} = \frac{\mathbf{V}\mathbf{c}_{tr} + \alpha\Delta t\mathbf{f}_{n+1}}{\mathbf{V} + \alpha\Delta t\mathbf{K}} \qquad (18)$$

Eq (18) contains wide range of numerical integration scheme: when ↦ = 0.5, integration scheme is Crank-Nicholson method; when ↦ = 0, integration scheme is forward difference method; and when ↦ = 1.0, integration scheme is backward difference method. Note that ↦ must be greater than 0.5 to achieve unconditional stability. In this paper, Crank-Nicholson method is used for accuracy and unconditional stability.

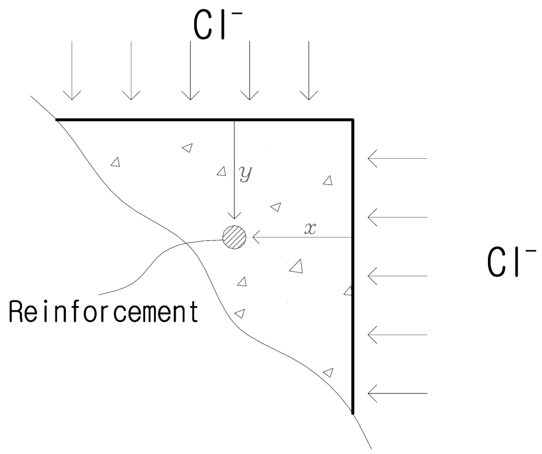


**Fig. 1** Physical model of 2D diffusion process

## 4. Numerical Applications

The numerical example of two-dimensional diffusion process yields chloride ion profile at the corner of rectangular-shaped concrete structure. In the numerical example, constant surface chloride ion concentration is assumed and two properties of diffusion; isotropic and orthotropic diffusion coefficients of concrete, are assumed. Fig. 1 shows physical model of two-dimensional diffusion process at the corner of rectangular-shaped concrete structure.

Fig. 2 shows the chloride ion profile obtained by assuming constant surface chloride ion concentration of 0.1% by weight percentage of concrete, the constant diffusion coefficient of 1.0 $cm^2$/yr. in both x and y directions and the service life of 20 years. Fig. 2 also shows severe underestimation of chloride ion concentration when chloride diffusion at the corner of rectangular-shaped concrete is treated as one-dimensional diffusion process. This, in fact, will cause considerable overestimation of corrosion-free life of reinforced concrete structure. Fig. 3 shows the field data of chloride profile from concrete bridge. Riple and Suprenant (1992) investigated chloride content in concrete bridges in Colorado for the verification of their computer program, which calculate chloride content in concrete and future concrete deterioration. Dotted line in is the field data of chloride content measured at the corner of concrete bridge pier and solid line is the numerical approximation using one and two-dimensional diffusion process. As can be

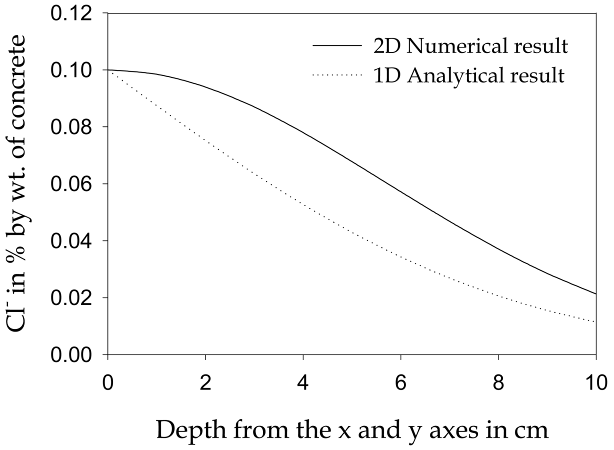


**Fig. 2** $Cl^-$ profile at the corner of rectangular-shaped concrete

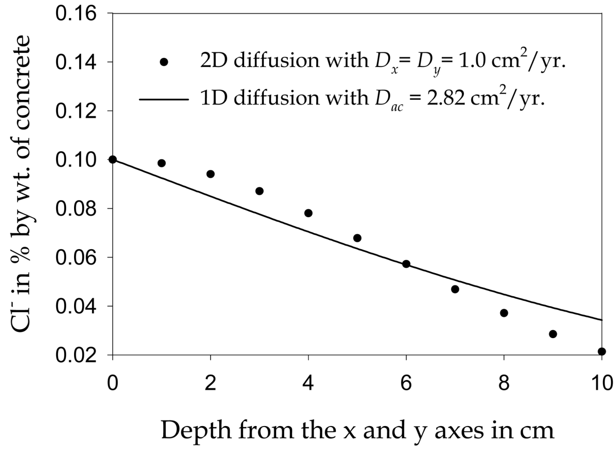


**Fig. 4** Example of transformed diffusion coefficient

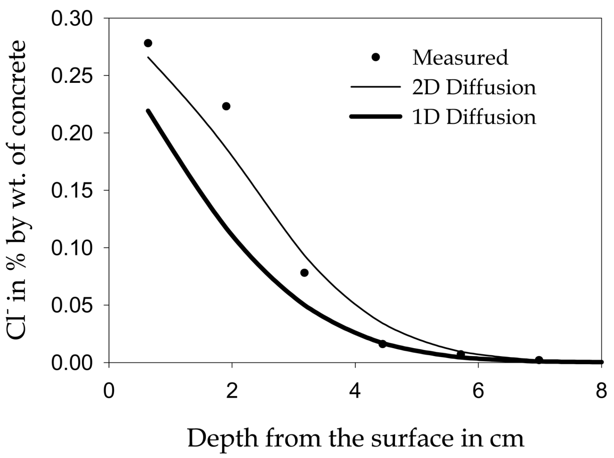


**Fig. 3** Chloride profile from concrete bridge in field

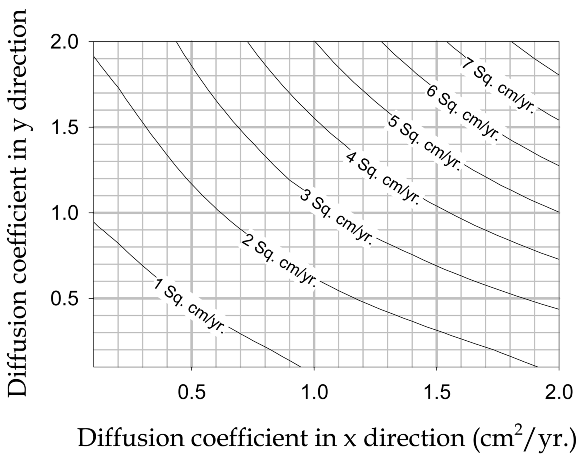


**Fig. 5** Transformed diffusion coefficient in concrete

seen, one-dimensional diffusion process can not capture chloride ion profile, whereas actual field data of chloride profile is in good agreement with two-dimensional diffusion process.

### 4.1 Transformed Diffusion Coefficient

The modeling of two-dimensional diffusion process is quite expensive and also unpractical. Capturing of this phenomenon with relatively cheap computational effort may be useful in practical use. For this purpose, transformed diffusion coefficient is proposed. Transformed diffusion coefficient is the chloride diffusion coefficient of concrete that capture two-dimensional diffusion process in one spatial dimension. It is acquired by least square fit analysis to the chloride ion profile obtained from numerical analysis of two-dimensional diffusion process. The chloride ion profiles are obtained by varying the orthotropic diffusion coefficients of concrete in $x$ and $y$ directions. Using these profiles, transformed diffusion coefficients are obtained by least square fit analysis with Eq (5).

Fig. 4 shows how the transformed diffusion coefficient can be calculated. The dotted line is the chloride ion profile from two-dimensional diffusion process with orthotropic diffusion coefficient of concrete as 1.0 cm$^2$/yr in both $x$ and $y$ axes and the solid line is the least square fit using Eq (5). The least square fit analysis with Eq (5) produces transformed diffusion coefficient of 2.82 cm$^2$/yr. Although, transformed diffusion coefficient can not exactly capture chloride profile for two-dimensional diffusion process, it captures general trend with relatively easy mathematical computation.

Fig. 5 shows the transformed diffusion coefficient for different orthotropic diffusion coefficient of concrete. Using this figure, two-dimensional chloride diffusion problem can easily be solved with one-dimensional diffusion equation of Eq (5). The use of transformed diffusion coefficient may extend to estimate the corrosion-free life of concrete structure.

### 4.2 Corrosion-free Life

The estimation of corrosion-free life of concrete structure would help the bridge engineers for their maintenance plan and allow a means of evaluating the success of various repair alternatives. However, estimating the time to onset of corrosion is not an easy task. The chloride threshold value to develop active corrosion of reinforcing steel does not seem to be the unique value and it depends on several influence factors such as concrete mix proportions, cement type, $C_3A$ content of cement, w/c ratio, temperature, relative humidity, etc. Numerous researches have been conducted to estimate the chloride threshold value for the depassivation of reinforcing steel in concrete (Alonso et al 1999; Funahashi 1990; Pfeifer et al 1987). Using data in these literatures on chloride threshold value, the first estimate of the probability density function of chloride threshold is formed. This distribution of chloride content has a mean value of 0.035% (by weight percentage of concrete) and a standard deviation of 0.012%. Note that the pdf of chloride threshold proposed here is for average concrete quality and average service conditions in highway bridges. This pdf will not apply to other structures, to other material or material quality.

Using the pdf of chloride threshold and the transformed diffusion coefficient, corrosion-free life of rectangular-shaped concrete structure can be estimated. Notice from Eq (5) that if *C(x, t)* is taken to be the chloride threshold for the onset of corrosion, $x$ the cover depth of reinforcing steel, $D_{ac}$ the transformed diffusion coefficient, then $t$ is the corrosion-free life of concrete structure. Solving Eq (5) for $t$ gives:

$$T_{cor} = \frac{d^2}{4D_T erf^{-1}\left[1-\frac{c_t}{C_o}\right]^2} \quad (15)$$

where, $T_{cor}$ is the corrosion-free life of concrete structure, $d$ is the cover depth of reinforcing steel at the corner of rectangular-shaped concrete, $D_T$ is the transformed diffusion coefficient, and $c_t$ is the chloride threshold value for the onset of corrosion. Since chloride threshold value in Eq (15), $c_t$, is expressed as probability density function, $f(c_t)$, corrosion-free life of concrete structure is also be expressed as probability distribution, $f(T_{cor})$. The mean value of possible distribution of $f(T_{cor})$ is:

$$E(T_{cor}) = \int_{-\infty}^{\infty} \frac{d^2 f(c_t)}{4D_T erf^{-1}[1-c_t/(C_o)]^2} dc_t \quad (16)$$

The variance of possible distribution of $f(T_{cor})$ is:

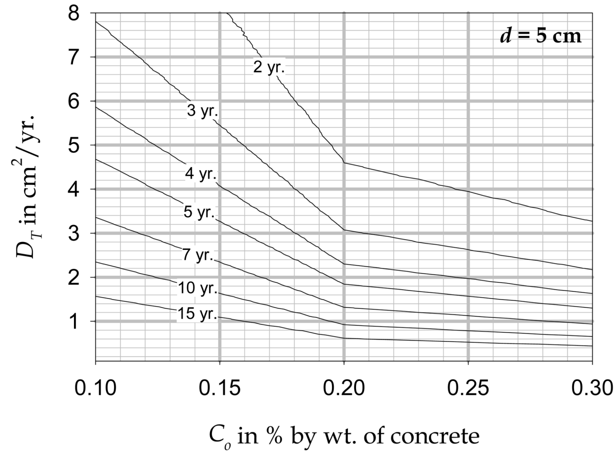


**Fig. 6** Corrosion-free life of concrete structure [$d$ = 5 cm]

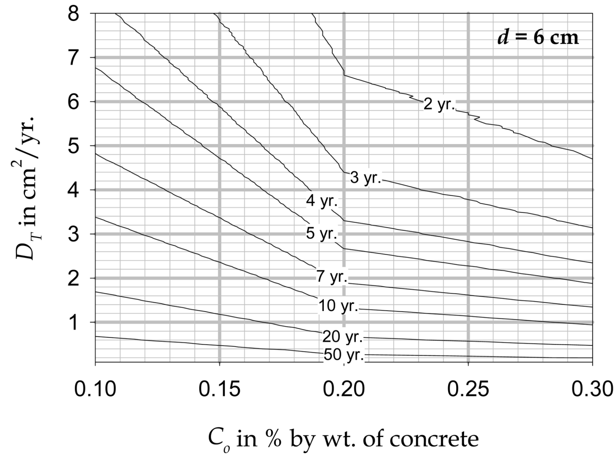


**Fig. 7** Corrosion-free life of concrete structure [$d$ = 6 cm]

$$Var(T_{cor}) = E(T_{cor}^{\;2}) - E(T_{cor})^2$$

$$= \int_{-\infty}^{\infty}\left[\frac{d^2}{4D_T erf^{-1}[1-c_t/(C_o)]^2}\right]^2 f(c_t)dc_t - E(T_{cor})^2 \quad (17)$$

For given values of orthotropic diffusion coefficients of concrete, transformed diffusion coefficient can be found from Fig. 5 and the possible distribution of corrosion-free life of concrete structure can then be formed using Eq (16) and Eq (17). Fig. 6 through Fig. 9 show the corrosion-free life of concrete structure as a function of transformed diffusion coefficient and surface chloride ion concentration. Here, corrosion-free life is estimated using 90% probability that corrosion of reinforcing steel at the corner of rectangular-shaped concrete commences after $T_{cor}$ years from construction. With the aid of Fig. 5 through Fig. 9, corrosion-free life can easily be estimated. As an example, cor-

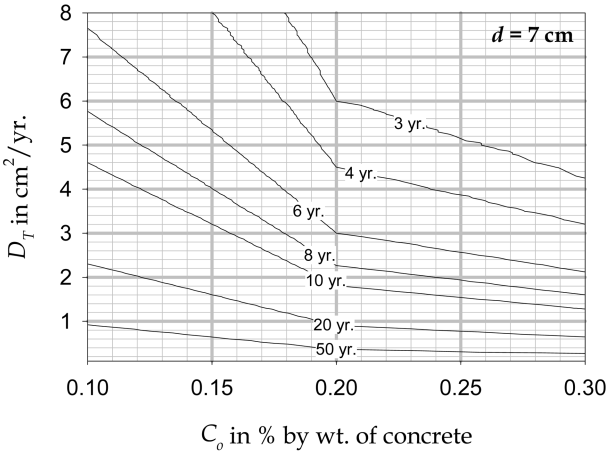


**Fig. 8** Corrosion-free life of concrete structure [$d$ = 7 cm]

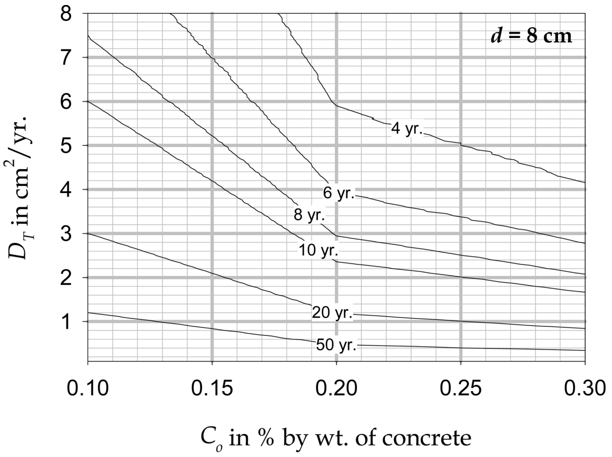


**Fig. 9** Corrosion-free life of concrete structure [$d$ = 8 cm]

rosion-free life of rectangular-shaped concrete structure having orthotropic diffusion coefficients of 0.1 $cm^2$/year in both $x$ and $y$ axes, constant surface chloride ion concentration of 0.2%, and reinforcing steel cover of 5 cm ($x = y = 5$ cm in Fig. 1) is evaluated. From the Fig. 5, transformed diffusion coefficient is found to be 2.8 $cm^2$/yr. and from Fig. 6, corrosion-free life is estimated as 3.3 years. If the diffusion process is treated as one-dimensional process, corrosion-free life of the same structure would increase to 9.2 years. This example shows significant overestimation on corrosion-free life when chloride diffusion process at the corner of rectangular-shaped concrete is treated as one-dimensional problem. The decreasing percentage of corrosion-free life in two-dimensional diffusion process relative to one spatial dimension is shown in Fig. 10. As diffusion coefficient of concrete increases, the percentage of decrease in corrosion-free life of concrete structure increases.

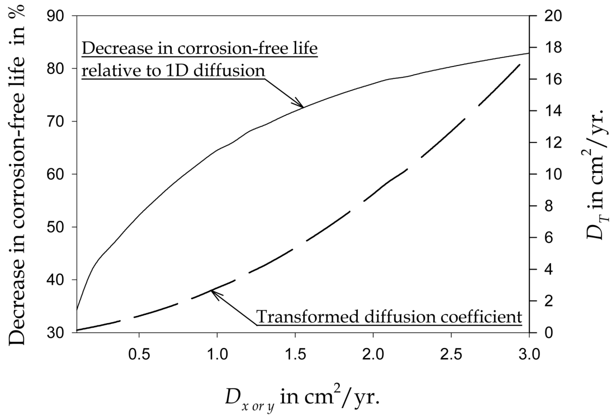


**Fig. 10** Decrease in corrosion-free life

## 5. Conclusion

Two-dimensional chloride diffusion at the corner of rectangular-shaped concrete structure is investigated using numerical procedure. The results obtained from numerical analysis and on comparison with field data indicate that chloride diffusion at the corner of rectangular-shaped concrete structure is in two-dimensional process and one-dimensional treatment of this problem results in considerable overestimation of corrosion-free life of structure. For the maintenance of reinforced concrete structure undergoing corrosion of reinforcing steel, the most important parameter estimated is the time to onset of corrosion, since corrosion related damages like spalls or delaminations occur soon after corrosion initiation. In this sense, the transformed diffusion coefficient and monographs proposed here may be useful for accurate estimation of corrosion-free life. With the better definitions of parameter values used in numerical analysis, more realistic chloride diffusion process at the corner of rectangular-shaped concrete can be simulated.

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