Journal of the Korea Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 15, No. 1, March 2011, 93–104

Just Maybe the Problem Lies in Reading the Pictorial Model[1]

PATTON, Barba
School of Education & Human Development, University of Houston –Victoria,
3007 N. Ben Wilson, Victoria, TX 77901, USA; Email: Pattonb@uhv.edu

(Received February 16, 2011, Revised March 26, 2011, Accepted March 28, 2011)

Sixty-four EC-6 teacher candidates were asked to elaborate on the method they used when interrupting seven mathematical pictorial models. The instrument used was research designed but modeled after a 4th grade released state tests. This study was conducted in the southern part of the United States. A panel of experts worked with the researcher to determine the most appropriate initial approach to interrupting the mathematical pictorial model. The pictorial models examined such concepts as measurement, place value and time. Since the questions were on a 4th grade level, it was expected that the teacher candidates would score 100%, however, this was not the case. The results will be given in the paper as each question is discussed. The results of the survey are being used to develop teaching modules for teacher candidates in mathematics methodology classes.

Keywords: teacher education, instruction of mathematics and interpretation of mathematical models
MESC Classification: B50; C70; D40
MSC2010 Classification: 97C70

INTRODUCTION

A trend exists for the government mandated mathematics tests in the United States of America to contain many situational problems accompanied with sketches or pictorial representation which provide additional information. The successful student will be one who is able to interpret the situational problem or text and evaluate the pictorial representation or sketch accompanying the text for additional information. The pictorial representation or sketch may require the learner to metacognitively process the sketches or pictorial representation in a variety of ways that contradicts how one reads informational

[1] This paper will be presented at the 46th Korean National Conference of Mathematics Education at Soongsil University, Dongjak-gu, Seoul 156-743, Korea; April 1–3, 2011.

or narrative text. Some sketches or pictorial representations may require the student to begin on the right and read to the left such as when a given place value problem while others require the student to even read around and around to determine the elapsed time problem when clock faces are provided such as the problem in number four. In order to effectively teach elementary mathematics, teacher candidates must develop and be fluent in mathematics content skills (Locangeli & Cornoldi 1997; Franke, Carpenter, Fennema, Answlll & Behrend, 1998).

This study was developed to examine the methods the Early Childhood – 6th grade teacher candidates utilize when attempting to interpret sketches or pictorial representations which often accompany the situational problems. Many teacher candidates do not see themselves as actively engaging in the creation of mathematical knowledge. Rather, they view a mathematics teacher as someone who merely delivers facts, procedures and mathematical formulas that should be memorized (Seaman, Nolan, & Corbin-Dwyer, 2001; Corbin-Dwyer & Patterson, 2001). Many teacher candidates maintain this traditional belief about the teaching and learning of mathematics (O'Laughlin, 1990; Wilson, 1990). Unfortunately, interrupting the sketches and pictorial representations cannot be memorized or viewed as a lower level thinking skill; it requires the teacher candidate to be able to apply knowledge. Thus it is very important the teacher candidate has a good to strong background in the mathematical content he/she is responsible for teaching. The teacher candidate must be able to answer correctly when student question "why do we do it this way?" The teacher must have enough knowledge of mathematics to not use little tricks which only work in a few cases. He/she must be able to present and teach concepts correctly as to not to present problems when a concept generalizations is needed.

Mathematics methods courses can have an influence on the perceptions of teacher candidates and the pedagogical practices they employ in the classroom (Ball, 1990; Simon & Schifter, 1991; Wilcox, Schram, Lappan, & Lanier, 1991). How a teacher candidate views his/her mathematics knowledge, understands the metacognitive nature of mathematics, and has confidence in the content area of mathematics will impact the pedagogical practices employed in the classroom as well as the content knowledge which is made available for the learner. One factor which greatly contributes to the pedagogical practices employed in the classroom is the teacher's personal experience which influences how mathematics teaching and learning is viewed (Brown and Borko, 1992; Ebby, 2000; Giroux, 1982; Lortie, 1975). With such personal experiences contributing to weak mathematical knowledge and narrow views of mathematical pedagogy, it is no wonder many teacher candidates see mathematics teaching as the telling and gathering of information (Ball, 1991; Borchetta & Dunn, 2010; Brown, Cooney, & Jones, 1990; Even, 1993; Frykholm, 1996; Thompson, 1992).

Much of mathematics prior to about 1980 was taught by the memorization method and

as a result, teachers in general do not know how to read sketches or pictorial representations. However, much as changed. The teacher of today is expected to teach for understanding of concept. These methods of yester year are not appropriate for the twenty-first century. This study was part of several which were developed to provide background information for teaching modules for the teacher candidates in mathematics concepts where appropriate.

METHODS

Setting

The teacher candidates participating in this study were all admitted into the Teacher Education Program at a university in South Texas in the United States of America. Sixty-four teacher candidates who were enrolled in a mathematics methodology class participated in the study.

The Participants

Teacher candidates participating in this study ages ranged from twenty to sixty years of age.	• 68% Aged 20-29 • 21% Aged 30-39 • 9% Aged 50-59 • 2% aged 60 or above
The ethnicity of the teacher candidate	• Caucasian – 67% • Hispanic – 24% • African American – 6% • Asian – 2% • Other – 1%

Instruments

The researcher found by doing an internet search a number of states in USA required achievement type tests to all students. Many states release previously used test items and these are listed for a time on their respective websites. The group of seven questions used in the study was selected from this research. A pilot study was conducted and revisions were made accordingly.

PROCEDURE

The participants (teacher candidates) were asked to work seven problems and to elaborate on the manner in which they initially approached the sketch or pictorial representa-

tion the problem thus reaching a solution while interpreting the pictorial representation that accompanied the problem. The questions were modeled after released 4th grade questions on the state mandated achievement test. Since these teacher candidates will be in an EC-6 classroom in the near future, the level was selected with the thought that these teacher candidates would become more qualified to teach in the EC-6 mathematics classroom if given the opportunity to take the same test as their future elementary students took.

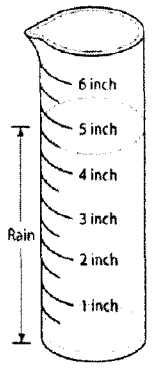
A panel of experts was used to equate the initial manner to approach the pictorial representation in order to solve the problems. The panel did not say the way selected was the best or the only way to read the pictorial representation. The panel did however; advocate that there is usually one and only one initial method. The initial way is one that is usually most successful in reaching most of the students in the math classroom. If that initial method was not successful with the students, then other methods should be utilized according to the panel.

The panel consisted of ten graduate students who were in the final course of a Master Mathematics Teacher program. Each member was currently teaching in a public school setting and had five or more years experience teaching mathematics. Each of the teachers was in a voluntary graduate program and had a very positive outlook about the teaching of mathematics. This panel had over 75 years combined experience in the classroom teaching mathematics. Each member of the panel was in an administrative mathematics position in their respective school district or campus. The combined years of experience in the mathematics classroom indicates that the panel members is respected school administrators and most likely have a positive outlook about mathematics.

RESULTS

Each of the problems was modeled closely to Texas (USA) 4th grade state achievement test released items. The panel of experts achieved 100% correct on all of the mathematical situational problems and agreed 98% on which was the correct way to read the information in the picture thus solving the problem.

Problem 1.

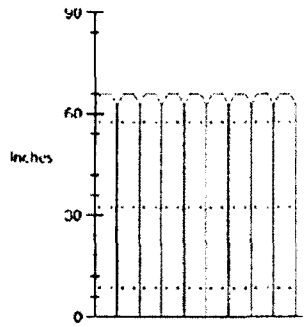
Last night, it rained at Jose's home. He checked the rain gauge this morning to see the amount of rainfall. How much did he find in the gauge? • 4 inches • 4 ½ inches • 5 inches • 5 ½ inches		When you were working on the solution to the question did you read the required information in the picture? • Left to right • Right to left • Top to bottom • Bottom to top

The teacher candidates were highly 100% successful with this however; only approximately 47% stated that they approached the problem in the best initial manner according to the panel of experts as bottom to top. Approximately 53% of the participants stated that they approached the problem in inappropriate different manner. Of course, due to learning styles there may always be some who will approach a problem from a different angle but a percentage of this size it may not be just the variation due to learning styles (Borchetta and Dunn). Since the teacher candidates were not interviewed about their answers, it would be pure speculation to say why the participants answered the way they did. Thus we will need to address this in a future study.

The panel expressed that the proper way to solve this one was to read the measurements from the bottom to the top or from 1, 2, 3,... When given a pictorial representation such as this one, if students do not read from the bottom to the top, they will most likely not read the amount in the beaker correctly but what it appears to be since the pictorial representation has a three-dimensional appearance. It is much easier for almost anyone to start either at one or at the whole number just below the exact answer on any pictorial representation such as this one. The correct answer is 4 ½ inches.

Problem 2 proved to be difficult for the teacher candidates even though the problem was on a 4th grade level. The percentage of teacher candidates who answered the question correctly was 80; however, 53% stated they approached the problem in the best initial manner according to the panel of experts as from bottom to top. The correct answer was 66 inches. Forty-seven percent of the subjects stated that they approached the problem in an assumed inappropriate manner.

Problem 2

Mary's Dad was planning to put a new fence around the back yard. He measured the fence at the right at the improvement center. How many inches tall will the fence be if he uses the fencing that he measured at the home improvement center? • 60 • 66 • 70 • 75		When you were working on the solution to the question did you read the required information in the picture? • Left to right • Right to left • Top to bottom • Bottom to top

This one involved several strategies and the complexity might have been part of the reason for the difficulties experienced. One would need to start from the bottom to the top but one would also need to realize that each of the marked intervals was 30 units. Students, young (inexperienced) and even older (more experienced), expect all measurement intervals to be in either 1 or 2 unit increments (Nagel, 2007, personal communication). Possibly this is something that comes from the early graphing experiences in the EC-1 grade classroom where the students and teacher create bar graphs based on one-to-one correspondence, etc. Whatever the reason, the student of today is faced with complex questions similar to the one above each year on the state achievement test thus we must have teachers in the classroom prepared to help those students prepare for the state test.

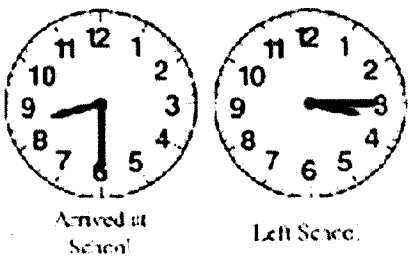
Problem 3

How much money does Maria have? 79 cents • 84 cents • $2.79 • $2.84		When you were working on the solution to the question did you read the required information in the picture? • Left to right • Right to left • Top to bottom • Bottom to top • Largest to smallest

While 92% of the participants (teacher candidates) answered the question correctly, 53% stated they approached the problem in the best manner according to the panel of

experts as largest to smallest. The amount shown in the pictorial representation is $2.84. These teacher candidates will be in the classroom very soon, therefore it is a great concern that approximately half of the teacher candidates stated they approached the problem in a manner other than the one identified by the panel as the best initial approach. The panel agreed if the student starts counting with the larger in the pictorial representation and in value amount and moves to next in size, the student will usually be able to determine the correct amount easily. This is simply an add-on basic counting activity which students are taught in the first and second grades (Nagel, 2008 personal communication).

Problem 4

Millie arrived at her Grandmother's house at 8:30 AM and left for home at 3:15PM. How much time did she spend at Grandmother's? • 5 hrs. 15 min. • 5 hrs. 45 min. • 6 hrs. 15 min. • 6 hrs. 45 min.	 Arrived at School Left School	When you were working on the solution to the question did you read the required information in the picture? • Left to right • Right to left • Top to bottom • Bottom to top • Middle to outside • around and around

Although one would expect the teacher candidates to be 100% correct, only 84% answered the problem correctly and only 39% stated they approached the problem in the best initial manner according to the panel of experts. The experts stated they believed the best initial approach was 'around and around'. The correct amount of elapsed time was 6 hours and 45 minutes. While it is informative, it is alarming that 61% of the subjects stated they approached the problem in an assumed inappropriate manner. Since these teacher candidates are only months from being eligible to be the teacher of record, it is very alarming as these problems were closely modeled after 4th grade released test items. Many of the teacher candidates stated they read the information from left to right or one of the other possibilities however, there were many loops drawn on their clocks to indicate they really did 'around and around' type method to solve the problem.

The panel of experts felt the best initial method to approach this one was to start at the time on the first clock and have the students to add one hour at a time until almost reaching the time on the second clock. Then proceed with minutes until the final amount of time is determined. Students were provided with hands-on clocks which have movable hands to use as manipulatives when they were learning to work with time and elapsed

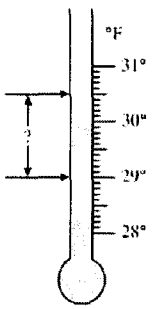
time. Thus the students were often able to transition from the concrete to the abstract but may continue with this 'around and around' method.

Problem 5

In the number to the right, what number is in the hundreds place? • 1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9	123,456,789	When you were working on the solution to the question did you read the required information (123,456,789)? • left to right • right to left • top to bottom • bottom to top

Almost every teacher candidate answered this problem correctly as 94% provided the correct response and 78% stated they approached the problem in the best initial manner according to the panel of experts. The panel identified the right to left approach as best initial. It is noteworthy that in the seven questions, the all teacher candidates responded on each of the questions except this one and four skipped it. This is a place value question and place value is determined from least to greatest or right to left. Even with this high percentage correct, still 22% of the subjects stated they approached the problem in an assumed inappropriate manner. It is fearful that these teacher candidates who have not internalized the concept of place value, if placed in a classroom will not be able to teach their students appropriately.

Problem 6

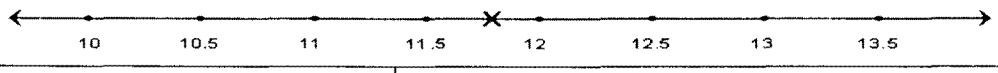
It was 29°F early this morning and the temperature shown is at 3 PM. What was the difference in the two temperatures? • 1.5 degrees • 1.4 degrees • 1.2 degrees • 1.1 degrees		When you were working on the solution to the question did you read the required information in the picture? • Left to right • Right to left • Top to bottom • Bottom to top

Only 85% of the teacher candidates answered the question correctly however, 75%

stated they approached the problem in the best initial manner according to the panel of experts. The panel advocated that the pictorial representation be approached from bottom to top. However, 25% of the participants stated they approached the problem in an assumed inappropriate manner.

The thermometer reading is not an easy as one would like to believe. Again the interval is not in single units, thus forcing the reader to not only look at the total change but to determine the value of each of the tick marks between the given intervals. The complexity of this problem may have been some of the reason the teacher candidates were not able to be 100% correct. If this was a problem for the teacher candidates who are adults, it will be an extremely difficult task for a young student and will need to have a prepared teacher in the classroom in order for that young student to master the likes of this problem.

Problem 7



What does the X represent?	When you were working on the solution to the question did you read the required information in the picture?
• Number smaller than 12 • Number larger than 12 • Number equal to 10 • Number equal to 13	• left to right • right to left • top to bottom • bottom to top • middle to the outside

Even though 98% of the participants answered the question correctly, only 26% stated they approached the problem in the best manner according to the panel of experts as from the middle to the outside. It is alarming that 74% of the teacher candidates stated they approached the problem in an assumed inappropriate manner.

The experts believed that if the student who was attempting to solve the problem were to first look at the location of the point in question and then he/she could focus on the readings on the number line to the left and right thus solving the problem correctly.

There is great concern as to how they will teach the students in their classrooms if even one teacher candidate does not approach the pictorial representations in the best initial manner.

CONCLUSIONS

The results of this study indicated a large difference between the teacher candidates and the expert teachers' strategies to interpret the mathematical pictorial representation. If

teacher candidates are to be successful in their teaching of mathematics, they need to be able to interpret pictorial representation accompanying mathematics problems. In the future, these teacher candidates will need to be able to construct many of their own pictorial representation (sketches) in order to provide illustrations for their students to solve complex problems. The teacher candidates could be the teacher of record in a few months and as such will be responsible for their students' ability to be able to answer correctly on the state or other achievement test.

The teacher candidates' methods of interpreting the pictorial representation significantly differed from the panel of experts. Their conditional knowledge about how to select appropriate strategies to adjust learning was inconsistent with the panel of experts. It is feared the teacher candidates will not be good mathematics teachers as a result.

Most of the teacher candidates were able to answer the 4th grade problems because they have the knowledge to answer the questions (declarative metacognition). However, many were not able to select the appropriate strategies to figure out the problem (procedural metacognition). For example, "Do I need to add or subtract to figure out the problem?" Since the mathematical problems were easy for the teacher candidates to solve and complete, effectively interpreting the illustration was not needed to select the correct answers. But, for a fourth grader interpreting both the situational problem and illustration will be difficult. All components of the metacognitive strategy process must be understood by the teacher and explicitly taught to the learners (Flavell, 1976; Doughtery, 1990).

A trend has existed for some time and appears will exist for a number of years in the future for the state academic tests to contain many situational problems accompanied with sketches or pictorial representation. The successful student will be one who is able to interpret the situational problem or text and evaluate the pictorial representation or sketch accompanying the text for additional information. The pictorial representation or sketch may require the learner to metacognitively process the sketches or pictorial representation in a variety of ways that contradicts how one reads informational or narrative text. Some may require the student to begin on the right and read to the left such as the given place value problem while others require the student to even read around and around to determine the elapsed time problem when clock faces are provided such as the problem in number four. In order to be an effective teacher the teacher candidate must be knowledgeable of these basic mathematics concepts and skills. In order to be able to effectively teach elementary mathematics, teacher candidates must develop mathematics content skills if the students in their respective classes to be successful.

REFERENCES

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *J. Res. Math. Educ.* **21(2)**, 132–144. ME **1990l.**00909 ERIC EJ409324

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter part of the equation. In: J. Brophy (Ed.), *Advances in Research on Teaching*, Vol. 2. (pp. 1– 48). Greenwich, CT: JAI. ME **1994a.**00078 ERIC ED301467

Borchetta, J. & Dunn, R. (2010). Helping Parents Understand How Children Learn. *Momentum* (Washington, D.C.) **41(2),** 40–43

Brown, C. A. & Borko, H. (1992). Becoming a mathematics teacher. In: D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillian

Brown, S.; Cooney, T. & Jones, D. (1990). Mathematics teacher education. In: W. R. Houston (Ed.), *Handbook of Research on Teacher Education*. New York, NY: Macmillan.

Corbin-Dwyer, S. & Patterson, D. (2001). Constructing portfolios together: Promoting Student/faculty metacognition. *Journal of Teaching and Learning* **1(1)**, 17–29.

Doughtery, B. J. (1990). Influences of teacher cognitive conceptual level on problem-solving instruction. In: G. Booker et al. (Ed.), *Proceedings of Fourteenth International Conference for Psychology of Mathematics Education* (pp. 119–126). Oaxtepec, Mexico: International study group For the Psychology of Mathematics Education.

Ebby, C. B. (2000). Learning to teach differently:The interaction between coursework and fieldwork for preservice teachers. *Journal of Mathematics Teacher Education* **3(1)**, 69–97. ERIC EJ607103

Even, R. D. (1993). Subject matter knowledge and pedagogical content knowledge: Prospective Secondary teachers and the function concept. *Journal for Research in Mathematics Education.* **24(2)**, 94–116. ERIC EJ478384

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In: L. B. Resnick (Ed.), *The Nature of Intelligence* (pp. 231–236). Hillsdale: NJ: Erlbaum.

Franke, M. L.; Carpenter, T.; Fennema, E.; Ansell, E. & Behrend, J. (1998). Understanding teachers' self-sustaining generative change in the context of professional development. *Teaching and Teacher Education* **14(1)**, 67–80. ERIC EJ568120

Frykholm, J. A. (1996). Preservice teachers in mathematics: Struggling with the standards. *Teaching and Teacher Education* **12**, 665–681.

Giroux, H. (1982). *Ideology, culture, and the Process of Schooling*. Philadelphia, Pa: Temple University.

Locangeli, D. & Cornoldi, C. (1997). Mathematics and Metacognition: What is the Nature of the Relationship? *Mathematical Cognition* **3**, 121–139. ERIC EJ570858

Lortie, D. (1975). Schoolteacher. Chicago, IL: University of Chicago Press.

O'Laughlin, M. (1990). Teachers' ways of knowing: A journal study of teacher learning in a dialogical and constructivist learning environment. Paper Presented at the annual meeting of the Association of Teacher Educators, Boston, MA. ERIC ED327477.

Seaman, R.; Nolan, K. & Corbin-Dwyer, S. (2001). Breaking the cycle: Only 1920 more years to equity. *Journal of Women and Minorities in Science and Engineering* 7.

Simon, M. A. & Schifter, D. (1991). Towards a constructivist perspective: An intervention study of mathematics teacher development. *Educational Studies in Mathematics* **22(4)**, 309–331. ERIC EJ436612

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In: D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillian

Wilcox, S. K.; Schram; P, Lappan, G. & Lanier, P. (1991). The role of a learning community in changing preservice teachers' knowledge and beliefs about mathematics education. East Lansing, MI: Michigan State University. ERIC ED330680

Wilson, S. M. (1990). The secret garden of teacher education. *Phi Delta Kappan* **72(3)**, 204–209. ERIC EJ416470