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Exploring Level Descriptors of Geometrical Thinking[1]

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The aim of this study was to explore the grade 1–3 students' geometrical thinking level descriptors based on van Hiele level descriptors. The data were collected through collection of geometric curriculum materials such as indicators and learning standards in Basic Education Core Curriculum and mathematics textbook for grades 1–3. The findings were found that

1) Inconsistency between descriptors appeared on mathematics curriculum and Thai mathematics textbooks.

2) Using topics on textbooks as criterion for exploring 5 of 7 descriptors appeared on Thai mathematics textbook indicated geometrical thinking levels based on van Hiele's model merely level 0 (Visualization) across textbooks for grades 1–3.

Keywords: Level Descriptors, Geometrical thinking, Thai mathematics textbook, National Core Curriculum
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INTRODUCTION

> How does the geometric thinking experienced by students in the primary grades? (Fuy & Liebov, 1993).

National Curriculum state guidelines recommend at Primary School that geometry play prominent role in school mathematics. NCTM (2000) states that mathematics curriculum for K–4 should include two- and three- dimensional geometry. In addition, geometry provides a context for the early development of mathematical thinking from lower-order thinking processes to higher-order thinking processes (NCTM, 2000; Fuy & Liebov, 1993).

In Thailand, geometry was specified as an integral part of Basic Education Curriculum (Basic Education Core Curriculum, 2001; 2008). In addition, mathematics was aimed at enabling all children and youths to continuously learn this subject in accord with their potentiality. The geometry contents prescribed for all learners are as follow: geometric figures and properties of one-dimensional geometric figures; visualization of geometric models; geometric theories; and geometric transformation through translation, reflection and rotation. Furthermore, the learner's quality when graduate from school system they have knowledge and understanding of triangle, quadrilateral, circle, ellipse, cuboids, sphere and cylinder as well as point, line segment and angle and they can apply diverse methods for problem-solving; can avail of mathematical knowledge, skills and processes appropriately for solving problems faced in various situations; can suitably present reasoning for decision-making and appropriately present the conclusion reached; can be used mathematical language and symbols for communication, as well as accurate and appropriate communication and presentation of mathematical concepts; can link various bodies of mathematical knowledge; can link mathematics with other disciplines; and have attained ability for creative thinking (Basic Education Core Curriculum, 2001; 2008).

Although geometry played an important role in mathematics curriculum in school, there were some studies showed that the students were not successful as they should be, both of their geometrical learning achievement and ability in mathematical process still were in rather low level. Some parts of the causes came from the teachers' teaching method and sequencing in mathematics content and material (Porter, 1989; Thomas, 1982 cited in Clement & Battista, 1992). It was supported by research findings in Thailand, found that these teaching did not enhance the students obtaining their thinking experience in analytical thinking, prove with reasoning, and see relationship of different dimensions for preparing the foundation in studying mathematics in higher level (Chamnankit, 2001, pp. 273–291; Sawangsri, 2002). As a result, the students had thinking level lower than it should be as well as difficulty in learning geometry as a formal prove in higher level

(Chaiyasang, 1987 cited in Noparit, 2005). Therefore, in research perspective, we needed to closely investigate the students' descriptors in order to access their intensive understanding as quality of thinking as well as difficulty in learning by not only judged their right or wrong answer (Fuy & Liebov, 1993).

In 1995, the Netherland mathematician, van Hiele, was worried about the students' difficulty facing in studying geometry. He believed that geometry taught in Secondary School Level was higher order thinking. Besides, the students did not have sufficient experience in low level which they should have before. So, he searched for guidelines in developing the students' intensive comprehension for studying geometry. Consequently, his research was focused on geometrical thinking level and teaching role to support the students moving their geometrical thinking level from one level to the next one. He designed his model for helping the students in obtaining intensive understanding. According to the theory studying the geometrical thinking and constructing the geometrical thinking model, still influenced and played a major role in the instructional management for developing the students' learning process, and curriculum development. According to the theory, the students' thinking level was improved on the hypothesis: 1) the geometry study was a process which was not continuous identifying the different characteristics of qualitative thinking level, 2) the thinking level was sequenced, stable, and hierarchy.

The progress depended on teaching management, not relating to one's age, 3) the concept of understanding as implication at one level, became explicit in the next level, 4) in each level, there were languages of level (Clement, 2004). In geometrical thinking model, five levels were used in describing the students' thinking level: level 0 is Visualization (e.g. shapes were judged by their appearance), level 1 is Analysis Level (e.g. sees figures in terms of their components and discovers properties of a class of shapes), level 2 is Informal Deduction (*e.g.*, logically interrelates previously discovered properties), level 3 is deduction (proves theorems deductively), level 4 is Rigor (*e.g.*, establishes theorems in different postulational systems) (Fuy & Liebov, 1993). According to this theory, there were many researchers conducted the extended research studies for developing the complete characteristic of level and example (fuller characterizations of the levels and examples) as well as collecting details of level descriptors. The descriptors could be used for investigating the teachers and students' language during the instruction, and describing the geometric activity in class (Fuy, Geddes & Tischler, 1995).

Most of descriptors in geometrical thinking level, van Hiele level descriptors were used as describing the students' example and responses referring to their geometrical thinking in each level (Fuy, Geddes & Tischler, 1995). In learning reform of Thailand, a study in students' thinking process and mathematical learning process, was given an importance (Inprasitha, 2006). Specifically, in geometry, there were no obvious descriptors and responses in geometric in each level from describing in different related contexts

including: the curriculum, textbook and instructional activity management in classroom.

For investigation of students' descriptors to access the students' understanding as quality of thinking and difficulty in learning, the objectives of the study were: 1) to search from aims, expository, exercise, test question in curriculum document in Primary Schools specifying the geometric thinking level by teaching geometry based on van Hiele's model, 2) to explore the descriptors of geometric thinking level by geometry teaching in Primary School Level using van Hiele level descriptor (Fuy, Geddes & Tischler, 1995). According to this research, the descriptors of geometrical thinking level from van Hiele's model would be obtained for using in describing the students' example and responses regarding to the existed geometric thinking in the present.

RESEARCH METHODOLOGY

The researcher collected data as following steps:

Making the van Hiele's level descriptors and sample student responses by adapting from "The van Hiele Model of Thinking in Geometry among Adolescent" in Journal for Research in Mathematics Education, Monograph Number 3, 1995. Moreover, the researcher also analyzes secondary data about geometric words used in mathematics textbook in grades 1–3.

The data were collected through collection of geometric curriculum materials such as indicators and standards in Basic Education Core Curriculum 2008 and mathematics textbook for grades 1–3 (expository material, exercises, and test questions) by list of quotations and key words that refer to geometrical thinking level descriptors.

DATA ANALYSIS AND RESULTS

van Hiele Geometric Thinking

Geometry in grades K–4 involves thinking mainly at levels 0 and 1 (Fuy & Liebov, 1993). Then, this research used van Hiele level descriptors and sample student responses from Journal for Research in Mathematics Education Monograph Number 3 (1995) to be framework as follows:

Level 0 (Visualization), student identifies and operates on shapes and other geometric configurations according to their appearance.

1) The student identifies instances of a shape by its appearance as a whole.
2) The student constructs draws or copies a shape.

3) The student names or labels shapes and other geometric configurations and uses standard and/or nonstandard names and labels appropriately.
4) The student compares and sorts shape on the basis of their appearance as a whole.
5) The student verbally describes shapes by their appearance as a whole.
6) The student solves routine problems by operating on shapes rather than by using properties which apply in general.
7) The student identifies parts of a figure but a) does not analyze a figure in terms of its components. b) does not think of properties as characterizing a class of figures. c) does not make generalizations about shapes or use related language.

Level 1 (Analysis). Student analyzes figures in terms of their components and relations between components, establishes properties of a class of figures empirically, and uses properties to solve problems.

The Basic Education Curriculum 2001 and 2008, which served as the core curriculum for national education at the basic level. This research used indicators, learning standards and mathematics textbook for grades 1–3 (expository material, exercises, and test questions) to explore level descriptors of geometrical thinking based on van Hiele's model.

Table 1. Indicators and learning standards in Basic Education Core Curriculum 2001 and 2008

Strand 3: Geometry Standard M3.1: Ability to explain and analyse two-dimensional and three-dimensional geometric figures		
Grade level indicators:		
Grade 1	Grade 2	Grade 3
1. Distinguish triangles, quadrilaterals, circles and ellipses.	1. Identify two-dimensional geometric figures whether in the form of triangles, quadrilaterals, circles or ellipses. 2. Identify three-dimensional figures whether in the form of cuboids, spheres or cylinders. 3. Distinguish between rectangles and cuboids, and between circles and spheres.	1. Identify two-dimensional geometric figures that are components of an object in the form of a three-dimensional geometric figure. 2. Identify two-dimensional geometric figures with axis of symmetry from a given figure. 3. Write linear points, straight lines, rays, parts of straight lines, angles, and symbols.
Standard M3.2: Ability for visualization, spatial reasoning and application of geometric models for problem-solving		
Grade level indicators:		
Grade 1	Grade 2	Grade 3
-	1. Draw two-dimensional geometric figures by using geometric models.	1. Draw two-dimensional geometric figures given in various models. 2. Identify various geometric figures in the surroundings.

The mathematics contents focus on geometry prescribed for all learners are as follow: geometric figures and properties of one-dimensional geometric figures; visualization of geometric models; geometric theories; and geometric transformation through translation, reflection and rotation that shown in Table 1.

From document analysis, aims of mathematics curriculum prescribed for students on grades 1–3 can explain and analyse two-dimensional and three-dimensional geometric figures. Furthermore, mathematics curriculum expected ability for visualization, spatial reasoning and application of geometric models for problem-solving; draw two-dimensional geometric figures by using geometric models, draw two-dimensional geometric figures given in various models, identify various geometric figures in the surroundings that shown descriptor on level 0 (visualization level) and level 1 (analysis level). This study searched for descriptors from the expository material, exercise, and test question in grades 1–3 mathematics textbook. The findings were found that geometry in mathematics textbooks for grades 1–3 involves thinking mainly at level 0 (visualization level). At level 0, the expository material started from informing definition, figure's property, yes-no question and more exercise that shown in Table 2.

Table 2a. Quotations about descriptors from mathematics textbooks of Grades 1–3

IPST Mathematics Textbooks	Descriptors: level 0 visualization
Grade 1 Chapter 11: Preparing with Geometry • Which figure is quadrilateral, triangle, circle or ellipse? • Which of these figures are quadrilateral or triangle or circle ellipse? • Let student tell, which picture have partially to be triangle or quadrilateral, ellipse? • Let student draw picture by used this model as follow (3 triangles, 3 quadrilaterals, 3 circles, 2 ellipses) • Let student search picture, which picture is partially triangle, quadrilateral, circle, ellipse? And to gum down on notebook.	1) The student names or labels shapes and answers yes/no question and answers routine questions about which is shapes? (Quadrilateral, triangle, circle, and ellipse) follows example. 2) The student draws triangles, quadrilaterals, circles, ellipses by patterns that assign to. 3) The student indentifies parts of a figure follow up example but does not analyze a figure in terms of its components.

Table 2b. Quotations about descriptors from mathematics textbooks of Grades 1–3

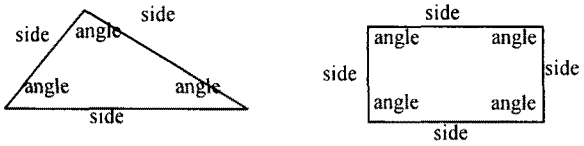
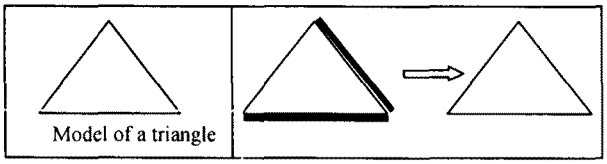
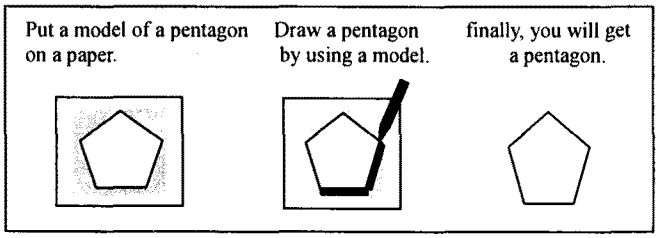
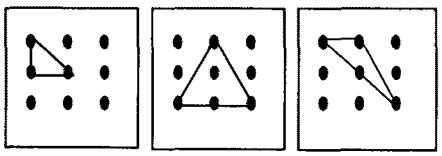
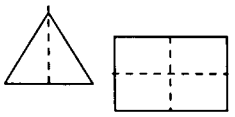
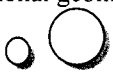
IPST Mathematics Textbooks	Descriptors: level 0 visualization
Grade 2 Chapter 11: Geometric Shapes Triangle, Quadrilateral, Circle, Ellipse Answer the questions: • Which figure are quadrilaterals? • Which figure is a triangle? • Which figure is a circle? • Which figure is an ellipse?  Triangle 3 sides and Quadrilaterals has 4 angles • Is this figure a triangle or a quadrilateral? • Drawing a figure of a triangle, quadrilateral, circle and ellipse by using models.  • Draw 2 different circles by using different materials as a model such as coins, bottom of a glass. • Draw 2 different quadrilaterals by using different materials as a model such as a box of toothpaste, a cover of a box of toothpaste. • Cuboids, sphere and cylinder. • Give 5 examples of cuboids shape, 3 examples of sphere shape and 3 examples of cylinder shape. • What shapes are these? Are they cuboids, sphere or cylinder? (books, tubes, balls, toothpaste boxes, milk cans, ping pong balls) • Quadrilateral, cuboids, circle and sphere. • Which picture is quadrilateral and which picture is sphere?	1) The student names or labels shapes and answers yes/no question and answers routine questions about which is shapes? (Quadrilateral, triangles, circle, and ellipse) follows example. 2) The student identifies which shapes is quadrilateral or triangle follow up properties that assign to. 3) The student draws triangles, quadrilaterals, circles, ellipses by patterns that assign to. 4) The student names or labels shapes with properties that is sphere, cylinder, and cuboids by demonstration 5) In a simple drawing, copy, constructing a shape by model objects that assign to and follow up step from demonstration.

Table 2c. Quotations about descriptors from mathematics textbooks of Grades 1–3

IPST Mathematics Textbooks	Descriptors: level 0 visualization
Grade 3 Chapter 10: Geometric Shapes Circle: Ellipse: Triangle have 3 sides and 3 angles: Quadrilateral have 4 sides and 4 angles: Pentagon have 5 sides and 4 angles: Hexagon have 6 sides and 6 sides: Octagon have 8 sides and 8 angles: • What are these geometric shapes called? Why? • What is a figure which there is 7 angles and 7 sides called? • How many sides and angles are there in a hexagon? • Give three examples of two-dimensional geometric figure and tell what they are called? Drawing geometric shapes by using models of geometric shapes.  • Drawing two-dimensional geometric figure by drawing a line between spots, for example, drawing a line bet ween spots to make triangle shapes as follow: 	1) The student names or labels shapes follow up example in expository but does not analyze a figure in terms of its components. 2) In a simple drawing, copy, constructing a shape by model objects that assign to and follow up step from demonstration. 3) Trace line connect point to be shape two dimension follow up step from demonstration. 4) The student compares and sorts shapes (three dimension) on the basis of their appearance as a whole.

• Symmetry shapes When we fold a paper according to the dashed line and a paper is divided into parts and they can be perfectly overlaid on each other, the trails are called symmetrical axes. Example of picture with symmetrical axes is as follows. There is one symmetrical axe ,  There are two symmetrical axes. • Three-dimensional geometric figures Sphere:  Cuboids:  Cylinder:  - What kind of three-dimensional geometric figures are these pictures similar to? Is this picture a two-dimensional geometric figure or a three-dimensional geometric figure?	

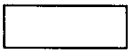
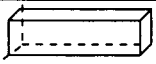
two-dimensional and three-dimensional geometric figures	three-dimensional geometric figures
 rectangular	 Cuboid

Table 3. Show level of descriptors on level 0 (Visualization)

Descriptors: level 0 visualization
1) The student names or labels shapes and uses standard and/or nonstandard names and labels appropriately.
♦ Students name geometric figures by touching outside of the figures and look at outside visualize of the figure.
2) The student constructs draws or copies a shape, does not analyze a figure in terms of its components. 3) The student names and writes the symbol regarding to the points, straight lines, radiations, the part of straight line, angle in contents. 4) The student groups the shapes and geometric figures. 5) The student compares and sorts shapes on the basis of their appearance as a whole. ♦ Use words for describing the picture by touching outside of the picture and seeing what it look like.

Table 2 indicates that the geometric thinking at level 0 (visualization) and 5 characteristics of descriptors. Thus, from the survey found that, grade 1–3 students' geometrical thinking level descriptors based on van Hiele level descriptors were found 5 characteristics of descriptors that shown on Table 3.

CONCLUSION

The important role of the geometry is a national curriculum and it is taken to teach in class. The objective of the study was to present the descriptors of geometrical thinking level of grade 1–3 students which revealed the experience of learning geometry and the difficulties in learning. It was clearly found that teaching geometry should not be based on only text books but the teacher should try to prepare mathematical activities or challenging problems and encourage students to think and solve the problem freely according to their capability. Moreover, the teacher should prepare questions that the students can discuss so that there will be a discussion in class whereas the teacher can evaluate the students' thinking methods. Therefore, the further research to explored grade 4–6 students' geometrical thinking level descriptors based on van Hiele level descriptors and using this as a reference, an initial draft of level descriptors was developed.

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REFERENCES

Clement, D. H. (2004). The Child's Thought and Geometry. In: T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in Mathematics Education Research.* (pp. 60–66). Virginia: NCTM, Inc.

Clement, D. H. & Battista, M. T. (1992). Geometry and Spatial reasoning. In: D. A. Grows (Ed.), *Handbook of research on mathematics teaching and learning.* (pp. 420–464). New York, NY: National Council of Teachers of Mathematics/Macmillan Publishing Co.

Fuy, D.; Geddes, D. & Tischler, R. (1995). *The Van Hiele Model of Thinking in Geometry among*

Adolescent. Journal for Research in Mathematics Education. Monograph Number 3. Reston, Virginia: The National Council of Teachers of Mathematics. ERIC ED294770

Fuy, D. & Liebov, A. (1993). Geometry and Spatial Sense. *Research Ideas for the Classroom: Early Childhood Mathematics.* In: Robert J. Jensen (Ed.), New York: Macmillan. National Council of Teachers of Mathematics.

Inprasitha, M. (2006). Improving Students' Level of Geometrical Thinking through Teacher's Regulating Roles. In: J. Novotná, H. Moraová, M, Krátká & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International group for the Psychology of Mathematics Education.* (pp. 262). Prague, Czech Republic: Faculty of Education, Charles University.

National Council of Teachers of Mathematics (NCTM.) (2000): *Principles and standards for school mathematics.* Reston, VA: NCTM. ME **1999f.**03937 for discussion draft (1998)

Ministry of Education. (2001). *The Basic Education Curriculum B.E.2001* (A.D.2001). Bangkok, Thailand: Agricultural Cooperative Association of Thailand.

Ministry of Education. (2008). *The Basic Education Curriculum B.E.2008* (A.D.2008). Bangkok, Thailand: Agricultural Cooperative Association of Thailand.

Van Hiele, P. M. (1986). *Structure and Insight.* A theory of mathematics education. New York, NY: Academic Press. ME **1988b.**03491

Chamnankit, Nuansri. (2001). *The Development of A Model to Create Teaching Competency on Geometric Concept Image for Pre-Service Teachers* (In Thai). Ed. D. Dissertation (Mathematics Education). Bangkok, Thailand: Graduate School, Srinakharinwirot University.

Noparit, T. (2005). *Developing Instruction based on Open Approach and its Impact on Levels of Geometric Thinking and Geometric Achievement of Eighth Grade Students.* Dissertation, Ed. D. (Mathematics Education). Bangkok: Graduate School, Srinakharinwirot University.

Sawangsri, B. (2002). *The Effect of Geometric Teaching with Teaching of Diana Van' s Hiele Phases on Level of Geometric Thinking and achievement of Mattayomasuksa 1 Student* (In Thai). Ed. M. Dissertation, (Education). Bangkok, Thailand: Graduate School, Nakhon Sawan Rajabhat University.