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# Bayesian estimations on the exponentiated half triangle distribution under Type-I hybrid censoring

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**Abstract**

The exponenetiated distribution has been used in reliability and survival analysis especially when the data is censored. In this paper, we derive Bayesian estimation of shape parameter and reliability function in the exponenetiated half triangle distribution based on Type-I hybrid censored data. Here we consider conjugate prior and noninformative prior and obtained corresponding posterior distributions. As an illustration, the mean square errors of the estimates are computed. Comparisons are made between these estimators using Monte Carlo simulation study.

*Keywords*: Bayesian estimation, exponenetiated half triangle distribution, hybrid Type-I censoring, reliability.

## 1. Introduction

The exponentiated half triangle distribution (EHTD), has the probability density function (pdf), and cumulative distribution function (cdf), given respectively, by

$$f(x) = 2\frac{\lambda}{\sigma}\left[1-\left(1-\frac{x}{\sigma}\right)^2\right]^{\lambda-1}\left(1-\frac{x}{\sigma}\right), \lambda, \sigma > 0, 0 < x < \sigma, \tag{1.1}$$

and

$$F(x) = \left[1-\left(1-\frac{x}{\sigma}\right)^2\right]^{\lambda}, \lambda, \sigma > 0, 0 < x < \sigma. \tag{1.2}$$

For the special case $\lambda = 1$, this distribution is the half triangle distribution. A triangle distribution was applied to a kernel function in non-parametric density estimation. Johnson (1997) studied the possibility of using the more intuitively obvious triangular distribution as

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a proxy for the beta distribution. Some properties of the triangular distribution was studied by Balakrishnan and Nevzorov (2003). Kang (2007) derived some explicit estimators of the scale parameter in a half-triangle distribution under multiply Type-II censoring by several approximate maximum likelihood estimation methods. Han and Kang (2008) derived some approximate maximum likelihood estimators (AMLEs) and maximum likelihood estimator (MLE) of the scale parameter in the half-triangle distribution based on progressively Type-II censored sample. From (1.2), it is easy to write the reliability function of EHTD, respectively, as

$$R(t) = 1 - \left[1 - \left(1 - \frac{t}{\sigma}\right)^2\right]^\lambda, \quad \lambda, \sigma > 0, \ 0 < t < \sigma. \tag{1.3}$$

The most commonly used censoring schemes are Type-I and Type-II censoring schemes. In the conventional Type-I censoring scheme, the experiment continues up to a pre-specified time $T$. On the other hand, the conventional Type-II censoring scheme requires the experiment to continue until a pre-specified number of failures $r \leq n$ occur. The mixture of Type-I and Type-II censoring schemes is known as a hybrid censoring scheme (see Figure 1.1).

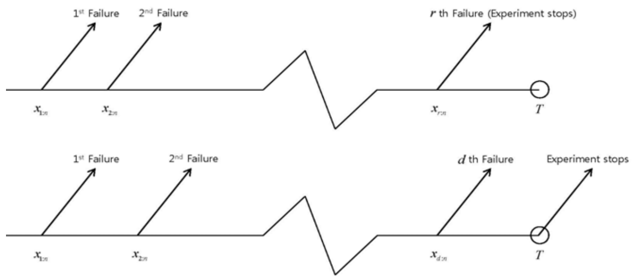


**Figure 1.1** The Type-I hybrid censoring scheme

Let us assume that the ordered lifetimes be denoted by $X_{1:n}, X_{2:n}, \cdots, X_{n:n}$. Epstein (1954) first introduced Type-I hybrid censoring scheme, and considered lifetime experiments assuming that the lifetime of each unit follows an exponential distribution. Kunda (2007) developed maximum likelihood estimators (MLEs) and the approximate maximum likelihood estimators (AMLEs) for the unknown parameters under Type-I hybrid censoring. He obtained the Bayes estimates and the corresponding highest posterior density credible intervals of the unknown parameters under suitable priors on the unknown parameters and using the Gibbs sampling procedure. Recently, Kang *et al.* (2009) derived some estimators of the scale parameter of the half triangle distribution based on Type-I hybrid censored samples.

In Bayesian estimation, we consider two types of loss functions. The first is the squared error loss function (quadratic loss) which is symmetrical, and associates equal importance to the losses due to overestimation and underestimation of equal magnitude. But in life testing and reliability problems, the nature of losses are not always symmetric. For example, if an overestimate is usually much more serious than an underestimate, the use of a symmetrical loss function might be inappropriate. As a useful alternative to the squared error loss function, the second is the linex (linear-exponential) loss function which is asymmetric. It was introduced by Varian (1975) and got a lot of popularity due to Zellner (1986). The linex loss function may be expressed as $l(\triangle) \propto \exp(c\triangle) - c\triangle - 1, c \neq 0$, where $\triangle = \widehat{\theta} - \theta$. The sign and magnitude of the shape parameter $c$ reflects the direction and degree of asymmetry, respectively. when $c$ is positive, the overestimation is more serious than underestimation and the situation is reverse when $c$ is negative. If $c$ tends to zero the linex loss function tends to squared error loss function. By Zellner (1986), the Bayes estimator of $\theta$, denoted by $\widehat{\theta}_l$ under the linex loss function is given by $\widehat{\theta}_l = -\dfrac{1}{c}\ln\{E_\pi[\exp(-c\theta)]\}$, provided that the expectation $E_\pi[\exp(-c\theta)]$ exists and is finite.

## 2. Maximum Likelihood Estimation

Under the Type-I hybrid censoring scheme, it is assumed that $r$ and $T$ are known in advance and the experiment is terminated at a random time $T^* = \min\{X_{r:n}, T\}$, where $1 \leq r \leq n$ and $T \in (0, \infty)$. Therefore, under this censoring scheme we have one of the two following types of observations:

Case I : $\{x_{1:n} < \cdots < x_{r:n}\}$ if $x_{r:n} < T$.
Case II : $\{x_{1:n} < \cdots < x_{d:n}\}$ if $d < r$ and $x_{d:n} < T < x_{d+1:n}$.

It may be mentioned that the $(d+1)^{th}$ failure does not take place before times point $T$ for Case II.

In this section, we provide the MLEs of $\lambda$ for two different cases. The likelihood functions for two different cases are given by

Case I

$$L(\lambda) \propto \lambda^r \prod_{i=1}^{r} U_i^{\lambda-1}(1 - U_r^\lambda)^{n-r}, \tag{2.1}$$

where $U_i = 1 - \left(1 - \dfrac{x_{(i)}}{\sigma}\right)^2$ and $U_r = 1 - \left(1 - \dfrac{x_{(r)}}{\sigma}\right)^2$.

Case II

$$L(\lambda) \propto \lambda^d \prod_{i=1}^{d} U_i^{\lambda-1}(1 - U_T^\lambda)^{n-d}, \tag{2.2}$$

where $U_T = 1 - \left(1 - \dfrac{T}{\sigma}\right)^2$. Therefore, combining Case I and Case II, we can obtain following likelihood function

$$L(\lambda) \propto \lambda^k \prod_{i=1}^{k} U_i^{\lambda-1}(1 - U^\lambda)^{n-k}, \tag{2.3}$$

where $k$ denotes the number of failures and $U = U_r$ if $k = r$, and $U = U_T$ if $k < r$. For now, we assume that the scale parameter $\sigma$ is known. But we will deal with it later. From (2.3), the natural logarithm of the likelihood function is given by

$$l(\lambda) = k \log \lambda + (\lambda - 1) \sum_{i=1}^{k} \log U_i + (n-k) \log(1 - U^{\lambda}). \tag{2.4}$$

The MLE of $\lambda$, denote by $\widehat{\lambda}$, is given by

$$\widehat{\lambda} = \frac{k}{(n-k)(U^{-\widehat{\lambda}} - 1)^{-1} \log U + \sum_{i=1}^{k} \log U_i^{-1}}. \tag{2.5}$$

Since the equation (2.5) can not be solved analytically for $\widehat{\lambda}$, some numerical method must be employed. The corresponding MLE $R(t)$ of the reliability function is given by (1.3) after replacing $\lambda$ by its MLE $\widehat{\lambda}$. We consider sample size $n = 20, 30, 40$ and we evaluate the mean squared errors of the MLEs $\widehat{\lambda}$ and $\widehat{R}$ using the Bisection method. These values are given in Tables 4.1 and 4.2.

## 3. Bayesian estimation

For a Bayesian inference, we consider a conjugate prior distribution and a noninformative prior for shape parameter $\lambda$. respectively.

### 3.1. Estimation based on a conjugate prior

A natural family of conjugate prior for shape parameter $\lambda$ is a gamma prior, given by

$$\pi(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}, \quad \alpha, \beta > 0, \ \lambda > 0. \tag{3.1}$$

Applying Bayes theorem, we obtain from Equation (2.3) and (3.1), the posterior density of $\lambda$ as

$$\pi(\lambda|\underline{x}) = \frac{w^{k+\alpha}}{M_1 \Gamma(k+\alpha)} \lambda^{k+\alpha-1} e^{-w\lambda} (1 - U^{\lambda})^{n-k}, \quad \alpha, \beta > 0, \ \lambda > 0, \tag{3.2}$$

where

$$M_1 = \sum_{j=0}^{n-k} (-1)^j \binom{n-k}{j} \left(1 + \frac{jV}{w}\right)^{-(k+\alpha)}, \ w = \beta + \sum_{i=1}^{k} V_i, \ V = \log U^{-1}, \ \textit{and} \ V_i = \log U_i^{-1}.$$

Consider the reliability $R = R(t)$ is parameter itself. Replacing $\lambda$ in terms of $R$ by that of (3.1), we obtain posterior density function of $R$ as

$$\pi(R|\underline{x}) = \frac{P^{k+\alpha}}{M_1 \Gamma(k+\alpha)} [G(R)]^{k+\alpha-1} e^{[(1-P)G(R)]} \left(1 - U^{G(R)/V_t}\right)^{n-k}, \ 0 < R < 1, \tag{3.3}$$

where $G(R) = \log(1-R)^{-1}$, $P = w/V_t$, $V_t = \log U_t^{-1}$ and $U_t = 1 - \left(1 - \frac{t}{\sigma}\right)^2$.

Under the squared error loss function, the Bayes estimators of $\lambda$ and $R$ can be obtained as

$$\widehat{\lambda}_s = \frac{k+\alpha}{w}\frac{M_2}{M_1} \tag{3.4}$$

and

$$\widehat{R}_s = 1 - \frac{M_4}{M_1}, \tag{3.5}$$

where

$$M_2 = \sum_{j=0}^{n-k}(-1)^j \binom{n-k}{j}\left(1+\frac{jV}{w}\right)^{-(k+\alpha+1)}$$

and

$$M_4 = \sum_{j=0}^{n-k}(-1)^j \binom{n-k}{j}\left(1+\frac{jV}{w}+\frac{1}{P}\right)^{-(k+\alpha)}.$$

The Bayes estimators of $\lambda$ and $R$ under linex loss function are given by

$$\widehat{\lambda}_l = \frac{1}{c}\log\left(\frac{M_1}{M_3}\right) \tag{3.6}$$

and

$$\widehat{R}_l = 1 - \frac{1}{c}\log\left(\frac{M_5}{M_1}\right), \tag{3.7}$$

where

$$M_3 = \sum_{j=0}^{n-k}(-1)^j \binom{n-k}{j}\left(1+\frac{c+jV}{w}\right)^{-(k+\alpha)}$$

and

$$M_5 = \sum_{s=0}^{\infty}\frac{c^s}{s!}\sum_{j=0}^{n-k}(-1)^j \binom{n-k}{j}\left(1+\frac{jV}{w}+\frac{s}{P}\right)^{-(k+\alpha)}.$$

### 3.2. Estimation based on a non-informative prior

For the situation where no prior information about the shape parameter $\lambda$ is available, one may use the quasi density as given by

$$\pi(\lambda) = \frac{1}{\lambda^m}, \quad \lambda > 0,\ m > 0. \tag{3.8}$$

This contains Jeffery's noninformative prior as a special case when $m = 1$. It follows, from (3.8), that the posterior distribution of $\lambda$ is given by

$$\pi(\lambda|\underline{x}) = \frac{Q^{k-m+1}}{N_1\Gamma(k-m+1)}\lambda^{k-m}e^{-Q\lambda}(1-U^{\lambda})^{n-k},\ \lambda > 0, \tag{3.9}$$

where $N_1 = \sum_{j=0}^{n-k}(-1)^j \binom{n-k}{j}\left(1+\frac{jV}{Q}\right)^{-(k-m+1)}$ and $Q = \sum_{i=1}^{k} V_i$.

Consider the reliability $R = R(t)$ is parameter itself. Replacing $\lambda$ in terms of $R$ by that of (3.9), we obtain posterior density function of $R$ as

$$\pi(R|\underline{x}) = \frac{Z^{k-m+1}}{N_1\Gamma(k-m+1)}[G(R)]^{k-m}\, e^{[(1-Q)G(R)]}\left(1-U^{G(R)/V_t}\right)^{n-k},\quad 0 < R < 1, \tag{3.10}$$

where $G(R)$ is given in (3.3) and $Z = Q/V_t$. Under the squared error loss function, the Bayes estimators of $\lambda$ and $R$ can be obtained as

$$\widehat{\lambda}_s = \frac{k-m+1}{Q}\frac{N_2}{N_1} \tag{3.11}$$

and

$$\widehat{R}_s = 1 - \frac{N_4}{N_1}, \tag{3.12}$$

where

$$N_2 = \sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left(1+\frac{jV}{Q}\right)^{-(k-m+2)}$$

and

$$N_4 = \sum_{j=0}^{n-k}(-1)^j\binom{n-k}{j}\left(1+\frac{jV}{Q}+\frac{1}{Z}\right)^{-(k-m+1)}.$$

The Bayes estimators of $\lambda$ and $R$ under linex loss function are given by

$$\widehat{\lambda}_l = \frac{1}{c}\log\left(\frac{N_1}{N_3}\right) \tag{3.13}$$

and

$$\widehat{R}_l = 1 - \frac{1}{c}\log\left(\frac{N_5}{N_1}\right), \tag{3.14}$$

where

$$N_3 = \sum_{j=0}^{n-k} (-1)^j \binom{n-k}{j} \left(1 + \frac{c + jV}{Q}\right)^{-(k-m+1)}$$

and

$$N_5 = \sum_{s=0}^{\infty} \frac{c^s}{s!} \sum_{j=0}^{n-k} (-1)^j \binom{n-k}{j} \left(1 + \frac{jV}{Q} + \frac{s}{Z}\right)^{-(k-m+1)}.$$

From in section 3.1 and 3.2 formula, the mean squared errors of the proposed estimators are simulated by Monte Carlo method based on 10,000 runs for sample size $n = 20, 30, 40$ and various choices of censoring under Type- hybrid censored samples. These values are given in Tables 4.1 and 4.2.

### 3.3. Estimation of the scale parameter $\sigma$

Assuming that the scale parameter $\sigma$ is known at the beginning, but basically we can consider a joint prior distribution of $(\lambda, \sigma)$ and then make a inference of it using Markov chain Monte Carlo (MCMC) algorithm. Since the shape parameter $\lambda$ is the parameter of interest here, we just estimate the scale parameter $\sigma$ and then plug it in. The nuisance parameter $\sigma$ can be estimated by maximizing its marginal likelihood,

$$L(\sigma) = \int f(x|\sigma, \lambda)\pi(\lambda)d\lambda. \tag{3.15}$$

That is,

$$\widehat{\sigma} = \arg\max L(\sigma)_{\sigma>0}. \tag{3.16}$$

Note that the marginal likelihood of $\sigma$ can be obtained by

$$\frac{\Gamma(k+\alpha)}{w^{k+\alpha}} \sum_{j=0}^{n-k} (-1)^j \binom{n-k}{j} \left(1 + \frac{jV}{w}\right)^{-(k+\alpha)} \tag{3.17}$$

or

$$\frac{\Gamma(k-m+1)}{w^{k-m+1}} \sum_{j=0}^{n-k} (-1)^j \binom{n-k}{j} \left(1 + \frac{jV}{Q}\right)^{-(k-m+1)}. \tag{3.18}$$

## 4. Simulation assesment

To compare the performance of the Bayes estimators of $\lambda$ and $R(t)$ under the squared error loss function and the linex loss function, we simulated the mean squared errors of all proposed estimators through Monte Carlo simulation method. The prior parameters chosen $(\alpha, \beta) = (3.0, 3.0)$, which yield the generated value of $\lambda = 0.83676$ as the true value. The true value of $R(t)$ is computed to be $R(0.5) = 0.21394$ when $(\alpha, \beta) = (3.0, 3.0)$. That is, gamma prior with $(\alpha, \beta) = (3.0, 3.0)$ is an informative prior for true shape parameter. As an alternative, a Jeffery's noninformative prior for $\lambda$ is considered (i.e. $m = 1$ ). Using the true value of $\lambda$, the Type-I hybrid censored data from the standard exponentiated half triangle distribution are generated for sample size $n = 20, 30, 40$ and different $r$ and $T$ values. Using this data, the mean squared errors of the Bayes estimators of $\lambda$ and $R(t)$ under the squared error loss function and the linex loss function are simulated by Monte Carlo method based on 10,000 runs for sample size $n = 20, 30, 40$ and different $r$ and $T$ values under Type-I hybrid censored samples. To see more clear effect of asymmetric loss function, we consider a large positive value $c = 10$ for reliability function while relatively small value $c = 2$ is considered for shape parameter. However, results vary slightly depending on the choice of $c$ value. These relative mean squared errors are given in Tables 4.1 and 4.2.

**Table 4.1** The relative mean squared errors for the estimators of $\lambda$ and $R(t)$ when prior is gamma distribution with $\alpha = 3.0$ and $\beta = 3.0$

| $T$ | $n$ | $r$ | $\widehat{\lambda}$ | $\widehat{\lambda}_s$ | $\widehat{\lambda}_l$ $(c=2)$ | $\widehat{R}$ | $\widehat{R}_s$ | $\widehat{R}_l$ $(c=10)$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 10 | 10 | 0.076736 | 0.067572 | 0.044679 | 0.003558 | 0.002887 | 0.002151 |
| | | 8 | 0.075657 | 0.068566 | 0.045416 | 0.003523 | 0.002926 | 0.002182 |
| | | 5 | 0.078602 | 0.069263 | 0.045424 | 0.003651 | 0.002958 | 0.002192 |
| | 20 | 20 | 0.041352 | 0.037319 | 0.029644 | 0.001975 | 0.001706 | 0.001487 |
| | | 15 | 0.042160 | 0.037694 | 0.029974 | 0.002007 | 0.001722 | 0.001501 |
| | | 5 | 0.047120 | 0.040754 | 0.031625 | 0.002239 | 0.001856 | 0.001584 |
| | 30 | 30 | 0.026361 | 0.024030 | 0.020502 | 0.001282 | 0.001136 | 0.001066 |
| | | 25 | 0.026686 | 0.024035 | 0.020503 | 0.001293 | 0.001135 | 0.001065 |
| | | 20 | 0.026870 | 0.024135 | 0.020666 | 0.001304 | 0.001143 | 0.001071 |
| 0.7 | 10 | 10 | 0.076589 | 0.067318 | 0.044588 | 0.003553 | 0.002878 | 0.002147 |
| | | 8 | 0.075658 | 0.068258 | 0.045275 | 0.003523 | 0.002915 | 0.002176 |
| | | 5 | 0.078607 | 0.069249 | 0.045422 | 0.003651 | 0.002958 | 0.002192 |
| | 20 | 20 | 0.041352 | 0.037219 | 0.029590 | 0.001974 | 0.001701 | 0.001484 |
| | | 15 | 0.042130 | 0.037692 | 0.029982 | 0.002006 | 0.001722 | 0.001501 |
| | | 5 | 0.047120 | 0.040754 | 0.031625 | 0.002239 | 0.001856 | 0.001584 |
| | 30 | 30 | 0.026451 | 0.023983 | 0.020469 | 0.001285 | 0.001134 | 0.001064 |
| | | 25 | 0.026635 | 0.023994 | 0.020470 | 0.001291 | 0.001134 | 0.001064 |
| | | 20 | 0.026869 | 0.024119 | 0.020610 | 0.001304 | 0.001142 | 0.001070 |
| 0.9 | 10 | 10 | 0.076593 | 0.067321 | 0.044594 | 0.003553 | 0.002878 | 0.002147 |
| | | 8 | 0.075659 | 0.068253 | 0.045273 | 0.003523 | 0.002915 | 0.002176 |
| | | 5 | 0.078607 | 0.069249 | 0.045422 | 0.003651 | 0.002958 | 0.002192 |
| | 20 | 20 | 0.041354 | 0.037225 | 0.029596 | 0.001974 | 0.001702 | 0.001485 |
| | | 15 | 0.042130 | 0.037688 | 0.029979 | 0.002006 | 0.001721 | 0.001501 |
| | | 5 | 0.047120 | 0.040754 | 0.031625 | 0.002239 | 0.001856 | 0.001584 |
| | 30 | 30 | 0.026398 | 0.023981 | 0.020468 | 0.001283 | 0.001134 | 0.001064 |
| | | 25 | 0.026633 | 0.023990 | 0.020468 | 0.001291 | 0.001133 | 0.001063 |
| | | 20 | 0.026869 | 0.024119 | 0.020610 | 0.001304 | 0.001142 | 0.001070 |

## 5. Concluding remarks

In this paper we present the Bayesian and Non-Bayesian estimators of the shape parameter $\lambda$ and reliability function $R(t)$ of exponentiated half triangle distribution under Type-I hybrid censoring. Bayes estimators under squared error loss function and linex loss function are derived. The MLEs are also obtained. The MLEs of $\lambda$ and $R$ are compared with Bayes

**Table 4.2** The relative mean squared errors for the estimators of $\lambda$ and $R(t)$ when prior is quasi density with $m = 1.0$

| $T$ | $n$ | $r$ | $\widehat{\lambda}$ | $\widehat{\lambda}_s$ | $\widehat{\lambda}_l\ (c=2)$ | $\widehat{R}$ | $\widehat{R}_s$ | $\widehat{R}_l\ (c=10)$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 10 | 10 | 0.076736 | 0.121400 | 0.073385 | 0.003558 | 0.004742 | 0.003337 |
| | | 8 | 0.075657 | 0.124780 | 0.075074 | 0.003523 | 0.004830 | 0.003404 |
| | | 5 | 0.078602 | 0.125620 | 0.074951 | 0.003651 | 0.004898 | 0.003416 |
| | 20 | 20 | 0.041352 | 0.047060 | 0.037251 | 0.001975 | 0.002125 | 0.001797 |
| | | 15 | 0.042160 | 0.047660 | 0.037743 | 0.002007 | 0.002149 | 0.001817 |
| | | 5 | 0.047120 | 0.053280 | 0.040872 | 0.002239 | 0.002388 | 0.001968 |
| | 30 | 30 | 0.026361 | 0.027490 | 0.023723 | 0.001282 | 0.001300 | 0.001184 |
| | | 25 | 0.026686 | 0.027530 | 0.023670 | 0.001293 | 0.001301 | 0.001184 |
| | | 20 | 0.026870 | 0.027620 | 0.024213 | 0.001304 | 0.001309 | 0.001190 |
| 0.7 | 10 | 10 | 0.076589 | 0.120060 | 0.072970 | 0.003553 | 0.004705 | 0.003321 |
| | | 8 | 0.075658 | 0.123000 | 0.074511 | 0.003523 | 0.004787 | 0.003382 |
| | | 5 | 0.078607 | 0.125360 | 0.074890 | 0.003651 | 0.004893 | 0.003414 |
| | 20 | 20 | 0.041352 | 0.046890 | 0.037166 | 0.001974 | 0.002118 | 0.001793 |
| | | 15 | 0.042130 | 0.047630 | 0.037745 | 0.002006 | 0.002148 | 0.001817 |
| | | 5 | 0.047120 | 0.053280 | 0.040872 | 0.002239 | 0.002388 | 0.001968 |
| | 30 | 30 | 0.026451 | 0.027440 | 0.023612 | 0.001285 | 0.001298 | 0.001183 |
| | | 25 | 0.026635 | 0.027470 | 0.023633 | 0.001291 | 0.001298 | 0.001182 |
| | | 20 | 0.026869 | 0.027610 | 0.023792 | 0.001304 | 0.001309 | 0.001190 |
| 0.9 | 10 | 10 | 0.076593 | 0.120000 | 0.072960 | 0.003553 | 0.004704 | 0.003320 |
| | | 8 | 0.075659 | 0.122970 | 0.074501 | 0.003523 | 0.004787 | 0.003381 |
| | | 5 | 0.078607 | 0.125360 | 0.074890 | 0.003651 | 0.004893 | 0.003414 |
| | 20 | 20 | 0.041354 | 0.046900 | 0.037170 | 0.001974 | 0.002118 | 0.001793 |
| | | 15 | 0.042130 | 0.047630 | 0.037741 | 0.002006 | 0.002147 | 0.001817 |
| | | 5 | 0.047120 | 0.053280 | 0.040872 | 0.002239 | 0.002388 | 0.001968 |
| | 30 | 30 | 0.026398 | 0.027430 | 0.023611 | 0.001283 | 0.001298 | 0.001182 |
| | | 25 | 0.026633 | 0.027470 | 0.023630 | 0.001291 | 0.001298 | 0.001182 |
| | | 20 | 0.026869 | 0.027610 | 0.023792 | 0.001304 | 0.001309 | 0.001190 |

estimates under squared error loss function and linex loss function in terms of estimated MSE. We can see that the Bayes estimates under squared error loss function is generally more efficient than Bayes estimates under linex loss function and their corresponding maximum likelihood estimates for the considered cases.

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