Optimizing Empty Container Repositioning at a Global Maritime Company

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글로벌 해운회사의 공컨테이너 재배치 최적화

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H maritime company in Korea had a long history of difficulty in obtaining enough empty containers to transport its commodities. Despite efforts to resolve the problem, the repositioning of empty containers between depots was still planned manually, which consumed much worker time. To automate the generation of reposition plans, we developed a planning system to optimize the company's reposition of empty containers between global depots with a linear programming model functioning as a key decision engine in the system. The model determines the route, volume and mode of transportation for repositioning empty containers from depots with surplus containers to those requiring containers, as well as the empty space of ships generated from the unloading or loading of full and empty containers. The system has been in operation in the company's global network since March 2008.

Keyword: container transportation, empty container, optimization

1. Introduction

H maritime company is an integrated logistics company, operating over 110 state-of-the-art vessels located in Korea. The company has formed a global business network with four international headquarters, more than 20 subsidiaries, and more than 50 branch offices. It is highly regarded as one of the world's top integrated-logistics companies with its targeted market prospects, efficient organization, top personnel, and advanced internet systems.

An important concern of the company is the repositioning of empty containers from depots with surplus containers to those requiring containers in the company's globally distributed network. The reposition of empty containers is difficult for three reasons : the empty containers available for reposition are typically scattered across the company's global network <Figure 1>, the empty containers are repositioned by a variety of ships, trucks, trains, and feeders with different schedules, and the planning horizon is fairly long (five weeks = 35 planning days in the company) due to deep-sea shipping services for trans-ocean transportation provided by the company.

Prior to the system developed in our project, repo-

Figure 1. Global network of the company

sition plans were developed manually by company container operators with more than three years experience due to the problem complexity. However, obtaining high-quality solutions by hand was difficult due to the absence of any tool to evaluate the impact of the container operators' decisions. This raised the costs and lowered the level of demand satisfactions. Therefore, the company decided to automate the repositioning process by incorporating an optimization model. The objective of our project was to develop a planning system that would be compatible with the company's operational practices and that could quickly formulate reposition plans. A key module in the planning system was the linear programming model generating reposition plans for several types of empty container.

Several previous research articles have examined the empty container repositioning problem. For the convenience of our literature review, we classify the previous studieswith respect to the type of transportation, i. e., inland transportation, shipping between seaports, and intermodal transportation. For the problem with inland transportation, Crainic et al. (1993) proposed linear and stochastic programming models for the problem with single container type and extended them into the problems with multiple container types by considering the substitution of containers. Shen and Khoong (1995) suggested a network optimization model instead of linear programs and extended it into more general models by considering backorders. Lai et al. (1995) provided a simulation model of the shipping company's ope-

rational activities, including empty container repositioning, and developed a heuristic algorithm. For the problem of shipping between seaports, Gao (1994) presented a two-stage deterministic model for a ship by considering the space restrictions on the ship and the limitation of lease amount. The first step of the model calculates the requirements(= demand minus supply) of empty containers of aship arriving at each seaport during the voyage and the second step repositions them by a linear program. Cheung and Chen (1998) represented the problem with single empty container type as a two-stage network model where the supply and demand of empty containers and the capacity of the ship's space are deterministic in the first stage and random in the second stage. Then, they proposed a quasi-gradient method and several heuristic algorithms to solve the problem. Lam et al. (2007) presented a stochastic dynamic program for the problem with two seaports and ships, extended it into the problem with multiple seaports and ships, and proposed a simulation-based heuristics. For the problem with intermodal transportation, Choong et al. (2002) developed a linear programming model for the problem with single container type and performed a case study. Olivo et al. (2005) also performed a case study using a linear programming model considering two container types. Finally, Li et al. (2004) and Li et al. (2007) considered inventory problems, Jula et al. (2006) considered a type of transportation problem, Shintani et al. (2008) considered a network design problem, and Chang et al. (2008) considered the substitution problem for empty container interchange.
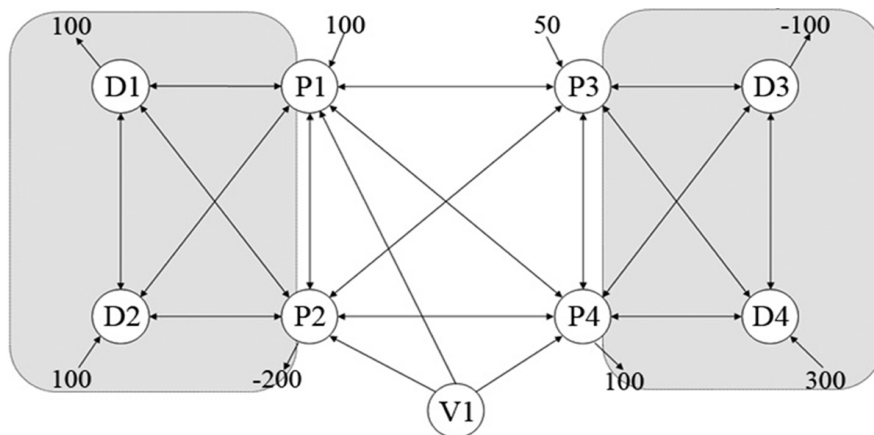
Figure 2. Example of the transportation network for repositioning empty containers

The problem represented in our model can be classified into the problem with intermodal transportation according to the above classification. The model extends models in previous research by considering the schedule and capacity of a liner ship, i.e., a container ship sailing on a regular schedule. Although several studies also considered the schedule and capacity of the ship in their models as parameters, the capacity of the ship in our model is a decision variable that can be determined by considering the (un)load of containers at previous seaports. In addition, our model is implemented in a rolling horizon framework in order to accommodate the uncertainties and any change of data such as supply and demand of empty containers. Hence, our model considers containers already sent or scheduled to be sent to a depot before newly planning as parameters as well as constraints.

The rest of the current paper is organized as follows. The following section describes the problem details and the model assumptions. Section 3 provides an overview of the linear programming model. Section 4 presents the study results and a discussion of the model's implementation. Finally, the study is summarized and suggestions are given for future research in Section 5.

2. Problem Description and Modeling Assumptions

2.1 Transport network and reposition types

<Figure 2> shows an example of the transport network. In the figure, the shaded area represents the inland and the area between the inlands represents the sea. In the network, each node corresponds to

seaports, non-port depots (not including the seaports) performing warehouse functions, and liner ships sailing at the beginning of the current planning horizon. For example, nodes D1, D2, D3 and D4 are non-port depots, nodes P1, P2, P3 and P4 are seaports, and node V1 is a sailing liner ship. The nodes corresponding to liner ships are included in the network only for a modeling purpose. As our model is implemented in a rolling horizon framework, it should consider containers presently on liner ships on their way to their destinations but whose original directions can be changed due to economic reasons that are described below. Therefore, our model assumes that empty containers on the liner ship have been sent from an imaginary depot corresponding to the ship and they are considered as initial inventories in the depot. In the network, each arc represents the transport mode connections between depots and the numbers beside the nodes show the supply and demand of empty containers if their value is positive and negative, respectively. Here, the supply of empty containers is available as soon as the shippers return them to the company's depots. The company estimates the supply and demand based on actual data and historical data and hence our model assumes that the demands and supplies are known and are deterministic in any given period.

There are two types of repositions : inland and inter-port. The former is the reposition of empty containers between non-port depots or between non-port depots and seaports that are reachable to each other (i.e., connected in the transport network) by inland transport modes such as truck, train, and barge. Second, the inter-port reposition implies the reposition of empty containers between seaports that are reachable to each other by feeders or liner ships.

2.2 Schedule and capacity of transport modes

We assume an unlimited trucking capacity since trucks are readily available in real practice in Korea. On the other hand, trains, barges, feeders, and liner ships are operated according to their regular schedules and their capacities are limited in real practice. In our model, we restricted our consideration of scheduling and capacity to those of liner ships and ignored those of trains, barges, and feeders for two reasons. First, when the planning system was developed, the company's existing database system did not contain the schedules of the modes. Therefore, an excessive time would have been required to insert the schedules of the modes in all depots in the company's global network. Second, although their schedules are available in the database, the capacity of the modes is difficult to estimate because the company buys the vehicles or some of their spaces and shares their capacity with other companies. To overcome the unreality of the plan obtained from our model, we decided to allow planners at the company to modify the model's solution, e.g., although our model suggests sending empty containers via train, the manager can send them via trucks unless a train is available. Therefore, we assume in our model that inland transport modes have infinite capacity. However, if the capacity of each transportation mode can be estimated, the schedule of inland transport modes can be incorporated into our model as a form of the capacity of the mode at a certain time period (the capacity constraint of each transport mode is described in the Appendix).

On the other hand, our model incorporates the capacity and schedule of liner ships as constraints since the capacity can be accurately estimated because the company owns the ships. In our model, the capacity of a liner ship is determined by the space assigned to the container type in the ship. Our model only considers one type of container at a time, i. e., the problem is a single commodity problem. That is, our model is independently applied to and solved for each container type according to the company's request based on their current practice. Although the estimation of the space assigned to the container type is another issue because the space may be shared with some other containers, our model assumes the space to be known and deterministic.

2.3 Transport state

The transport state is used in our model to consider the volume of containers already sent or scheduled to be sent to a depot before planning. The direction and volume of containers already sent cannot be changed in inland transportation, except for an urgent situation, but they can be changed in ship transportation in order to prevent stock-out at some depots. That is, because ship transportation typically takes three to five weeks to arrive at a destination an inflexible situation could lead to unsatisfied demand at some depots. Moreover, changing their direction to a demanding depot may be more economical than transporting from the other surplus depots. Furthermore, even if containers are not yet sent, it is very difficult to change an original plan if the contract with the carriers has been made since changing the contract necessitates additional time, work and pay for penalties.

To incorporate this concept, we define three transport states : planning, approved, and acknowledged. If a reposition is in the planning state, it is newly determined by our model. Second, the approved state represents that the direction, volume, and transport mode of the containers in the state can be changed by paying some penalties. Finally, the acknowledged state represents that these container variables are fixed values in our problem.

2.4 Costs and other assumptions

Four cost factors are considered in the model : inventory holding, transportation, penalty, and lease. The inventory holding cost occurs when containers are held in order to satisfy future demand. The transportation cost includes the cost to transport containers and the terminal handling charge if the destination is a seaport. The terminal handling charge is the cost of the stevedoring services of containers at seaport. The penalty cost is a cost proportional to the changed volume when there is some change of container volume in the approved state. The lease cost is incurred when the leasing of containers is unavoidable in order to satisfy the demand.

The other assumptions made in this model can be summarized as follows : (a) the planning horizon is divided into discrete periods; (b) the volume of the other containers (full containers and the other type of empty containers) unloaded from and loaded onto a liner ship is known and deterministic; (c) transit times are given and deterministic; (d) backlogging is not allowed and hence demand should be satisfied on time; (e) all transport modes are perfect in state, i.e., they are not out of order over the planning periods (f) containers are ready to be used and no repairs or discards of

containers occur within the planning horizon; (g) all depots have infinite capacity; (h) inventory holding costs are computed based on the end-of-period inventory; and (i) leased empty containers are used by the planning horizon, i.e., they are never returned to the leaseholder during the planning periods.

3. Optimization Model

The optimization model presented in this paper can be regarded as a special case of a time-expanded minimum-cost flow problem. The minimum-cost flow problem can be formulated as a linear program owing to its total unimodularity (Wolsey, 1998). The company's desire for rapid generation of solutions prevented us from using many of the models in the literature. Therefore, we developed a valid, tractable, and simple model for the company by incorporating practical requirements important to the company. Although heuristics could be developed by considering complex and realistic constraints such as batching the transportation size (its corresponding constraint is described in the Appendix), we decided to develop a linear programming model due to its simplicity in extending and its promise of rapidly generating solutions.

Before presenting the model, we summarize the notations used in the model below.

Sets

D set of non-port depots

P set of seaports

ID_i set of non-port depots reachable from depot i by inland transport modes

IP_i set of seaports reachable from depot i by inland transport modes

P_i set of seaports reachable from depot i by any transport mode except for liner ships

V set of liner ships

V_{it} set of liner ships embarking at seaport i in period t

V_{ijt} set of liner ships leaving in period t from seaport i to j

BP_{vt} set of seaports that liner ship v visits before period t on its route

AP_{vt} set of seaports that liner ship v visits after period t on its route

B_{vq} set of seaports that liner ship v visits before its route sequence q

A_{vq} set of seaports that liner ship v visits after its route sequence q

T set of periods

Q_v sequenced set of seaports on the route of liner ship v

W set of transport states $\{1, 2\}$ where 1 represents the planning state and 2 represents the approved state

M_{ij} set of inland transport modes operated from depot i to j

Parameters

h_i inventory holding cost per container at depot i

$c_{ij}^{\,m}$ transportation cost per container from depot i to j via transport mode m. Here, if depot j is a seaport, the cost includes the terminal handling charge at the seaport

p_{ij} penalty cost per container occurred by changing the approved quantity from depot i to j

b_i lease cost per container

$ap_{ijt}^{\,m}$ volume of containers in approved state from (imaginary) depot i to depot j in period t via transport mode m

r_{it} netrequirement at depot i in period t, denoted as

$$r_{it} = d_{it} - s_{it} - ac_{it}$$

where d_{it} and s_{it} are demand and supply, respectively, at depot i in period t, and ac_{it} is the volume of containers in acknowledged state arriving at depot i in period t

$l_{ij}^{\,m}$ transit time of transport mode m from depot i to depot j or remaining time of the mode arriving at depot j after leaving depot i

at_{vq} period that liner ship v arrives in route sequence q

lt_{vq} period that liner ship v leaves in route sequence q

z_{vq} volume of the other containers (full containers and the other type of empty containers) unloaded from liner ship v subtracted by loaded volume onto the ship in its route sequence q

I_{i0} initial inventory level at depot i

s_{v0} initial remaining capacity of liner ship v

Decision variables

I_{it} inventory level at depot i in period t

$X_{ijt}^{\,wm}$ volume transported in period t from depot i

to j via transport mode m at the transport state w. We preliminarily set $X_{ijt}^{1m} = 0$ if $ap_{ijt}^{m} > 0$ and $X_{ijt}^{2m} = 0$ if $ap_{ijt}^{m} > 0$.

L_{it} leased container volume at depot i in period t.

$CX_{ijt}^{m^+}$ increased volume of containers from approved volume ap_{ijt}^{m}. Note that it is defined only if $ap_{ijt}^{m} > 0$

$CX_{ijt}^{m^-}$ decreased volume of containers from approved volume ap_{ijt}^{m}. Note that it is defined only if $ap_{ijt}^{m} > 0$

S_{vq} remaining capacity of liner ship v in the route sequence q

The linear programming model implemented in the system is given below.

Minimize

$$\sum_{i\in D\cup P}\sum_{t\in T} h_i \cdot I_{it} + \sum_{i\in D}\sum_{j\in IP_i\cup ID_i}\sum_{t\in T}\sum_{w\in W}\sum_{m\in M_{ij}} c_{ij}^{m}\cdot X_{ijt}^{wm}$$
$$+\sum_{i\in P}\sum_{j\in P_i\cup ID_i}\sum_{t\in T}\sum_{w\in W}\sum_{m\in M_{ij}\cup v_{ijt}} c_{ij}^{m}\cdot X_{ijt}^{wm}$$
$$+\sum_{i\in D\cup P}\sum_{t\in T} b_i\cdot L_{it}$$
$$\sum_{i\in D}\sum_{j\in IP_i\cup ID_i}\sum_{t\in T}\sum_{m\in M_{ij}} p_{ij}\cdot(CX_{ijt}^{m+}+CX_{ijt}^{m-})$$
$$\sum_{i\in P}\sum_{j\in P_i}\sum_{t\in T}\sum_{m\in M_{ij}\cup V_{ijt}} p_{ij}\cdot(CX_{ijt}^{m+}+CX_{ijt}^{m-})$$

subject to

$$I_{it}=I_{i,t-1}+\sum_{j\in IP_i\cup ID_i}\sum_{w\in W}\sum_{m\in M_{ij}}(X_{ji,t-l_{ji}^m}^{wm}-X_{ijt}^{wm})$$
$$+L_{it}-r_{it}$$
$$\text{for } i\in D \text{ and } t\in T \quad (1)$$

$$I=I_{i,t-1}+\sum_{j\in IP_i\cup ID_i}\sum_{w\in W}\sum_{m\in M_{ij}}(X_{ji,t-l_{ji}^m}^{wm}-X_{ijt}^{wm})$$
$$+\sum_{v\in V}\sum_{j\in BP_{vt}}\sum_{w\in W}X_{ji,t-l_{ji}^v}^{wv}-\sum_{v\in V}\sum_{j\in AP_{vt}}\sum_{w\in W}X_{ijt}^{wm}$$
$$+L_{it}-r_{it}$$
$$\text{for } i\in P \text{ and } t\in T \quad (2)$$

$$S_{vq}=S_{v,q-1}+\sum_{j\in B_{vq}}\sum_{w\in W}X_{j,i_{vq},at_{vq}-l_{ji_{vq}}^v}^{wv}$$
$$-\sum_{j\in A_{vq}}\sum_{w\in W}X_{i_{vq},j,lt_{vq}}^{wm}+z_{vq}$$
$$\text{for } v\in V \text{ and } q\in Q_v \quad (3)$$

$$X_{ijt}^{2m}-ap_{ijt}^{m}=CX_{ijt}^{m+}-CX_{ijt}^{m-}$$
$$\text{for } i\in D, j\in IP_i\cup ID_i, t\in T \text{ and } m\in M_{ij}$$
$$\text{or } i\in P, j\in P_i\cup ID_i, t\in T \text{ and } m\in M_{ij}\cup V_{ijt} \quad (4)$$

$$I_{it}, L_{it}\geq 0$$
$$\text{for } i\in D\cup P \text{ and } t\in T \quad (5)$$

$$S_{vq}\geq 0$$
$$\text{for } v\in V \text{ and } q\in Q_v \quad (6)$$

$$X_{ijt}^{wm}\geq 0$$
$$\text{for } i\in D, j\in IP_i\cup ID_i, t\in T, w\in W$$
$$\text{and } m\in M_{ij} \text{ or } i\in P, j\in P_i\cup ID_i, t\in T\ w\in W$$
$$\text{and } m\in M_{ij}\cup V_{ijt} \quad (7)$$

$$CX_{ijt}^{m+}\geq 0, CX_{ijt}^{m-}\geq 0$$
$$\text{for } i\in D, j\in IP_i\cup ID_i, t\in T \text{ and } m\in M_{ij}$$
$$\text{or } i\in P, j\in P_i\cup ID_i, t\in T \text{ and } m\in M_{ij}\cup V_{ijt} \quad (8)$$

The objective function denotes the minimization of the sum of inventory holding, transportation, lease, and penalty costs. Constraints (1) and (2) are inventory balance constraints, with the former representingthe inventory balance at each non-port depot. That is, for each non-port depot, the inventory level at the end of each period is computed by the sum of the previous inventories, the transported volume in and out from the depot and the leased volume subtracted by the net requirement. Constraint (2) represents the inventory balance at each seaport, which is the same as constraint (1) except for the incoming and outgoing volumes transported from/to its reachable seaports and non-port depots. The fourth and fifth terms in the constraint represent the volume transported via liner ships.

Constraint (3) corresponds to the flow conservation of the remaining space of each liner ship. The remaining capacity of a liner ship in a route sequence is obtained by : the sum of the remaining capacity at the previous sequence, the empty container volume loaded at the port in the present sequence, and the volume of the other containers (full containers and the other type of empty containers) unloaded from and loaded onto liner ship at the seaport and the sum is subtracted by the empty container volume transported from the seaports in the earlier route sequences. Note that constraint (3) and the non-negative constraint (6) act as the capacity restriction of each liner ship.

Finally, constraint (4) is used to compute the volume increased and decreased from the approved volume, and the other constraints (5), (6), (7), and (8) represent the conditions on the decision variables.

4. Implementation and Results

We implemented the system during the winter of 2007~2008 with one-month practice period to check the validity of the system's results. Lingo 10.2, which is a commercial linear programming software program, was used to solve the model and generate the solution.

Figure 3. Sample output report

During the practice periods, the experienced planners at the company compared the plan by the system with the plan by their former trial-and-error method using the past data to verify the system's results. In addition, the planning system was verified over one year while they used the plan in real operations. The system has been in continuous operation in the company's global network since March 2008. A sample screen of an output report is shown in <Figure 3>.

The system generates the reposition plan of each container type within 8 minutes, which is less than the ten minutes requested by the company, and hence approximately one hour is taken to generate the plans for all 11 container types at the company since the system generates the plan of each container sequentially until all plans are generated. The one-hour planning time is shorter than the required time of the manual trial-and- error method that needs approximately one day for one container type. This drastic reduction in planning time allowed the company to respond quickly to any changes related to the plan. Moreover, the planning burden of experienced planners was significantly reduced which freed the planners for more productive works.

Prior to our project, the planners had seldom used the space of the company's own liner ships newly generated by unloading full containers when repositioning empty containers between short shipping-distance ports, e. g., Busan port and Shanghai port. Nor had they often changed the original direction of empty containers even when the change may reduce related costs. However, the model in the planning system explicitly considers the space of liner ships and the change of directions of empty containers, as described in Section 2. The planning system enabled the company to improve its usage of liner ships' space by as much as 11 % in 2008, compared to that in 2007. Although we wished to quantify the cost reduction by the system's implementation, the company did not wish to publish these details. However, we anticipated that the system significantly reduced the related costs, especially lease cost owing to on-time reposition of empty containers.

Although forecasting the supply and demand of empty containers was beyond the scope of our project, the forecasting accuracy was also improved partially owing to our work. Before implementing the system, the supply and demand of empty containers was forecasted by a simple moving average and sales force-estimate methods. In addition, the salespeople had sometimes not inserted their forecasts into the company's existing database. However, owing to our system, the company realized the importance of the forecast and hence continuously educated the sales-

people on how to forecast and insert the forecast data. In addition, the moving average method was intensified by considering the statistics of the supply and demand data, which improved the forecasting accuracy from approximately 50% to 78%.

Based on the success of our planning system, the company plans to extend the model and system by considering further realistic constraints, e.g., batch transportation, minimum and maximum transportation amount, schedule and available space of a train, and consideration of multiple container types in a single model. In this case, a heuristic algorithm willbe a viable tool for the system to satisfy the quick running requirement in real operations.

5. Concluding Remarks

In order to solve the problem of obtaining enough empty containers at a global maritime company in Korea, we developed a planning system to optimize the company's reposition of empty containers between global depots by a linear programming model. The problem formulated in the model is the determination of the transportation volume, route, and mode for repositioning empty containers in order to satisfy the demand at globally scattered depots. The objective is to minimize the sum of the inventory holding, transportation, penalty, and lease costs. We extended previous research by incorporating the space of ships dynamically changing due to the unloading or loading of containers. The proposed planning system benefited the company by improving the space usage of liner ships and is expected to reduce related costs. In addition, the forecasting accuracy was improved owing to the lesson of our work. The system has been in continuous operation in the company's global network since March 2008.

Due to the company's request for quick implementation, we only developed a simple linear programming, which therefore needs to be extended by considering batch transportation given in the Appendix. In this case, a heuristic algorithm should be developed to generate the plan since the problem with batch transportation is highly likely to be an NP-hard problem. In addition, the single container-type model developed in our project should be extended to the multiple container-type model, in which the vessel space allocation between containers is an additional model consideration. Finally, this research should be extended by considering random factors such as random demand, random supply, and random capacity of transport modes.

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Appendix

Let v_{ijt}^m and u_{ijt}^m be lower and upper limits, respectively, on the transportation of transport mode m from depot i to j in period t, Y_{ijt}^m be 1 if any transportation from depot i to j occurs by transport mode m, and 0 otherwise, and s_{ij}^m be the fixed ordering cost (related to the batch transport) of transport mode m from depot i to j. Then, the transportation is bounded as the following constraints :

$$v_{ijt}^m \cdot Y_{ijt}^m \leq \sum_{w \in W} X_{ijt}^{wm} \leq u_{ijt}^m \cdot Y_{ijt}^m$$
$$\text{for } i \in D, \ j \in IP_i \cup ID_i, \ t \in T, \ w \in W$$
$$\text{and } m \in M_{ij} \text{ or } i \in P, \ j \in P_i \cup ID_i, \ t \in T, \ w \in W$$
$$\text{and } m \in M_{ij} \tag{A1}$$

$Y_{ijt}^m \in \{0, 1\}$
 for $i \in D$, $j \in IP_i \cup ID_i$, $t \in T$, $w \in W$
 and $m \in M_{ij}$ or $i \in P$, $j \in P_i \cup ID_i$, $t \in T$, $w \in W$
 and $m \in M_{ij} \cup V_{ijt}$ (A2)

In constraint (A1), the binary variable $Y_{ijt}^m > 0$ is multiplied to the limits so as to make the constraint activate once $X_{ijt}^m > 0$. The constraints (A1) and (A2) determine the transportation batch size if the objective function of the optimization model in Section 3 is modified as follows :

Minimize

$$
\sum_{i \in D \cup P} \sum_{t \in T} h_i \cdot I_{it} + \sum_{i \in D} \sum_{j \in IP_i \cup ID_i} \sum_{w \in W} \sum_{t \in T} \sum_{m \in M_{ij}}
$$
$$
(c_{ij}^m \cdot X_{ijt}^{wm} + s_{ij}^m \cdot Y_{ijt}^{wm})
$$
$$
+ \sum_{i \in P} \sum_{j \in P_i \cup ID_i} \sum_{w \in W} \sum_{t \in T} \sum_{m \in M_{ij} \cup V_{ijt}}
$$
$$
(c_{ij}^m \cdot X_{ijt}^{wm} + s_{ij}^m \cdot Y_{ijt}^{wm}) + \sum_{i \in D \cup P} \sum_{t \in T} b_i \cdot L_{it}
$$
$$
+ \sum_{i \in D} \sum_{j \in IP_i \cup ID_i} \sum_{t \in T} \sum_{m \in M_{ij}} p_{ij} \cdot (CX_{ijt}^{m+} + CX_{ijt}^{m-})
$$
$$
+ \sum_{i \in P} \sum_{j \in P_i} \sum_{t \in T} \sum_{m \in M_{ij} \cup V_{ijt}} p_{ij} \cdot (CX_{ijt}^{m+} + CX_{ijt}^{m-})
$$

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