Warranty Analysis Based on Different Lengths of Warranty Periods

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Abstract

Global companies can sell their products with different warranty periods based on location and times. Customers can select the length of warranty on their own if they pay an additional fee. In this paper, we consider the warranty period and the repair time limit as random variables. A two-dimensional warranty policy is considered with repair times and failure times. The repair times are considered within the repair time limit and the failure times are considered within the warranty period. Under the non-renewable warranty policy, we obtain the expected number of warranty services and their variances in the censored area by warranty period and repair time limit to conduct a warranty cost analysis. Numerical examples are discussed to demonstrate the applicability of the methodologies and results using field data based on the proposed approach in the paper.

Keywords: Random variable, non-renewable warranty, two-dimensional warranty, warranty period.

1. Introduction

Manufacturers use warranty policies as a marketing tool with hopes to increase the sales and to minimize the related warranty costs. An an appropriate warranty period is an important measure for the manufacturers to minimize the warranty costs. For example, if a warranty period is too long, then manufacturers are vulnerable to the higher cost of more claims and responsibilities. If the warranty period is too short, it could be a weak link to attract customers to purchase the product. As such, warranty becomes an important factor for consumers and manufacturers.

One of the main interests that arise from the warranty policy is to obtain the optimal warranty period and its corresponding warranty cost analysis. The range of warranty cost analysis needs to consider the characteristic of the warranty policy and replacement/repair cost as well as the distribution of the number of product failures. Different models (Blischke, 1994; Blischke and Murthy, 1996) have been studied to provide guidance in selecting the optimal warranty plans. Further, usage and age have been studied as two dimensions by many researchers. Using a two-dimensional warranty policy, we calculate the warranty cost and investigate the statistical properties of warranty models. Several researchers (Chen and Popova, 2002; Chukova and Johnston, 2006; Chun and Tang, 1999) have proposed two-dimensional models under warranty. Chukova and Johnston (2006) consider that the warranty has options in choosing the degree of repair applied to an item that has failed within the warranty period and develop a particular warranty repair strategy, related to the degree of the warranty repair, for non-renewing, two-dimensional, free of charge to the consumer warranty policy. Iskandar *et al.* (2005) investigate a new warranty servicing strategy for items sold with two-dimensional warranty where the failed item is replaced by a new one when it fails for the first time in a specified region

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of the warranty and all other failures are repaired minimally. In Chen and Popova (2002)'s paper, they suggest a new maintenance policy which minimizes the total expected servicing cost for an item with a two-dimensional warranty.

In this paper, we regard that the repair times and failure times are two dimensions. Whenever a product fails, it is delivered to the customer service center. Then the manufacturer is assumed to provide the repair service first; however, if they cannot fix it within the repair time limit, then they provide a replacement service and discontinue the repair option. Under the idea, we develop two-dimensional warranty model and investigate the warranty cost analysis. If the failure times and the repair times are independent, marked Poisson process could be used to develop the two-dimensional warranty model. If they are dependent, bivariate distributions could be used to model the failure times and the repair times. In either case, the number of warranty services can be determined under warranty.

In this paper, we assume that the warranty period and the repair time limits are not fixed, but random variable in the two dimensional warranty policy. It is because the warranty period may not be fixed but could be different based on the locations and the times. For example, a big global company like Hyundai Motors wants to sell their products in different countries with different periods of warranty. Hyundai Motors sells their cars with a 10 years and 100,000 miles warranty in the U.S.; however, Hyundai's car has been sold with a much shorter warranty duration and warranty miles in Korea. In addition, this can be different based on the period that the warranty is offered. For instance, their warranty periods were not 10 years and 100,000 miles even in the U.S until 1990s. When a customer wants to buy a car, they can select the warranty period and can extend the warranty period if they would like to pay the additional money. We consider that warranty period and the repair time limit random variables because they could be changed based on locations, times and customer's selection. Furthermore, failures times and repair times are random variables, too. So four random variables are considered and we obtain the expected number of warranty services during the warranty period.

Under the two-dimensional warranty policy, two kinds of warranty services are considered. One is a repair service and the other is a replacement service. The typical well-known two-dimensional warranty policy is to use usage and age/time as two-dimensions. In this paper, totally different two-dimensions are used such as the failure times and the repair times. Many researchers have studied age and product usage as two-dimensional warranty model. A vehicle is very good example for the two-dimensional warranty policy using age and product usage; however, in many cases, it is not easy to obtain the information regarding the product usage. For example, if we consider electric appliances like laptops, refrigerators and radios as well as facilities like nuclear reactor, it is easy to obtain their ages but it may not easy to know their usages. But, we may easily obtain the information regarding failure times and repair times if they have experienced failure and repair that enables their use for a two-dimensional warranty policy. Therefore, in the study, we consider failure times and repair times as two dimensions for warranty policy. Using the field data, the product's failure times and repair times have been investigated for the two-dimensional warranty policy in this study. In the developed two-dimensional warranty policy, if a customer's failed product is delivered to the customer service center for repair services, the customer service center is to return the repaired product back within the threshold time for the customer's satisfaction. Therefore, if the failed product cannot be repaired after the time being, the replacement service is provided instead of the repair service. The failure times would be censored by the warranty period, while the repair times would be censored by the limitation of the repair time. If the repair time for a failed system exceeds the time limit, then it is replaced, rather than being continued for repair.

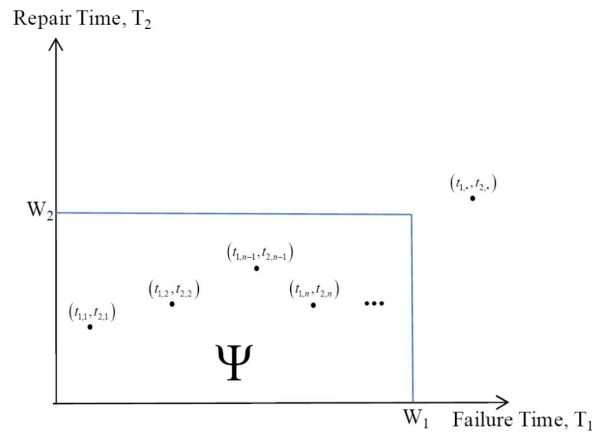In Figure 1, a two-dimensional warranty policy is described. W_1 represents the warranty period

Repair Time, T$_2$

$(t_{1,\cdot}, t_{2,\cdot})$

W$_2$

$(t_{1,n-1}, t_{2,n-1})$

$(t_{1,2}, t_{2,2})$ $(t_{1,n}, t_{2,n})$

$(t_{1,1}, t_{2,1})$ $\cdots$

Ψ

W$_1$ Failure Time, T$_1$

Figure 1: *Two-dimensional warranty service model under a non-renewable policy*

and W_2 represents a time limit for the repair service. The horizontal axis is the failure time and the vertical axis is the repair time. Repair times which are less than the repair time limit are only considered in the warranty period and they are not included in the warranty period for the customer's satisfaction. The failure time, repair time, warranty period, and repair time limit are all assumed to be random variables . We investigate how many times the warranty services happen in the censored area and examine the distribution of the number of failures for a warranty cost analysis.

In this paper, we develop a two-dimensional warranty models with repair times and failure times. The failure time is the interval between product's recovery time for previous failure and next failure time; the repair time is the interval between a failure time and its recovery time. For this study, the following assumptions are needed to develop the cost model.

1) Each failure of a component of the system during the warranty period is immediately detected.

2) A repair time is not included in the warranty period because the repair times is relatively short compared to the warranty period. It increases customer satisfaction.

3) Repair and replacement do not happen simultaneously.

4) All warranty claims are executed and all claims are valid.

5) When a product fails, the repair service would be provided first.

2. Warranty Cost Analysis

2.1. Two-dimensional bivariate distribution

Let $M(W_1, W_2)$ be the bivariate renewal function. Two-dimensional renewal function plays an important role in the analysis of two-dimensional warranty policies; however, it is difficult to obtain analytic expressions for $M(W_1, W_2)$ and computational procedures are generally required. Hunter (1974) obtains the analytical expression for $M(W_1, W_2)$ using Downton's bivariate exponential distribution(BED) (1970). It is rare that the transform is invertible in closed form. For most of the bivariate models, closed Laplace transform inversions are not available (Nachlas, 2005). In the paper, two- dimensional renewal function is developed. Let $M(W_1, W_2)$ be the number of warranty services within

the warranty period and let (x, y) be failure times and repair times respectively. Later, their parameters could be calculated using the field data. A bivariate extension of the exponential distribution is proposed as a model for certain problems in reliability engineering. The exponential distribution plays a fundamental role as a model in a variety of applications, typically connected with survival time, in some of its many forms of appearance. Unfortunately (unlike the normal distribution) the exponential distribution does not have a natural extension to the bivariate or the multivariate case. Therefore, a large number of classes of bivariate distributions with exponential marginals have been proposed since 1960. Among them, the BED with the memoryless property are Marshall and Olkin's (1967), Freund's (1961) and Block and Basu's (1974) from Kotz and Singpurwalla (1999). On the contrary, the BED without the memoryless property is Raftery's (1984). In addition, if the marginal distributions of BED are exponential, then we can use the BED for the field data. Marshall and Olkin's BED (1967) and Raftery's BED (1984) have exponential marginals. Freund's BED (1961) and Block and Basu's BED (1974) have marginals which are a mixture of exponential distributions. In the study, Marshall and Olkin's BED is chosen for the warranty cost analysis because it has a memoryless property and exponential marginal. In Marshall and Olkin's BED, both the marginals have exponential distribution and can be equal with a positive probability. Because of that reason, if in a bivariate data set, for some cases two dimensions take values with positive probabilities, the Marshall and Olkin's BED can be used quite effectively to analyze such data set (Kundu and Dey, 2009). The Marshall and Olkin's (1967) BED's joint probability density function is given by

$$f(x, y) = \theta_1 (\theta_2 + \theta_3) \exp\left(-\theta_1 x - (\theta_2 + \theta_3) y\right), \tag{2.1}$$

for $0 < x < y$ and

$$f(x, y) = \theta_2 (\theta_1 + \theta_3) \exp\left(-\theta_2 x - (\theta_1 + \theta_3) y\right), \tag{2.2}$$

for $0 < y < x$ and

$$f(x, y) = \theta_3 \exp\left(-(\theta_1 + \theta_2 + \theta_3) y\right), \tag{2.3}$$

for $0 < x = y$, when $x > 0$, $y > 0$, $\theta_1 > 0$, $\theta_2 > 0$, $\theta_3 > 0$.

The marginal pdfs of X and Y are exponential with parameters $\theta_1 + \theta_3$ and $\theta_2 + \theta_3$, respectively; so,

$$E(X) = \frac{1}{\theta_1 + \theta_3}, \qquad E(Y) = \frac{1}{\theta_2 + \theta_3}. \tag{2.4}$$

The correlation coefficient $\rho = \text{Cor}(X, Y)$ is given by

$$\rho = \frac{\theta_3}{\theta_1 + \theta_2 + \theta_3}. \tag{2.5}$$

2.2. Expected number of warranty services

We start by determining $E[N(W_1, W_2)]$ the expected number of warranty services in the censored area of (W_1, W_2). First, we condition on X_1 and Y_1, the times of the first failure renewal and the first repair renewal. Using the conditional probability, $E[N(W_1, W_2)]$ can be written as follows:

$$E[N(W_1, W_2)] = E[E[N(W_1, W_2)|X_1, Y_1]]$$
$$= \int_0^\infty \int_0^\infty E[N(W_1, W_2)|X_1 = x, Y_1 = y] f(x, y) dx dy \tag{2.6}$$

where $f(x, y)$ is the joint inter-arrival density. To determine $E[N(W_1, W_2)|X_1 = x, Y_1 = y]$, we now condition on whether or not the two constants (W_1, W_2) exceed (x, y), respectively. Therefore, we consider 4 cases as follows:

$$
\begin{aligned}
&1) \quad W_1 < x \quad \text{and} \quad W_2 < y, \\
&2) \quad W_1 \geq x \quad \text{and} \quad W_2 < y, \\
&3) \quad W_1 < x \quad \text{and} \quad W_2 \geq y, \\
&4) \quad W_1 \geq x \quad \text{and} \quad W_2 \geq y.
\end{aligned}
\tag{2.7}
$$

If we are given that $W_1 \geq x$ and $W_2 \geq y$ then the number of renewals by time will equal 1 plus the number of additional renewals between W_1 and x and between W_2 and y. However, if the inter-failure intervals follow a BED that has the bivariate lack of memory property, it follows $W_1 < x$ and $W_2 < y$ given that the amount by which they exceed x and y is a bivariate exponential. Given that the number of renewals between W_1 and x and between W_2 and y will have the same distributions as $N(W_1, W_2)$ by the memoryless property of exponential random variables. In addition, if the repair time exceeds the repair time limit in the warranty period, *i.e.* $W_1 \geq x$ and $W_2 < y$ then the warranty service center provides the replacement service instead of continuing to fix the failed product for the customer's satisfaction. Therefore, the expected number of warranty services includes the repair services and the replacement services together. On the other hand, for other cases, as the first renewal occurs by times x and y, it follows that the number of renewals by times (W_1, W_2) is equal to zero. Hence,

$$
\begin{aligned}
&E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 < x, W_2 < y\right] = 0, \\
&E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 \geq x, W_2 < y\right] = 0, \\
&E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 < x, W_2 \geq y\right] = 1, \\
&E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 \geq x, W_2 \geq y\right] = 1 + E\left[N\left(W_1, W_2\right)\right].
\end{aligned}
\tag{2.8}
$$

Using Equation (2.8) if the first failure time is X_1 and its repair time is Y_1 the expected number of warranty services within repair service time limitation W_2 and the warranty period W_1 is given by

$$
\begin{aligned}
&E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y\right] \\
&= E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 < x, W_2 < y\right] P(W_1 < x, W_2 < y|X_1 < x, Y_1 < y) \\
&\quad + E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 \geq x, W_2 < y\right] P(W_1 \geq x, W_2 < y|X_1 < x, Y_1 < y) \\
&\quad + E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 < x, W_2 \geq y\right] P(W_1 < x, W_2 \geq y|X_1 < x, Y_1 < y) \\
&\quad + E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 \geq x, W_2 \geq y\right] P(W_1 \geq x, W_2 \geq y|X_1 < x, Y_1 < y) \\
&= E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 < x, W_2 \geq y\right] P(W_1 < x, W_2 \geq y) \\
&\quad + E\left[N\left(W_1, W_2\right)|X_1 = x, Y_1 = y, W_1 \geq x, W_2 \geq y\right] P(W_1 \geq x, W_2 \geq y) \\
&= P(W_1 < x, W_2 \geq y) + (1 + E[N(W_1, W_2)])P(W_1 \geq x, W_2 \geq y).
\end{aligned}
\tag{2.9}
$$

P_1 denotes $P(W_1 < x, W_2 \geq y)$ and P_2 denotes $P(W_1 \geq x, W_2 \geq y)$. Let $E[N(W_1, W_2)]$ be the expected number of the warranty services in the area which censored by warranty period W_1 and repair time limit W_2. Substituting Equation (2.9) into Equation (2.6), we obtain

$$
\begin{aligned}
E[N(W_1, W_2)] &= \int_0^\infty \int_0^\infty (P_1 + (1 + E[N(W_1, W_2)])P_2)f(x, y)dxdy \\
&= \int_0^\infty \int_0^\infty (P_1 f(x, y) + (1 + E[N(W_1, W_2)])P_2 f(x, y))\, dxdy
\end{aligned}
$$

$$= \int_0^\infty \int_0^\infty P_1 f(x,y) dx dy + (1 + E[N(W_1, W_2)]) \int_0^\infty \int_0^\infty P_2 f(x,y) dx dy \qquad (2.10)$$

or

$$E[N(W_1, W_2)] = \frac{\int_0^\infty \int_0^\infty P_1 f(x,y) dx dy + \int_0^\infty \int_0^\infty P_2 f(x,y) dx dy}{1 - \int_0^\infty \int_0^\infty P_1 f(x,y) dx dy}. \qquad (2.11)$$

Equation (2.11) can be written by

$$M(W_1, W_2) = \frac{P(W_1 \geq X, W_2 \geq Y) + P(W_1 < X, W_2 \geq Y)}{1 - P(W_1 \geq X, W_2 \geq Y)}. \qquad (2.12)$$

To obtain the variance of the warranty system cost, we first need to calculate the second moment. Similarly to the first moment, we consider the first failure during the warranty period. We separate four cases such as Equation (2.8). Then, similarly to Equation (2.9),

$$E\left[N(W_1, W_2)^2 \middle| X_1 = x, Y_1 = y, W_1 \geq x, W_2 \geq y\right] = E\left[(1 + N(W_1, W_2))^2\right],$$
$$E\left[N(W_1, W_2)^2 \middle| X_1 = x, Y_1 = y, W_1 < x, W_2 \geq y\right] = 1 \qquad (2.13)$$

and remaining two cases equal to zero. Therefore,

$$E\left[N(W_1, W_2)^2 \middle| X_1 = x, Y_1 = y\right]$$
$$= \left(1 + 2E[N(W_1, W_2)] + E\left[N(W_1, W_2)^2\right]\right) P(W_1 \geq x, W_2 \geq y) + P(W_1 < x, W_2 \geq y). \qquad (2.14)$$

Using Equation (2.14), we obtain the second moment as follows:

$$E\left[N(W_1, W_2)^2\right]$$
$$= E\left[E\left[(N(W_1, W_2))^2 \middle| X_1 = x, Y_1 = y\right]\right]$$
$$= \int_0^\infty \int_0^\infty \left(E\left[(N(W_1, W_2))^2 \middle| X_1 = x, Y_1 = y\right]\right) f(x,y) dx dy$$
$$= \int_0^\infty \int_0^\infty \left(\left(1 + 2E[N(W_1, W_2)] + E\left[N(W_1, W_2)^2\right]\right) P_1 + P_2\right) f(x,y) dx dy$$
$$= \left(1 + 2E[N(W_1, W_2)] + E\left[N(W_1, W_2)^2\right]\right) \int_0^\infty \int_0^\infty P_1 f(x,y) dx dy + \int_0^\infty \int_0^\infty P_2 f(x,y) dx dy. \qquad (2.15)$$

After simplification, the second moment is given by

$$E\left[N(W_1, W_2)^2\right] = \frac{(1 + 2E[N(W_1, W_2)]) \int_0^\infty \int_0^\infty P_1 f(x,y) dx dy + \int_0^\infty \int_0^\infty P_2 f(x,y) dx dy}{1 - \int_0^\infty \int_0^\infty P_1 f(x,y) dx dy}, \qquad (2.16)$$

where $E[N(W_1, W_2)]$ is given as Equation (2.11).

Using the first moment and the second moment, we easily obtain the variance of the number of warranty services. Using similar ways for previous case, we obtain the variance as follows:

$$\text{Var}(N(W_1, W_2))$$
$$= \frac{P(W_1 \geq X, W_2 \geq Y)(P(W_1 < X, W_2 \geq Y) + P(W_1 \geq X, W_2 \geq Y) + 1) + P(W_1 < X, W_2 \geq Y)}{(1 - P(W_1 \geq X, W_2 \geq Y))^2}. \qquad (2.17)$$

Table 1: Failure times and repair times for nuclear power plants(failure time, date, repair time, and hours)

	Starting date	Failure date	Restarting date	Failure(day)	Repair(hours)
1	1978-04-29	1985-01-27	1985-01-27	2465.68	2.21
2	1983-07-25	1983-08-13	1983-08-13	19.41	5.54
3	1985-09-30	1985-10-19	1985-10-21	20.00	67.58
4	1986-04-29	1986-06-29	1986-07-01	61.96	68.29
5	1988-09-10	1988-09-12	1988-09-12	2.71	1.79
6	1989-09-30	1989-10-11	1989-10-12	11.67	67.83
7	1998-08-11	1999-01-22	1999-01-23	164.75	46.25
8	1999-12-31	2000-09-11	2000-09-12	255.66	40.46
9	2004-07-29	2004-11-30	2004-12-02	124.11	168.46
10	2005-04-22	2005-06-29	2005-07-01	68.68	137.42
11	1983-04-22	1983-05-01	1983-05-02	9.23	82.46
12	1997-07-01	1997-07-15	1997-07-15	14.85	0.04
13	1998-07-01	1998-12-18	1998-12-19	170.01	97.67
14	1999-10-01	2003-04-19	2003-04-20	1296.21	104.04
15	1986-08-25	1986-09-17	1986-09-19	23.90	119.50
16	1987-06-10	1987-06-16	1987-06-17	6.81	19.08
17	1995-03-31	1995-04-27	1995-04-27	27.49	15.75
18	1996-01-01	1996-05-15	1996-05-15	135.47	20.63
19	2002-05-21	2002-11-03	2002-11-05	166.57	89.33
20	2002-12-24	2005-07-02	2005-07-05	921.17	180.46

(From the operational performance information system for nuclear power plant)

3. Real Application and Numerical Examples

In Korea, there are four nuclear sites and, in 2010, there are 20 nuclear power plants in operation with a total licensed output amount to 17,716 MWe (MegaWatt electrical) and 8 nuclear power plants under construction, for a total of 28 units in operation by the end of 2016 from Safety and Operational Status of Nuclear Power Plants in Korea (2008). When 20 nuclear power units' first failure times and their repair times and the warranty period and repair time limit are considered as random variables, we obtain the expected number of warranty services in the warranty period. We briefly describe the field data and investigate them to check their dependency using Kendall's τ method. We implement our proposed approaches to conduct a warranty cost analysis using the field data.

3.1. Data description

There are 20 nuclear failures/repair data in Table 1. And their starting dates, first failure dates and restarting dates are described, the failure times can be obtained by that starting dates are subtracted from failure dates. Similarly, the repair times can be obtained in that the failure dates are subtracted from the restarting dates. The unit failure time is in days and the unit repair time is in hours.

From the operational performance information system of nuclear power plants, we obtain the failure data and repair data; however there is no information regarding warranty period and repair time limits. The data for the repair times and failure times are available but the data for the warranty period and repair time limit are not available. Therefore, we consider that by central limit theorem, the warranty periods and repair time limits are assumed to follow normal distribution, respectively. We investigate the warranty cost analysis using repair times and failure times of the nuclear power plants in the warranty period. Based on the dependency between the failures times and repair times, the ways for warranty cost analysis could be changed.

Using a Kendall's τ, we test the hypothesis that the failure times and repair times are dependent. Kendall's rank correlation measures the strength of monotonic association between the failure times and repair times. It may also be noted that usual Pearson correlation is fairly robust and it usually

Table 2: Estimated parameters in the Marshall and Olkin's BED

	θ_1	θ_2	θ_3	ρ
Estimated Parameters	0.014938	0.003306	0.000046	0.002492

Table 3: Expected number of warranty services under different μ_1 and μ_2 with $\sigma_1^2 = \sigma_2^2 = 1$, for the warranty period and repair time limit

μ_2	$\mu_1 = 1000$			$\mu_1 = 2000$			$\mu_1 = 3000$			$\mu_1 = 5000$		
	E	Var	CV	E	Var	CV	E	Var	CV	E	Var	CV
10	0.16	0.21	2.86	0.16	0.21	2.86	0.16	0.21	2.86	0.16	0.21	2.86
20	0.34	0.57	2.20	0.35	0.59	2.20	0.35	0.59	2.21	0.35	0.59	2.21
30	0.56	1.15	1.93	0.57	1.21	1.94	0.57	1.21	1.94	0.57	1.21	1.94
40	0.57	2.02	2.50	0.82	2.16	1.79	0.82	2.17	1.79	0.82	2.17	1.79
50	1.07	3.28	1.69	1.11	3.59	1.70	1.11	3.60	1.70	1.12	3.60	1.70
100	3.08	21.38	1.50	3.46	27.34	1.51	3.47	27.60	1.51	3.47	27.61	1.51

agrees well in terms of statistical significance with results obtained using Kendall's rank correlation. Based on the result of Kendall's τ method using R software (McLeod, 2005), τ is -0.284 and the p-value is 0.085515. Therefore, it is concluded that the failure times and repair times are dependent. In the numerical example, we consider that they are dependent and use bivariate distribution.

3.2. Expected number of warranty services

To illustrate the proposed method, we assume that a two-dimensional warranty has been provided by the manufacturer to have the warranty period and the time limitation of the repair services. Using the repair times and the failure times, we conduct a warranty cost analysis by Marshall and Olkin's BED. From the nuclear power plant field data, we calculate their BED's parameters in Table 2 using Equation (2.4) and (2.5).

By central limit theorem, we consider that the warranty period and repair time limit follow normal distribution, respectively. Further, W_1 and W_2 are dependent from Section 3.1. Based on the parameters in Table 2, we show the numerical example and the sensitivity analysis.

Table 3 shows the expected number of failures under warranty for the limitation parameters. Using Equation (2.11) and (2.17), we investigate the repair cost. We obtain the expected number of warranty services and its variance. As a result of the sensitivity analysis, the expected number of warranty services and its variance are described in Table 3. If W_1 and W_2 are normally distributed, then μ_1 and σ_1^2 stand for the expectation and variance of W_1 and μ_2 and σ_2^2 stand for the expectation and variance of W_2. In Table 3, based on the proposed cost models, Equation (2.11) and (2.17), we obtain the expected number of warranty services, their variance and coefficient of variation for different values which start at 10 and finish at 100 by 10 unit. We change the values μ_1 as 1000, 2000, 3000 and 5000 with $\sigma_1^2 = \sigma_2^2 = 1$. Figure 2 for the expected number of warranty services and their variance under different parameters for the warranty period and repair time limit are provided. Using Figure 2, we find out the changes of expected values of warranty services and their variances by changing of the parameters.

4. Concluding Remarks

In this paper, we showed the methodology for two-dimensional warranty policy using the failure times and repair times. The warrant period and repair time limit are considered random variables because they could be different based on their location, times, and customers' own selection. We develop the cost models to understand the expected values of warranty services and their variances in the warranty
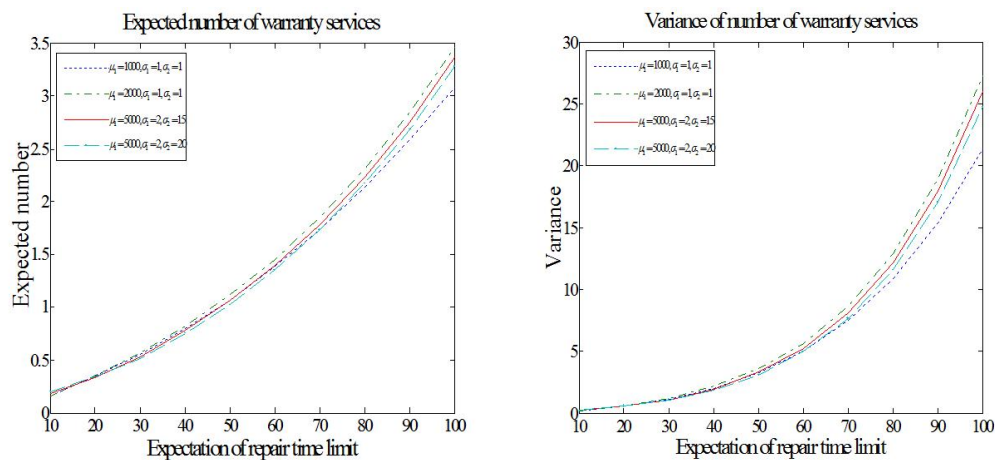
Figure 2: *Expected number of warranty services and its variance under different parameters for the warranty period and repair time limit*

period. In addition, our proposed approaches provide practical tools for practitioners and help them make important decisions for their companies. Further, as future research topics, if the failure times and repair times are not exponentially distributed, then we have to develop another models. They would be interesting topics.

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