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NURBS Interpolation Strategies of Complex Surfaces in High Speed Machining

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Abstract – The increase in the productivity and the assurance of quality machining on the NC machines depends on, amongst other things, the perfection of the programming using adequate methods of interpolation. The programming language is until now based on the code ISO 6983 which defines the principles of the code G. This latter is not well adapted to the new strategies of machining imposed by the machining of complex surfaces and machining at high speed with the increasingly more severe requirements of precision. The CNC which adopt the interpolation of NURBS (Non Uniform Rational B-spline) are very rare (FANUC Siemens…). Based on the advantages of NURBS (continuity, flexibility, smoothing....), new formats G are currently developed but their use is still very limited. Our work consists on putting forward these new approaches of programming using the interpolation of NURBS. For this reason, a program capable to trace NURBS trajectories under Visual BASIC 6.0 was developed. This program was used thereafter in CAM software for the generation of NURBS formats like their new formats NC.

***Keywords*: geometrical modeling, code G, NURBS interpolation, machining of the complex forms, generation of trajectory**

1. Introduction

When using modern CAD/CAM systems, designers adopt more free-form or contoured geometric shapes for the design and modeling of complex parts [1]. Since conventional CNC (Computer Numerical Control) machines only provide linear (G01) and circular (G02, G03) interpolations, CAM systems have to create many linear and circular segments to approximate the contoured geometry under given tolerances before ending NC codes to CNC machines. However, when the part accuracy becomes tighter, the conventional approach suffers from the following problems:

- Tighter tolerances result in shorter data segments, hence a larger volume of data transferred.
- Data transmission cannot catch up with the large amount of data transferred during high-speed machining.
- Feed rate fluctuation and velocity discontinuity exist at the junction of two connected line segments.
- Acceleration discontinuity and excessive jerk occurring at high speed can cause system vibration and reduce the machining quality.

These drawbacks indicate that it is difficult for the conventional approach to satisfy the requirements of high-speed and high-accuracy machining in modern manufacturing systems. In order to overcome the drawbacks, parametric curves and surfaces from early Bezier to more recent B-splines and NURBS [2,3] have been adopted by CAD/CAM and CNC systems. Since NURBS can represent both free-form and analytical geometry, it has emerged as standard geometry representation tool.

Today, though NURBS technology has found wide application in CAD/CAM, its uses in CNC seem to lag behind. So far, there are only a few top grade CNC systems like FANUC and SIEMENS supporting NURBS spline curve interpolation [4-5].

As shown in [6], the NURBS interpolation has many advantages over the linear interpolation, so it valuable to study it.

The work undertaken within the framework of this paper concerns the implementation of the interpolation by format NURBS in generation of trajectories of complex surfaces. This concept was developed in order to take into account the constraints of manufacturing associated with milling complex forms in a step with integrated design and to decrease the errors generated in generation of trajectories. We use this new modeling to implement a specific strategy of machining known as strategy of machining per format of NURBS.

2. B-splines curves

A Spline of degree N is a polynomial function per segments of degree N which is continuous of class C^{n-1} at each node. A curve Spline is defined by n+1 control points and n+1 weight functions [1-7-8].

The Spline basic uniforms are defined by the following expressions [1-7-8]. :

$$N_{i,1}(u)=\begin{cases}1 & \text{if } \; t_i \le u \le t_{i+1} \\ 0 & \text{otherwise}\end{cases} \tag{1}$$

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$$N_{i,k}(u)=\frac{(u-t_i)N_{i,k-i}(u)}{t_{i-k-1}-t_i}+\frac{(t_{i-k}-u)N_{i+,k-i}(u)}{t_{i-k}-t_{i+1}} \quad (2)$$

Where k controls the degree (k–1) of the resulting polynomial in u and also the continuity of the curve.

$$N+k+1=T \quad (3)$$

Where T the number of knots.

The final equation of the uniform B-Spline curve when $k = 4$ is [9]:

$$p_i(u)=\frac{1}{6}\begin{bmatrix}u^3 & u^2 & u & 1\end{bmatrix}\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 0 & 3 & 1\\ 1 & 4 & 1 & 0\end{bmatrix}\begin{bmatrix}p_{i-1}\\ p_i\\ p_{i+1}\\ p_{i+2}\end{bmatrix} \quad (4)$$

$i\in[1:n-2]$

2.1 Case study (1)

Suppose the initial control points of the desired curve are:

$p1 = (-10, -5, 0)$, $p2 = (4, -5, 0)$, $p3 = (-10, 3.6, 0)$, $p4 = (4, 3.6, 0)$.

And $k = 4$, draw the uniform B-Spline curve. Rearrange the control points in the matrix form as follows:

$p11 = (-10, -5, 0)$, $p12 = (4, -5, 0)$, $p13 = (-10, 3.6, 0)$, $p14 = (4, 3.6, 0)$.

Use the equation (4) to determine $p(u)$, under Visual BASIC 6, plot the control points and the curves result from the equation (4) as shown in figure (Fig. 1), with (u) ranging from (0) to (1).

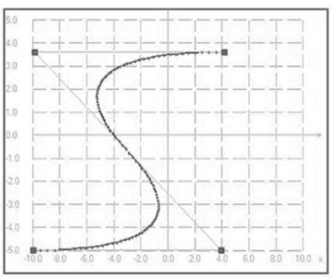


Fig. 1. curve B-spline (3rd degree), $k = 4$, form of Matrix (4×4) with control-points.

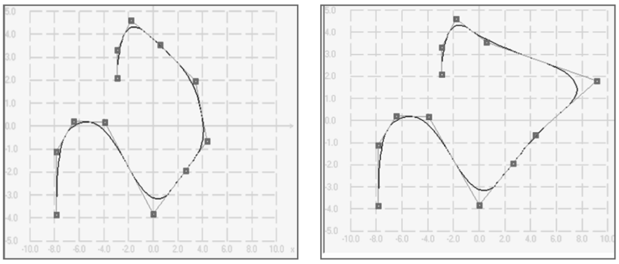


Fig. 2. Influence of the position of control points on B-spline curve.

A B-spline curve exhibits local control, a controls point is connected to four segments (in the case of a cubic) and moving a control point can influence only these segments (figure 2).

3. Generation of trajectories

At the present time, new tool machines with parallel structure are used in the industry. Their dynamic performances and their weak inertias have opened large perspectives for the machining of complex surfaces. For more than two decades, the programming language of the CNC machines was based on the ISO 6983 code defining the principles of the G code. We nowadays a reawakening of interest in the main interpolation G functions of trajectories. Thus the Formats by NURBS whose developed. This change of vision in the programming of the NC machines will be able to then allow a better integration CNC machines on flexible systems manufacturing. We thus will interest us to analyze work undertaken in this field.

4. NURBS parametric curves

The general form of a NURBS curve is defined as follows [1-10]

$$C(u)=\sum_{i=0}^{n}R_{i,p}(u)P_i=\frac{\sum_{i=0}^{n}N_{i,p}(u)w_iP_i}{\sum_{i=0}^{n}N_{i,p}(u)w_i}=\frac{A(u)}{w(u)} \quad (5)$$

$$R_{i,p}(u)=\frac{N_{i,p}(u)w_i}{\sum_{i=0}^{n}N_{i,p}(u)w_i} \quad (6)$$

Where $\{P_i\}$ are the control points, $\{w_i\}$ are the corresponding weights of $\{P_i\}$, and $\{w_iP_i\}$ are the weighted control points. In addition, $w(u)$ is the weighting function, $A(u)$ is the weighted B-spline function, (n+1) is the number of control points and p is the degree of the NURBS curve. $\{N_{i,p}(u)\}$ and $\{R_{i,p}(u)\}$ are the pth-degree B-spline basis functions and the rational B-spline basis functions defined on the non-uniform knot vector

$U = \{u_0, u_1, ..., u_{n+p+1}\}$, respectively. The pth-degree B-spline basis function is recursively defined as follows

$$N_{i,0}(u)=\begin{cases}1 & \text{if } u_i\le u\le u_{i+1}\\ 0 & \text{otherwise}\end{cases} \quad (7)$$

$$N_{i,p}(u)=\frac{u-u_i}{u_{i+p}-u_i}N_{i,p-1}(u)+\frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}}N_{i+1,p-1}(u) \quad (8)$$

$i=0,1,\ldots,n.$

The *m*th derivative of a NURBS curve [10] is given as:

$$N_{i,p}^{(m)}(u)=p\left(\frac{N_{i,p-1}^{(m-1)}(u)}{u_{i+p}-u_i}-\frac{N_{i+1,p-1}^{(m-1)}(u)}{u_{i+p+1}-u_{i+1}}\right) \quad (9)$$

$$\binom{m}{i}=\frac{m!}{(m-i)!i!}=\binom{m-1}{i}+\binom{m-1}{i-1} \quad (10)$$

$$C^{(m)}(u)=\frac{A^{(m)}(u)-\sum_{i=1}^{m}\binom{m}{1}w^{(i)}(u)C^{(m-1)}(u)}{w(u)} \quad (11)$$

$$w^{(m)}(u)=\sum_{i=0}^{n}N_{i,p}^{(m)}(u)w_i \quad (12)$$

$$A^{(m)}(u)=\sum_{i=0}^{n}N_{i,p}^{(m)}(u)w_iP_i$$

Where $w^{(m)}(u)$, $A^{(m)}(u)$ and $N_{i,p}^{(m)}(u)$ are the *m*th derivatives of the weighting functions, the weighted B-spline functions and the B-spline basis functions, respectively. Eq.(9) is called Pascal's formula.

4.1 NURBS interpolation (G06.2)

Many computer aided design (CAD) systems used to design metal dies for automobiles and airplanes utilize non uniform rational B-spline (NURBS) to express a sculptured surface or curve for the metal dies. This function enables NURBS curve expression to be directly specified to the CNC. This eliminates the need for approximating the NURBS curve with minute line segments [2]. This offers the following advantages:

- No error due to approximation of a NURBS curve by small line segments
- Short part program
- No break between blocks when small blocks are executed at high speed
- No need for high-speed transfer from the host computer to the CNC When this function is used, a computer-aided machining (CAM) system creates a NURBS curve according to the NURBS expression output from the CAD system, after compensating for the length of the tool holder, tool diameter, and other tool elements. The NURBS curve is programmed in the NC format by using these three defining parameters: control point, weight, and knot. NURBS interpolation must be specified in high-precision contour control mode (between G05 P10000 and G05 P0). The CNC executes NURBS interpolation while smoothly accelerating or decelerating the movement so that the acceleration on each axis will not exceed the allowable maximum acceleration of the machine. In this way, the CNC automatically controls the speed in order to prevent excessive strain being imposed on the machine.

4.2 NURBS interpolation mode

NURBS interpolation mode is selected when G06.2 is programmed in high-precision contour control mode. G06.2 is a modal G code of group 01. NURBS interpolation mode ends when a G code of group 01 other than G06.2 (G00, G01, G02, G03, etc.) is specified. NURBS interpolation mode must end before the command for ending high–precision contour control mode is programmed.

4.2.1 Format

G06.2 P_P;	G06.2: the G codes of NURBS
X_Y_Z_R_K_F_;	Interpolation
X_Y_Z_R_K_;	P: degree of the curve
....	F: feed rate
K_;	XYZ: coordinated control points
K_;	R: weight of the curve
....	K: value of the nodal vector

Fig. 6. Block programs for the interpolation by format NURBS [23].

4.3 Generation of trajectory per format of NURBS interpolation

Non Uniform Rational B-Splines (NURBS) have been used by CAD systems for some time [10]. That's why it seems so natural that CNCs should be able to employ tool paths that are also defined in terms of NURBS. However, most CNCs today instead require contoured tool paths to be defined using straight lines, or chords. And this long-practiced approach can lead to inefficiencies familiar to almost any die or mold shop. Using chords to define complex geometries accurately results in large, data-dense program files, files that historically have been difficult to manage and slow to execute. The development of NURBS-interpolating CNCs promised programs that could define the same complex geometries with fewer blocs of code, and thus could provide some relief for the data-flow bottlenecks.

Some researchers have proposed different approaches for precisely approximating planar parametric curves in CAD/CAM systems [11][12-13][14], and some investigators have proposed different interpolation algorithms for precisely interpolating parametric curves on CNC machines [15-16] [17][18-19]. The approximation [11][12-13][14] in CAD/CAM systems is usually implemented on off-line computation systems and thus the computation time may not be a critical problem. However, for obtaining good machining results, the interpolation algorithms [15-16] [17][18-19] must be implemented on CNC machines with limited interpolation time. Thus, the copious and complicated operations of NURBS curves usually limit the machining performances of interpolation algorithms

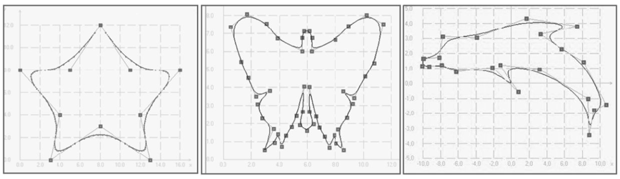


Fig. 7. a) The Star-shape NURBS curve (solid). b) The Butterfly-shape NURBS curve (solid) [25], c) The Dolphin-shape NURBS curve (solid).

[15-16] [17][18-19] in actual machining applications.

In this approach Works of [1-20-21-22-24-25], are interested by this theory.

4.3.1 Test and result

In this section, performances of NURBS interpolation algorithm by tree NURBS curves cases are contoured to validate the feasibility of the proposed method (Fig. 7).

Case 1: Star-shape curve (degree 2)

Control points:

$$\begin{bmatrix} 8 \\ 12 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix}\begin{bmatrix} 0 \\ 8 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \end{bmatrix}\begin{bmatrix} 13 \\ 0 \end{bmatrix}\begin{bmatrix} 12 \\ 4 \end{bmatrix}\begin{bmatrix} 16 \\ 8 \end{bmatrix}\begin{bmatrix} 11 \\ 8 \end{bmatrix}\begin{bmatrix} 8 \\ 12 \end{bmatrix}$$

Knot vector:

[0 0 0 0.111 0.222 0.333 0.444 0.555 0.666 0.777 0.888 1 1 1]

Weights: [1 1 1 1 1 1 1 1 1 1 1]

After inserting these points in our implementation we obtained the Star-shape NURBS curve [25], (Fig.7).

These points are inserted in software of CAM, and a program NC is generated.

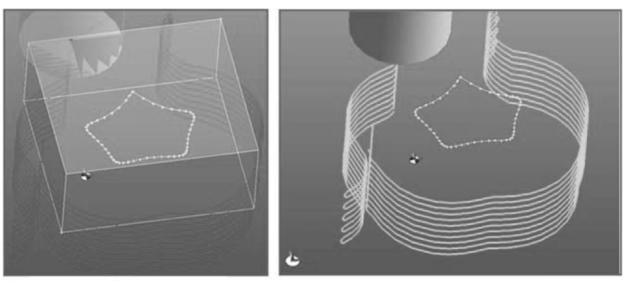
4.3.2 Validation

The programs that have developed can generate NURBS curves having complex shapes and send ASCI data to CAM software. Having chosen the strategies and the conditions of machining, we end up with a NC program which can be directly run on CNC tool machines (Fig. 8).

5. Conclusion

In this paper, we have analyzed new approaches of NC programming using NURBS interpolation. We have developed a Visual Basic program that plots NURBS curves and surfaces exploiting the advantages of this type of curves (continuity, flexibility, smoothing). For a chosen part, the geometric data are generated and sent to CAM software which generates the tool trajectories for the machining of the part with new format.

The great advantage of this technique is to follow with a minimum of information a curve which, to be approximate with sufficient smoothness, would require the definition of a great number of consecutive segments. The interpolator NURBS allow to define complex curves using a reduced number of parameters what also makes it possible to reduce considerably the length of the program of machining, to avoid the decomposition in segments of the courses of tool, and to have advances of the machine spindles without discontinuity.



a) Trajectory of tool with stock part

b) trajectory of tool without stock part

Fig. 8. Trajectory of tool in format of NURBS interpolation.

Appendix

Appendix A

Thus code NC for the machining of complex surface "**stat** NURBS curves" obtained on

ISO 3 Axis Mill machine tool is:

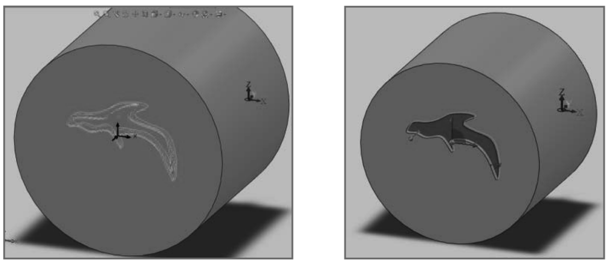
…….

N420 **G6.2** P4 K0.0 X-4.88 Y2.055
N430 K0.0 X-4.889 Y2.079
N440 K0.0 X-4.898 Y2.102
N450 K0.0 X-4.914 Y2.146
N460 K91.666 X-4.922 Y2.165
N470 K91.666 X-4.937 Y2.204
N480 K166.665 X-4.944 Y2.223
N490 K166.665 X-4.958 Y2.258
N500 K241.664 X-4.963 Y2.273
N510 K241.664 X-4.975 Y2.303
N520 K299.997 X-4.981 Y2.318
N530 K299.997 X-4.991 Y2.344
N540 K358.33 X-4.995 Y2.355
N550 K358.33 X-5.003 Y2.377
N560 K399.996 X-5.008 Y2.388
N570 K399.996 X-5.014 Y2.405
N580 K441.662 X-5.017 Y2.411
N590 K441.662 X-5.022 Y2.424
N600 K466.662 X-5.024 Y2.431
N610 K466.662 X-5.028 Y2.439
N620 K491.662 X-5.028 Y2.442
N630 K491.662 X-5.03 Y2.446
N640 K499.995 X-5.028 Y2.439
N650 K499.995 X-5.024 Y2.431
N660 K508.328 X-5.022 Y2.424
N670 K508.328 X-5.017 Y2.411
N680 K533.328 X-5.014 Y2.405
N690 K533.328 X-5.008 Y2.388
N700 K558.328 X-5.003 Y2.377
N710 K558.328 X-4.995 Y2.355
N720 K599.994 X-4.991 Y2.344
N730 K599.994 X-4.981 Y2.318
N740 K641.66 X-4.975 Y2.303

N750 K641.66 X-4.963 Y2.273
N760 K699.993 X-4.958 Y2.258
N770 K699.993 X-4.944 Y2.223
N780 K758.326 X-4.937 Y2.204
N790 K758.326 X-4.922 Y2.165
N800 K833.325 X-4.914 Y2.146
N810 K833.325 X-4.898 Y2.102
N820 K908.324 X-4.889 Y2.079
N830 K908.324 X-4.88 Y2.055
N840 K999.99
N850 K999.99
N860 K999.99
N870 K999.99
…………………

The total machining time including tool change is **1.327** minutes.

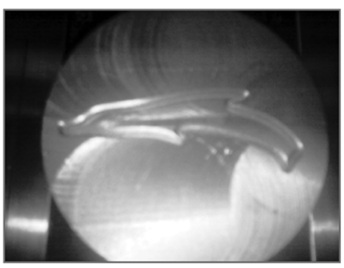
Appendix B



a) Contour trajectory of Dolphin-shape by NURBS curve tools b) Final contour of Dolphin-shape

Fig. 9. Trajectory of tool in format of NURBS interpolation.

a) Four axes milling cnc machine. b) Part in position of working



c) part Dolphin-shape by NURBS curve.

Fig. 10. Machining of Dolphin-shape with four axes milling cnc.

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