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Noninformative priors for the reliability function of two-parameter exponential distribution

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Abstract

In this paper, we develop the reference and the matching priors for the reliability function of two-parameter exponential distribution. We derive the reference priors and the matching prior, and prove the propriety of joint posterior distribution under the general prior including the reference priors and the matching prior. Through the simulation study, we show that the proposed reference priors match the target coverage probabilities in a frequentist sense.

Keywords: Matching prior, nonregular case, reference prior, reliability function.

1. Introduction

Consider X has an exponential distribution with scale parameter λ and the location parameter μ. Then the exponential distribution of X is given by

$$f(x|\mu,\lambda)=\lambda^{-1}\exp\left\{-\frac{x-\mu}{\lambda}\right\}, x\ge\mu>0, \lambda>0. \tag{1.1}$$

Then the reliability function $R(t)$ is given by

$$R(t)=P(X>t)=\exp\left\{-\frac{t-\mu}{\lambda}\right\}, t>\mu. \tag{1.2}$$

The present paper focuses on the noninformative priors for the reliability function $R(t)$.

In recent years, the notion of a noninformative prior has attracted much attention. Among several types of noninformative priors, reference prior proposed by Berger and Bernardo (1992) and probability matching prior initiated by Stein (1985) and Tibshirani (1989) are very useful in objective Bayesian inference. Most of the work has been centered around a

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smooth parametric families (regular families). Kim *et al.* (2009) developed reference priors and probability matching priors for regular family of Pareto distribution.

However, nonregular families, such as the uniform or shifted exponential, are also important in many practical problems. Ghosal and Samanta (1995) studied such families extensively. Ghosal and Smanta (1997) developed the reference priors for the case of one parameter families of discontinuous densities in the sense of Bernardo (1979), and Ghosal (1997) derived the reference priors for the multiparameter nonregular cases that the family of densities have discontinuities at some points which depend on one component of the parameter, while the family is regular with respect to the other parameters. Also Ghosal (1999) developed the probability matching prior for one parameter and two parameter cases under nonregular families. Quite often reference prior is the probability matching prior. Recently, Kang *et al.* (2010) developed the reference priors for the common location parameter in the half-normal distributions, and showed that the proposed reference prior matches the target coverage probabilities.

The problem of making inference about $R(t)$ of nonregular family of distribution has received much attention in literature. Guenther *et al.* (1976) gave a procedure which produces exact confidence limits for $R(t)$. Engelhardt and Bain (1978) proposed an approximate method for computing confidence limit with censored data. Also see Engelhardt (1995) and Lawless (1982). In particular, we refer to Engelhardt (1995) for a review and related references. Roy and Mathew (2005) proposed a confidence limit using the concept of a generalized confidence interval based on type II censored data. In numerical results of Roy and Mathew (2005), they showed that the proposed generalized confidence limit is an extremely satisfactory confidence limit in the sense of the coverage of confidence interval. On the other hand for the confidence interval of Engelhardt and Bain (1978), the coverage can be unsatisfactory and the interval can be conservative when t is large compared to μ. Varde (1969) and Sinha and Guttman (1976) gave the Bayes estimator for reliability function based on conjugate prior and noninformative prior for each parameters, respectively.

In this paper, we develop the reference priors and the matching prior for the reliability function when X has two-parameter exponential distribution. The outline of the remaining sections is as follows. In Section 2, we develop reference priors and matching prior for the reliability function. In Section 3, we provide that the propriety of the posterior distribution for the general prior including the reference priors and the matching prior. In Section 4, simulated frequentist coverage probabilities under the derived priors are given.

2. The noninformative priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. Ghosal (1997) derived the reference prior in sense of Bernardo (1979) for multiparameter nonregular cases. We derive the reference priors for different groups of orderings of (μ, θ) by following Ghosal (1997).

Let X be a random sample from an exponential distributions with scale parameter λ and the location parameter μ. Then the reliability function is $R(t) = \exp\{-(t-\mu)/\lambda\}$. The parameter $\theta = (t-\mu)/\lambda$ is of interest. Note that the problem for reliability function is equivalent to θ in developing noninformative priors.

Let $\theta = \frac{(t-\mu)}{\lambda}$ and $\mu = \mu$. Then with this reparameterization, likelihood function from the (1.1) is given by

$$f(x|\mu,\theta) = \frac{\theta}{t-\mu}\exp\left\{-\frac{(x-\mu)}{t-\mu}\theta\right\} \tag{2.1}$$

where $\theta > 0$ and $x > \mu$.

We firstly derived the reference prior when θ is parameter of interest.

The reference prior is developed by considering a sequence of compact subsets of the parameter space, and taking the limit of a sequence of priors as these compact subsets fill out of the parameter space. The compact subset was taken to be Cartesian products of sets of the form $\mu \in [a_1, b_1]$.

From the likelihood function (2.1), the quantity $F(\mu,\theta)$ is given by

$$F(\mu,\theta) = \theta^{-2}, \tag{2.2}$$

where $F(\mu,\theta) = \{4J_{11}(\mu,\theta)\}$,

$$J_{11}(\mu,\theta) = \int g_\theta(x;\mu,\theta) g_\theta(x;\mu,\theta)dx,$$

$g_\theta = \partial g/\partial\theta$ and $g = f^{\frac{1}{2}}$. And also $c(\mu,\theta)$ is given by

$$c(\mu,\theta) = E_{\mu,\theta}[\partial \log f/\partial\mu] = \frac{\theta}{t-\mu}.$$

Thus the conditional reference prior for μ given θ is

$$\pi(\mu|\theta) = c(\mu,\theta) = \frac{\theta}{t-\mu}. \tag{2.3}$$

The normalizing constant $K_l(\theta)$ of the reference prior $\pi(\mu|\theta)$ is given by

$$K_l(\theta) = \left(\int_{a_1}^{b_1}\pi(\mu|\theta)d\mu\right)^{-1} = \left(\int_{a_1}^{b_1}\frac{\theta}{t-\mu}d\mu\right)^{-1} = \theta^{-1}[\log(t-a_1)/(t-b_1)]^{-1}. \tag{2.4}$$

and so we obtain

$$p_l(\mu|\theta) = K_l(\theta)\pi(\mu|\theta) = [\log(t-a_1)/(t-b_1)]^{-1}(t-\mu)^{-1}. \tag{2.5}$$

Thus the marginal reference prior for θ is given by

$$\pi_l(\theta) = \exp\left\{\int_{a_1}^{b_1} p_l(\mu|\theta)\log\theta^{-1}d\mu\right\} = \theta^{-1}. \tag{2.6}$$

Therefore the reference prior for (μ,θ), when θ is parameter of interest, is given by

$$\begin{aligned}\pi_1(\mu,\theta) &= \lim_{l\to\infty}\left[\frac{K_l(\theta)\pi_l(\theta)}{K_l(\theta_0)\pi_l(\theta_0)}\right]\pi(\mu|\theta)\\ &\propto (t-\mu)^{-1}\theta^{-1}, \end{aligned} \tag{2.7}$$

where θ_0 is a fixed point. Also when both μ and θ are parameters of interest, the reference prior for (μ, θ) is given by

$$\begin{aligned}\pi_2(\mu,\theta) &= c(\mu,\theta)[\det F(\mu,\theta)]^{\frac{1}{2}} \\ &\propto (t-\mu)^{-1}.\end{aligned} \tag{2.8}$$

When θ is parameter of interest, the reference prior for (μ, θ) based on an appropriate penalty term of Ghosh and Mukerjee (1992) (and also see Ghosal, 1997) given by

$$\pi_3(\mu,\theta) = [\det F(\mu,\theta)]^{\frac{1}{2}} = \theta^{-1}. \tag{2.9}$$

Next we consider the matching prior for θ. Ghosal (1999) developed the probability matching prior for one parameter and two parameter cases under nonregular families. We derive the probability matching prior when θ is parameter of interest by following Ghosal (1999). A probability matching prior is a solution that satisfies a differential equation of Ghosal (1999), that is

$$\frac{1}{\lambda(\mu,\theta)}\frac{\partial}{\partial\theta}\{\log\pi(\mu,\theta)\} + \frac{\partial}{\partial\theta}\{\frac{1}{\lambda(\mu,\theta)}\} = 0, \tag{2.10}$$

where

$$\lambda(\mu,\theta) = \sqrt{-2E\left[\frac{\partial^2}{\partial\theta^2}\log f(X;\mu,\theta)\right]}.$$

Then since $\lambda(\mu,\theta) = \theta^{-1}$, (2.10) simplifies to

$$\theta\frac{\partial}{\partial\theta}\{\log\pi(\mu,\theta)\} + \frac{\partial}{\partial\theta}\{\theta\} = 0. \tag{2.11}$$

Hence the set of solution of (2.11) is of the form

$$\pi(\mu,\theta) = \theta^{-1}d(\mu), \tag{2.12}$$

where $d(\mu) > 0$ is an arbitrary function of μ.

Remark 2.1 Note that the reference priors (2.7) and (2.9) satisfy a matching criterion.

3. Implementation of the Bayesian procedure

We investigate the propriety of posteriors for a general class of priors which include the reference priors (2.7), (2.8) and (2.9). We consider the class of priors

$$\pi_g(\mu,\theta) \propto \theta^{-a}(t-\mu)^{-b}. \tag{3.1}$$

where $a \geq 0$ and $b \geq 0$. The following general theorem can be proved.

Theorem 3.1 The posterior distribution of (μ, θ) under the general prior (3.1) is proper if $n - a + 1 > 0$.

Proof: Under the general prior (3.1), the joint posterior for μ, θ given $\mathbf{x}$ is

$$\pi(\mu, \theta|\mathbf{x}) \propto (t-\mu)^{-n-b}\theta^{n-a} \exp\left\{-\frac{\theta}{t-\mu}\sum_{i=1}^{n}(x_i-\mu)\right\}, \tag{3.2}$$

Then integrating with respect to θ in (3.2) if $n-a+1>0$, we have the posterior

$$\begin{aligned}\pi(\mu|\mathbf{x}) &\propto (t-\mu)^{-a-b+1}(\bar{x}-\mu)^{-(n-a+1)} \\ &\propto (t-\mu)^{-n-b}\left[1+\frac{\bar{x}-t}{t-\mu}\right]^{-(n-a+1)},\end{aligned} \tag{3.3}$$

where $\bar{x} = \sum_{i=1}^{n} x_i/n$. Then

$$\int_0^{x_{(1)}} \pi(\mu|\mathbf{x})d\mu \le \int_0^{x_{(1)}} (t-\mu)^{-n-b}d\mu < \infty. \tag{3.4}$$

This completes the proof. □

Theorem 3.2 Under the general prior (3.1), the marginal posterior density of θ_1 is given by

$$\pi(\theta|\mathbf{x}) \propto \int_0^{x_{(1)}} (t-\mu)^{-n-b}\theta^{n-a} \exp\left\{-\frac{\theta}{t-\mu}\sum_{i=1}^{n}(x_i-\mu)\right\} d\mu. \tag{3.5}$$

Note that normalizing constant for the marginal density of θ requires an one dimensional integration. Therefore we can have the marginal posterior density of θ and so we compute the marginal moment of θ. In Section 4, we investigate the frequentist coverage probabilities for the reference priors π_1, π_2 and π_3, respectively.

4. Numerical study

We investigate the frequentist coverage probability by investigating the credible interval of the marginal posteriors density of θ under the noninformative prior π given in Section 3 for several configurations (μ, λ) and n. That is to say, the frequentist coverage of a $100(1-\alpha)\%th$ posterior quantile should be close to $1-\alpha$. This is done numerically.

Table 4.1 Frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for θ

μ	λ	t	n	π_1	π_2	π_3
0.1	0.1	1.0	10	0.050 (0.951)	0.101 (0.976)	0.050 (0.951)
			20	0.051 (0.950)	0.082 (0.971)	0.050 (0.950)
			30	0.051 (0.948)	0.077 (0.965)	0.051 (0.948)
		3.0	10	0.054 (0.947)	0.105 (0.972)	0.054 (0.947)
			20	0.052 (0.951)	0.083 (0.970)	0.052 (0.951)
			30	0.049 (0.952)	0.072 (0.967)	0.049 (0.952)
		5.0	10	0.047 (0.950)	0.097 (0.975)	0.047 (0.950)
			20	0.046 (0.949)	0.077 (0.969)	0.046 (0.949)
			30	0.051 (0.953)	0.076 (0.967)	0.051 (0.953)
		10.0	10	0.049 (0.953)	0.100 (0.976)	0.049 (0.953)
			20	0.051 (0.952)	0.081 (0.970)	0.051 (0.952)
			30	0.052 (0.950)	0.075 (0.966)	0.052 (0.950)
	1.0	3.0	10	0.058 (0.958)	0.110 (0.979)	0.058 (0.958)
			20	0.052 (0.951)	0.083 (0.968)	0.052 (0.951)
			30	0.044 (0.951)	0.071 (0.965)	0.044 (0.951)
		5.0	10	0.059 (0.961)	0.112 (0.978)	0.059 (0.961)
			20	0.052 (0.956)	0.086 (0.974)	0.052 (0.956)
			30	0.048 (0.955)	0.070 (0.968)	0.047 (0.955)
		10.0	10	0.060 (0.960)	0.120 (0.980)	0.060 (0.960)
			20	0.057 (0.956)	0.090 (0.973)	0.057 (0.956)
			30	0.050 (0.953)	0.073 (0.966)	0.050 (0.953)
		20.0	10	0.060 (0.962)	0.114 (0.982)	0.060 (0.962)
			20	0.057 (0.956)	0.087 (0.972)	0.057 (0.956)
			30	0.051 (0.947)	0.075 (0.964)	0.051 (0.947)
	10.0	15.0	10	0.063 (0.950)	0.117 (0.971)	0.062 (0.950)
			20	0.058 (0.950)	0.088 (0.968)	0.058 (0.950)
			30	0.053 (0.949)	0.077 (0.963)	0.053 (0.949)
		20.0	10	0.070 (0.956)	0.126 (0.976)	0.070 (0.956)
			20	0.058 (0.952)	0.090 (0.970)	0.058 (0.952)
			30	0.055 (0.959)	0.082 (0.970)	0.055 (0.959)
		30.0	10	0.071 (0.956)	0.127 (0.977)	0.071 (0.956)
			20	0.058 (0.962)	0.091 (0.976)	0.058 (0.962)
			30	0.057 (0.957)	0.083 (0.970)	0.057 (0.957)
		50.0	10	0.073 (0.960)	0.127 (0.980)	0.073 (0.960)
			20	0.065 (0.960)	0.099 (0.977)	0.065 (0.960)
			30	0.062 (0.957)	0.086 (0.971)	0.062 (0.957)

Table 4.2 Frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for θ

μ	λ	t	n	π_1	π_2	π_3
1.0	0.1	3.0	10	0.047 (0.955)	0.101 (0.977)	0.047 (0.955)
			20	0.050 (0.952)	0.079 (0.971)	0.050 (0.952)
			30	0.052 (0.948)	0.077 (0.964)	0.052 (0.948)
		5.0	10	0.050 (0.953)	0.096 (0.973)	0.049 (0.953)
			20	0.050 (0.953)	0.083 (0.969)	0.050 (0.953)
			30	0.048 (0.948)	0.070 (0.964)	0.048 (0.948)
		10.0	10	0.051 (0.948)	0.102 (0.974)	0.051 (0.948)
			20	0.051 (0.954)	0.081 (0.970)	0.051 (0.954)
			30	0.051 (0.950)	0.074 (0.966)	0.051 (0.950)
		20.0	10	0.049 (0.950)	0.099 (0.974)	0.049 (0.950)
			20	0.053 (0.944)	0.084 (0.965)	0.053 (0.944)
			30	0.046 (0.948)	0.070 (0.964)	0.046 (0.948)
	1.0	5.0	10	0.049 (0.953)	0.099 (0.976)	0.049 (0.953)
			20	0.052 (0.952)	0.082 (0.969)	0.052 (0.952)
			30	0.050 (0.952)	0.074 (0.964)	0.050 (0.952)
		10.0	10	0.049 (0.951)	0.097 (0.976)	0.049 (0.951)
			20	0.050 (0.949)	0.082 (0.968)	0.050 (0.949)
			30	0.051 (0.951)	0.073 (0.966)	0.051 (0.951)
		20.0	10	0.048 (0.952)	0.100 (0.975)	0.048 (0.952)
			20	0.049 (0.952)	0.078 (0.972)	0.048 (0.952)
			30	0.052 (0.954)	0.075 (0.969)	0.052 (0.954)
		30.0	10	0.050 (0.952)	0.099 (0.975)	0.050 (0.952)
			20	0.049 (0.948)	0.079 (0.967)	0.049 (0.948)
			30	0.050 (0.949)	0.075 (0.966)	0.050 (0.949)
	10.0	15.0	10	0.058 (0.945)	0.108 (0.970)	0.058 (0.946)
			20	0.053 (0.951)	0.080 (0.970)	0.053 (0.951)
			30	0.050 (0.951)	0.075 (0.967)	0.050 (0.951)
		20.0	10	0.057 (0.952)	0.108 (0.973)	0.057 (0.952)
			20	0.051 (0.956)	0.084 (0.970)	0.051 (0.955)
			30	0.052 (0.951)	0.078 (0.967)	0.052 (0.951)
		30.0	10	0.057 (0.955)	0.111 (0.977)	0.057 (0.955)
			20	0.052 (0.953)	0.084 (0.970)	0.052 (0.953)
			30	0.051 (0.956)	0.076 (0.970)	0.051 (0.956)
		50.0	10	0.060 (0.962)	0.111 (0.980)	0.060 (0.962)
			20	0.053 (0.955)	0.082 (0.971)	0.053 (0.955)
			30	0.052 (0.950)	0.077 (0.966)	0.052 (0.950)

Tables 4.1, 4.2 and 4.3 give numerical values of the frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for the proposed priors. The computation of these numerical values is based on the following algorithm for any fixed true (μ, λ) and any prespecified value α. Here α is 0.05 (0.95). Let $\theta^{\pi}(\alpha|\mathbf{x}, \mathbf{y})$ be the posterior α-quantile of θ given $\mathbf{x}$. That is to say, $F(\theta^{\pi}(\alpha|\mathbf{x})|\mathbf{x}) = \alpha$, where $F(\cdot|\mathbf{x})$ is the marginal posterior distribution of θ. Then the

frequentist coverage probability of this one sided credible interval of θ is

$$P_{(\mu,\lambda)}(\alpha;\theta) = P_{(\mu,\lambda)}(0 < \theta < \theta^{\pi}(\alpha|\mathbf{x})). \tag{4.1}$$

The estimated $P_{(\mu,\lambda)}(\alpha;\theta)$ when $\alpha = 0.05(0.95)$ is shown in Tables 4.1, 4.2 and 4.3. In particular, for fixed (μ,λ), we take 10,000 independent random samples of $\mathbf{X}$ from the model (2.1).

For the cases presented in Tables 4.1, 4.2 and 4.3, we see that the reference priors π_1 and π_3 match the target coverage probability much more accurately than the reference prior π_2 for values of (μ,λ) and values of t. In particular, the reference priors π_1 and π_3 meet very well the target coverage probabilities in small samples and give almost same results. Note that the results of tables are not much sensitive to change of the values of (μ,θ) and t. Thus we recommend to use the reference priors π_1 and π_3 in the sense of asymptotic frequentist coverage property.

Table 4.3 Frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for θ

μ	λ	t	n	π_1	π_2	π_3
10.0	0.1	15.0	10	0.048 (0.955)	0.096 (0.978)	0.048 (0.955)
			20	0.053 (0.948)	0.084 (0.966)	0.053 (0.948)
			30	0.048 (0.949)	0.072 (0.966)	0.048 (0.949)
		20.0	10	0.055 (0.948)	0.105 (0.974)	0.055 (0.948)
			20	0.049 (0.951)	0.078 (0.969)	0.049 (0.951)
			30	0.050 (0.950)	0.075 (0.965)	0.050 (0.950)
		30.0	10	0.050 (0.950)	0.100 (0.973)	0.050 (0.950)
			20	0.049 (0.950)	0.085 (0.969)	0.049 (0.950)
			30	0.052 (0.955)	0.074 (0.969)	0.052 (0.955)
		50.0	10	0.049 (0.954)	0.098 (0.976)	0.049 (0.954)
			20	0.051 (0.949)	0.084 (0.969)	0.051 (0.949)
			30	0.054 (0.949)	0.078 (0.967)	0.054 (0.949)
	1.0	20.0	10	0.046 (0.949)	0.095 (0.973)	0.046 (0.948)
			20	0.050 (0.949)	0.085 (0.969)	0.050 (0.949)
			30	0.048 (0.951)	0.073 (0.968)	0.048 (0.951)
		30.0	10	0.051 (0.952)	0.103 (0.975)	0.051 (0.952)
			20	0.051 (0.952)	0.081 (0.969)	0.051 (0.952)
			30	0.051 (0.950)	0.073 (0.967)	0.051 (0.950)
		50.0	10	0.049 (0.949)	0.096 (0.972)	0.049 (0.949)
			20	0.050 (0.948)	0.081 (0.968)	0.050 (0.948)
			30	0.053 (0.950)	0.076 (0.964)	0.053 (0.950)
		70.0	10	0.046 (0.948)	0.097 (0.972)	0.046 (0.948)
			20	0.048 (0.949)	0.078 (0.966)	0.048 (0.949)
			30	0.048 (0.948)	0.071 (0.963)	0.048 (0.948)
	10.0	30.0	10	0.052 (0.952)	0.103 (0.974)	0.051 (0.952)
			20	0.051 (0.952)	0.077 (0.970)	0.051 (0.952)
			30	0.049 (0.950)	0.072 (0.966)	0.049 (0.950)
		50.0	10	0.047 (0.951)	0.095 (0.974)	0.046 (0.950)
			20	0.051 (0.950)	0.081 (0.969)	0.051 (0.950)
			30	0.050 (0.948)	0.073 (0.965)	0.050 (0.948)
		70.0	10	0.053 (0.949)	0.106 (0.975)	0.052 (0.948)
			20	0.047 (0.951)	0.076 (0.969)	0.047 (0.951)
			30	0.045 (0.950)	0.067 (0.966)	0.045 (0.950)
		100.0	10	0.048 (0.950)	0.095 (0.974)	0.048 (0.950)
			20	0.051 (0.950)	0.080 (0.969)	0.050 (0.949)
			30	0.050 (0.955)	0.074 (0.968)	0.050 (0.955)

Example. This example taken from Sinha and Guttman (1976) and the data set is given in Grubbs (1971). The summary of data set is $x_{(1)} = 162$ and $\bar{x} = 997.2105$. For this data set, we compute the 90% and the 95% lower confidence limit of the reliability function $R(t)$.

Sinha and Guttman (1976) gave the $100(1-\alpha)\%$ lower confidence limits for $R(t)$ using the prior $\pi(\mu,\lambda) \propto \lambda^{-a}, a > 0$. For $a = 1$ and $a = 2$, the 90%(95%) lower confidence limits are given in Table 4.4. Also the 90%(95%) Bayesian credible intervals based on the reference priors are given. Note that the prior with $a = 1$ of Sinha and Guttman (1976) is our reference prior π_3 and the prior with $a = 2$ of Sinha and Guttman (1976) is our reference prior π_2

From the results of Table 4.4, the reference priors π_1 and π_3 give a shorter 90% and 95% confidence intervals than the reference prior π_2, and the reference prior π_1 has the shortest confidence interval. Note that the reference prior π_2 had not good coverage probabilities in our simulation results.

Table 4.4 Lower 90% (95%) confidence limits of $R(t)$

t	S&G($a=1$)	S&G($a=2$)	π_1	π_2	π_3
180	0.8819 (0.8603)	0.8789 (0.8565)	0.9144 (0.8932)	0.8789 (0.8565)	0.8819 (0.8603)
240	0.8192 (0.7974)	0.8134 (0.7911)	0.8329 (0.8119)	0.8134 (0.7911)	0.8192 (0.7974)
300	0.7526 (0.7341)	0.7479 (0.7256)	0.7635 (0.7423)	0.7479 (0.7256)	0.7526 (0.7341)
360	0.6951 (0.6723)	0.6848 (0.6617)	0.6995 (0.6772)	0.6846 (0.6617)	0.6951 (0.6723)
420	0.6376 (0.6136)	0.6254 (0.6014)	0.6405 (0.6167)	0.6254 (0.6014)	0.6376 (0.6136)
480	0.5843 (0.5591)	0.5707 (0.5456)	0.5862 (0.5611)	0.5707 (0.5456)	0.5843 (0.5591)
540	0.5351 (0.5089)	0.5204 (0.4944)	0.5364 (0.5102)	0.5204 (0.4944)	0.5351 (0.5089)
600	0.4898 (0.4628)	0.4744 (0.4477)	0.4907 (0.4638)	0.4744 (0.4477)	0.4898 (0.4628)

5. Concluding remarks

We have found noninformative priors for the reliability function of two parameter exponential distribution. We derived the reference priors and the matching prior when θ is parameter of interest is parameter of interest. We showed that the reference priors π_1 and π_3 are a matching prior and meet well the target coverage probabilities. Thus we recommend the use of the reference priors π_1 and π_3 for Bayesian inference of the reliability function in this distribution.

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