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A GIS-based Geometric Method for Solving the Competitive Location Problem in Discrete Space

Gunhak Lee*

이산적 입지 공간의 경쟁적 입지 문제를 해결하기 위한 GIS 기반 기하학적 방법론 연구

이건학*

Abstract : A competitive location problem in discrete space is computationally difficult to solve in general because of its combinatorial feature. In this paper, we address an alternative method for solving competitive location problems in discrete space, particularly employing deterministic allocation. The key point of the suggested method is to reducing the number of predefined potential facility sites associated with the size of problem by utilizing geometric concepts. The suggested method was applied to the existing broadband marketplace with increasing competition as an application. Specifically, we compared computational results and spatial configurations of two different sized problems: the problem with the original potential sites over the study area and the problem with the reduced potential sites extracted by a GIS-based geometric algorithm. The results show that the competitive location model with the reduced potential sites can be solved more efficiently, while both problems presented the same optimal locations maximizing customer capture.

Key Words : competitive location problem, deterministic allocation, GIS-based geometric algorithm, weighted Voronoi diagram

요약 : 일반적으로 이산적 입지 공간에서 경쟁적 입지 문제는 입지 후보지에 따라 수많은 조합의 경우가 발생하는 의사결정 문제이기 때문에, 수리적으로 계산하기가 쉽지 않다. 따라서 본 연구에서는 결정적 배분 형태를 가정한 이산적 입지 공간의 경쟁적 입지 문제를 보다 효율적으로 해결하기 위한 대안적 방법에 대해 논의한다. 제안된 방법론의 핵심은 입지 문제의 크기와 관련되는 잠재적 입지 후보지의 개수를 기하학적 개념을 이용하여 줄이는 것이다. 사례 분석으로 경쟁이 가열화되고 있는 초고속 인터넷 시장을 대상으로 제안된 방법론을 적용하였는데 두 가지 다른 크기의 문제, 즉 연구 지역 전체에 대해 정의된 잠재적 입지 후보지와 GIS 기반의 기하학적 알고리즘에 의해 추출된 보다 적은 수의 잠재적 입지 후보지에 대해 계산 결과와 공간적 배열을 비교하였다. 사례 분석 결과, 두 문제 모두 고객 유치를 최대화시키는 동일한 최적 입지를 보여주는 한편, 적은 수의 잠재적 입지 후보지를 가진 경쟁적 입지 모델이 보다 효율적으로 해결될 수 있었다.

주요어 : 경쟁적 입지 문제, 결정적 배분, GIS 기반 기하학적 알고리즘, 가중화된 보로노이 다이어그램

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* Assistant Professor, Department of Geography Education, Chonnam National University, gunhlee@jnu.ac.kr

1. Introduction and background

The competitive location problem is to optimally locate new facilities of a firm to maximize the market share or profits in a competitive market environment. There are many considerations for this problem such as competitive equilibrium, patronizing behavior, decision space, market characteristics, and so on (Plastria, 2001). Among them, attraction function of a particular facility is particularly important because it is directly related to customer preference to facilities and thus eventually influences estimated market share by a firm. In many location models, the proximity based attraction function is considered to be the most popular type because of its simplicity. According to the proximity rule, the closest facility is the most attractive to customers. That is, it assumes the customer's preference to the closest facility. Hotelling's (1929) seminal work employed this proximity based attraction. However, the proximity based function is often unrealistic because it takes into account only spatial separation between customer and facility. In reality, other non-spatial attributes such as facility image, sales price, and service level would be influential to customer choice as well. Therefore, to incorporate such non-spatial attributes into the function, a well-known spatial interaction model might be an alternative form including a composite of attributes as facility attractiveness in the functions. Many previous studies have utilized the spatial interaction function in competitive location modeling (Reilly, 1931; Lakshmanan and Hansen, 1965; Achabal *et al.*, 1982; O'Kelly, 1987; O'Kelly and Miller, 1989; Drezner *et al.*, 2002; Drezner and Drezner, 2006).

Once attraction function is defined, it is necessary to determine the patronage rule of customers. The patronage rule defines a way how customers are assigned to a particular facility. The two types of patronage rules (or allocation rules) have widely used in the literature: i) deterministic allocation that customers patronize the nearest or most attractive single facility (Hakimi, 1983; Goodchild, 1984; ReVelle, 1986; ReVelle and Serra, 1991; Drezner, 1994; Plastria and Carrizosa, 2004) and ii) probabilistic allocation that patronage of customers are split to multiple facilities according to predefined facility attraction (Huff, 1964; Achabal *et al.*, 1982; Ghosh and Craig, 1986; Eiselt and Laporte, 1989; Drezner, 1995; Fernández *et al.*, 2007; Zhang and Rushton, 2008). In this paper, we focus on the deterministic allocation rule for competitive location model.

Another important consideration for competitive location problem is the choice of decision space: continuous space allowing facility to be located anywhere and discrete space limiting facility locations with a finite number of eligible sites (Plastria, 2001). More frequently, a single facility location is addressed in the plane because of computational complexity. Many location problems are discrete in nature because there is a specific set of potential sites in reality (Current *et al.*, 1990). However, those discrete problems are often hard to solve optimally by combinatorial feature (e.g., selection r facilities out of n) (ReVelle and Eiselt, 2005). Although many heuristic algorithms have been developed for solving combinatorial optimization problems, reducing the problem size a prior is also an viable option to release the computational burden for discrete location problems (Church, 2002).

In this paper, we address an efficient way of solving the competitive location problem under deterministic allocation assumption in discrete space. A deterministic customer patronage might make the model simplistic but is reasonably employed here, regarding human rationale to maximize their satisfaction by using a particular facility. Specifically, we attempt to reduce discrete decision space by exploring geometric aspects of market capture among competing facilities. To do so, we first formulate the competitive location model with deterministic allocation rule. Then we present a geometrical method for solving competitive location problem. As an illustrative example, our approach is applied to the broadband market which recently becomes more competitive and then computational results of the models with differently sized decision space are compared.

2. Hypothetical broadband market and optimization model

In order to illustrate broadband market situations, suppose multiple competing broadband providers operating multiple serving units in the market. Among existing firms, one firm is called A as the entering firm planning to add more facilities to its existing system in order to extend the market share. Other firms are labeled Bs. Consider the initial market configuration consisting of two sets of points representing aggregate demand and potential sites (Figure 1). Five demand nodes and seven locations (four of existing systems and three of potential sites) are shown.

Intuitively, several situations of location-allocation under deterministic allocation rule are identified for demand points. The first case is a demand point captured only by existing facilities of the entering firm (*A*), as indicated by points I and IV. The second case is a point captured only by existing facilities of the competitor (*Bs*), as represented by point II. The third case is a demand point which buying power is evenly shared by multiple facilities, as point III. In fact, this is a special case adopting a tie rule while “all or nothing” deterministic rule is reserved. When multiple facilities with equal attractiveness are available to a demand point, potential buying power of a demand point will be split evenly following the tie rule. A point III is served by both existing facility 3 of the competitor and facility 5 of the entering firm. The last case represents an unserved demand point which is not within coverage of any facility, as point V. This case makes sense under the assumption of physical limits of broadband services[1]. Given the entering firm wishing to add more new facilities

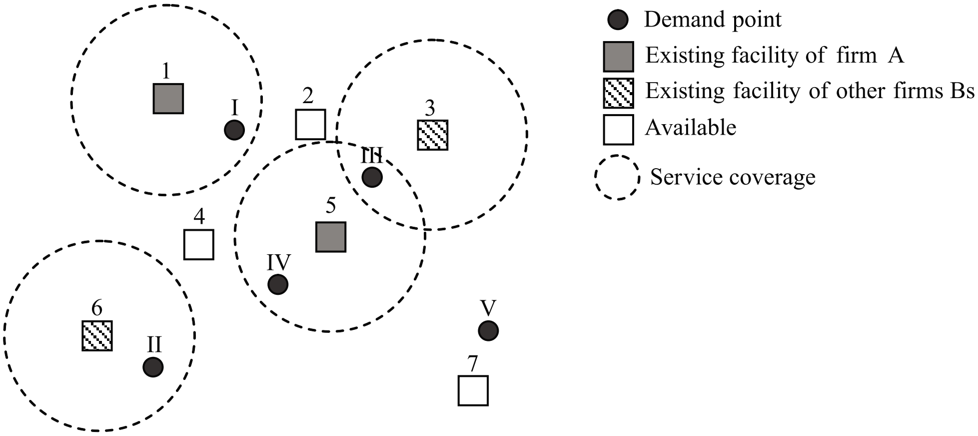

Figure 1. Hypothetical market configuration

to an existing system, the first case needs to be excluded in model because there is no need to consider demands already patronizing one of the facilities of the entering firm. Thus, the entering firm concerns only the second case, some proportion captured by the competitor in the third case, and the last one. Those demand points are predefined for building the competitive location model as sets, AL_{Bs}, AL_{ABs}, and AL_{NONE}, respectively. Considering given facility locations, facilities 2, 4, and 7 are regarded as unoccupied sites. Since this paper addresses the deterministic patronage rule of customers, potential co-location of a new facility is not allowed. Thus, the eligible candidate sites for a new facility refer to a set of unoccupied locations. This set is represented as L_{EMPTY}.

Regarding the attraction function, we formulate the spatial interaction based function. Let $D_e(i, j)$ be the Euclidean distance from a demand point i to a facility j. Note that $D_e(i, j) \neq 0$ because (i, j) is a distinctive point set. Usually the interaction between a demand point and a facility is proportional to attractiveness of a facility, measured by attributes describing the facility's characteristics; it is inversely proportional to the spatial separation between a demand point and a facility. The interaction between a demand point i and facility j (A_{ij}) is written as follows (Nakanishi and Cooper, 1974; Achabal *et al.*, 1982):

$$A_{ij}=\prod_j x_{kj}^{\alpha k}/D_e(i, j)^{\beta} \tag{1}$$

where x_{kj} is kth attribute of facility located at j; α_k is the parameter of kth attribute of a facility j influencing attraction; β is the distance decay effect[2)].

The spatial interaction model would be useful to represent various criteria which influence customer choice of broadband services, such as monthly charge, the number of bundles, any promotions, and the guaranteed bandwidth. Those contributing attributes to customer choice can be readily included as a multiplicatively combined single measure in the formulation.

Based upon the attraction function defined above, we formulate the deterministic competitive location model in the broadband application context using the following binary decision variables, parameters, and indices.

a_i: initial population of demand point i;
a_i^A: potential demand of demand point i to be captured by the entering firm A;
S^A: maximum coverage standard of the entering firm A;
P^A: desired number of new facilities of the entering firm A;

$$x_i^A=\begin{cases}1 & \text{if a new facility of the entering firm a is sited at } j\\ 0 & \text{otherwise}\end{cases};$$

$$y_{ij}^A=\begin{cases}1 & \text{if a demand } i \text{ is assingned to a new facility of the entering firm } A \text{ at } j\\ 0 & \text{otherwise}\end{cases}.$$

Since customers already captured by the entering firm A are not considered as potential demands, it must be reasonably filtered out by excluding those customers of a particular demand point (ReVelle, 1986). Therefore, the potential demand for a demand point i (a_i^A) is computed by multiplying original aggregate population (a_i) by the variable, τ_i^{Bs} which indicates the estimated portion served by competing firms.

$$a_i^A=\begin{cases}a_i\tau_i^{Bs} & \text{if } i \in AL_{Bs} \cup AL_{ABs}\\ a_i & \text{if } i \in AL_{NONE}\end{cases},$$

where $\tau_i^{Bs}=\dfrac{|E_i^{Bs}|}{|E_i^A|+|E_i^{Bs}|}$; $|E_i^A|$ is the number of equally attractive existing facilities of the entering firm A serving demand point i; $|E_i^{Bs}|$ is the number of equally attractive existing facilities of the competitive firms Bs serving demand point *i*.

Note that an unserved demand point is presumed as a full potential demand for the entering firm because there are no competing facilities and thus it can be fully captured by a new facility sited within coverage distance.

The developed location model below is a modified version of classical *p*-median problem (Hakimi, 1964). While *p*-median problem aims to minimize total cost of traveling from customer locations to the nearest facility, the objective function (2) here is to maximize the portion of potential demands captured by new facilities of the entering firm. The expected portion to be captured or allocation coefficient (Ψ_{ij}^{A}) would reflect the deterministic patronage or allocation rule, representing "all or nothing" feature. We will discuss this with geometric concepts later.

$$\text{Maximize } Z^A = \sum_{i \in AL_{Bs} \cup AL_{ABs} \cup AL_{NONE}} \sum_{j \in EMPTY} a_i^A \Psi_{ij}^A y_{ij}^A \qquad (2)$$

Subject to

$$\sum_j y_{ij}^A \leq 1 \quad \forall i \in AL_{Bs} \cup AL_{ABs} \cup AL_{NONE}$$
$$j \in L_{EMPTY}, \text{ where } D_r(i,j) \leq s^A \qquad (3)$$

$$y_{ij}^A \leq x_j^A \quad \forall i \in AL_{Bs} \cup AL_{ABs} \cup AL_{NONE}$$
$$j \in L_{EMPTY}, \text{ where } D_r(i,j) \leq s^A \qquad (4)$$

$$\sum_j x_j^A \leq P^A \quad \forall j \in L_{EMPTY} \qquad (5)$$

$$x_j^A \in \{0,1\} \quad \forall j \in L_{EMPTY} \qquad (6)$$

$$y_{ij}^A \in \{0,1\} \quad \forall i \in AL_{Bs} \cup AL_{ABs} \cup AL_{NONE}$$
$$j \in L_{EMPTY} \qquad (7)$$

Constraints (3) - (5) are similar to the classical p-median problem but with modification in the broadband context. Specifically, inequality constraints (3) relax the mandatory assignment for each demand point *i*, allowing for unserved demand points where no facilities are available within service coverage. Due to the limited geographic coverage of broadband service, these constraints are necessarily required. The situation of not being captured by newly constructed facilities, however, may occur (e.g., $y_{i1}^A = y_{i2}^A = \cdots = y_{ij}^A = 0$, $j \in L_{EMPTY}$, where $D_r(i,j) \leq s^A$). But the objective function to maximize total market share captured by new facilities prohibits this situation. Constraints (4) specify the relationship between facility siting and demand capturing. There is no assignment of demand points to a facility which is not placed. The desired number of facilities to be sited is specified exogenously by constraint (5). Constraints (6) and (7) impose the integer restriction of decision variables.

3. A GIS-based geometric approach

1) Exploring geometric relationship in discrete space

Although a number of geometric approaches have been suggested for solving location models, Church (1984) and Drezner (1994) are worth noting for competitive location models. Church (1984) presented an approach to solve the maximal covering problem (Church and ReVelle, 1974) on continuous decision space using the circle intersect point set (CIPS). He proved that on the planar maximal covering location problem at least one optimal solution exists in CIPS. A more significant contribution of this paper is to discretizing continuous decision space into a finite candidate set for facility sites. However, this approach may not be applicable to competitive location problem. An optimal solution is not necessarily found in exact locations of CIPS, rather it depends on the situation of customer capture. We will discuss this later. Similar to this approach, recent article by Murray and Tong

(2007) introduced a geometric approach for the maximal covering problem with polygon based sets, named the polygon intersect point set (PIPS).

Meanwhile, Drezner (1994) introduced the break-even distance (BED) and relevant circle geometry. The BED is the minimum distance to existing facilities (equivalently proximity based attraction of facilities), which provides the critical value of determining the success of market capture under deterministic allocation. The BED for a demand point i (B_i) can be mathematically expressed as follows:

$$B_i = \min_j \{D(i,j)\},\ i \neq j,$$

where j is an index of existing facilities.

With a radius of the BED, a circle centered at demand point can be defined for each individual demand point (Figure 2). If a new facility is placed within the circle (e.g., star a), it captures all customer at a demand point. If a new facility is placed on the circumference of the circle (e.g., star b), it captures an equal portion of customers with an existing facility. If a new facility, however, is placed on the exterior of the circle (star c), it cannot capture any customers because of its inferiority over an existing facility.

Since the original concept of the BED is only defined for the demand point already captured by any of existing facilities, a special treatment is required for an unserved demand point without existing facilities. In this case, maximum service distance would be thought of as the break-even distance. It means that any facility within service coverage can capture customers without competition. Accordingly the BED for an unserved demand point is redefined using a service coverage standard as follows:

$$B_i = s^E,\ \forall i \in AL_{NONE} \tag{8}$$

On the other hand, Drezner's BED based on the Euclidean distance metric may not be directly applicable to the spatial interaction based deterministic allocation which includes non-spatial attributes as well as physical distance. By

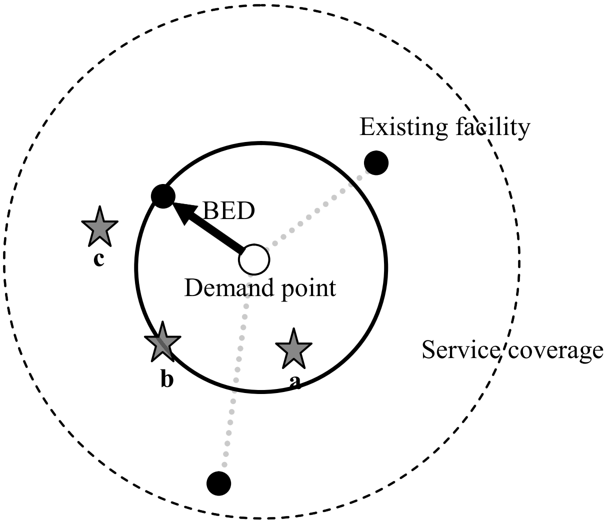

Figure 2. Illustration of the break-even distance

employing a multiplicative weight in the Euclidean distance, we can obtain the multiplicatively weighted distance projected on the Euclidean space, $D_{mw}(i,j)$ as follows:

$$D_{mw}(i,j)=\frac{D_e(i,j)}{w_j},\ i\neq j,$$

where $w_j=\prod_j(\mathbf{x})$, $\mathbf{x}$ is a vector of attributes involved in attractiveness of facility at j.

Now the spatial interaction based BED or referred to as a Weighted BED (WBED) hereafter, is the minimum weighted distance to existing facilities3). Thus, the WBED of a demand point i (B_i^{SI}) can be mathematically written as follows:

$$B_i^{SI}=\min_j\{D_{mw}(i,j)\},\ i\neq j,$$

The WBED for an unserved demand point is also defined using coverage standard as the same as the equation (8).

Once the WBED is identified for each demand point, the circle, called a capturing circle (CC), centered at a demand point with a radius of the WBED can be easily drawn as shown in Figure 3.

When we look at the geometric relationship between point locations (e.g., A, B, C, D, E, F, G, P, Q1, Q2, and Q3) and CCs, some interesting observations are found for demand allocation. Depending on the locations in circles, the capture by a new facility varies. A point will be placed in four possible locations with reference to the CCs: *complete interior, boundary*, simultaneous *interior and boundary*, and *complete exterior*. For the first case, customers of a demand point are fully captured by the new facility placed inside a single capturing circle (e.g., A). When a facility is interior of the intersection of overlaying circles (e.g., B), the facility can capture customers of all demand points which are centers of overlying circles. However, when a facility is on the boundary of a capturing circle (e.g., D), the capture is shared with existing facilities. For the third case, a facility might be sited interior of a circle and lie on the boundary of another circle simultaneously (e.g., F). In this case, one demand

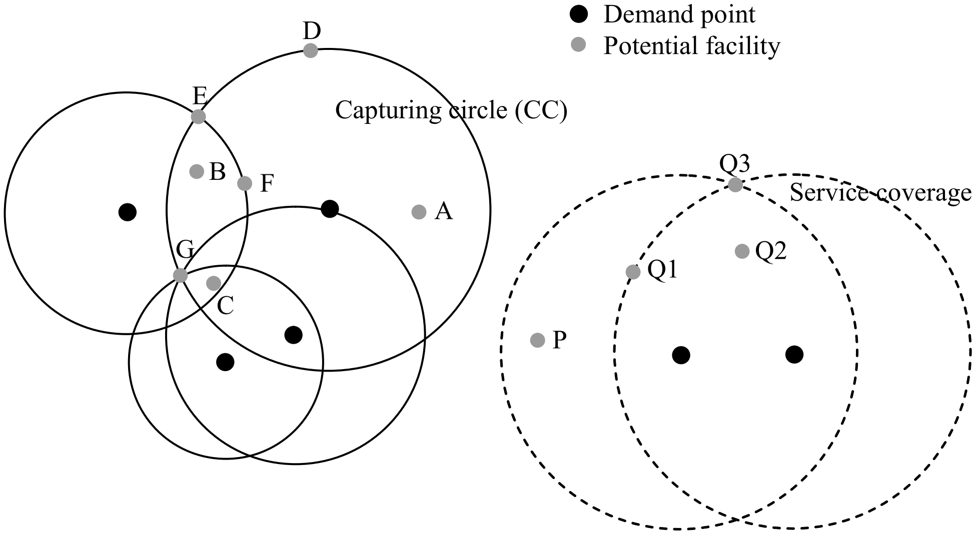

Figure 3. Capturing circles with radii of the WBED for each demand point

Note: the maximum distance of service coverage is regarded as the WBED for demand points without existing facilities available.

Table 1. Geometric relationship between capturing circle and point position

Points in circles	The number of overlay		
	1-overlay	2-overlay	n-overlay
Interior	a_i (e.g., A, P)	$a_{i_1}+a_{i_2}$ (e.g., B, Q2)	$\sum_{m=1}^{n} a_{i_m}$ (e.g., C (n=4))
Boundary	$\left(\frac{1}{\lvert E_{i_k}^{Bs}\rvert+1}\right)a_{i_k}$ (e.g., D)	$\left(\frac{1}{\lvert E_{i_1}^{Bs}\rvert+1}\right)a_{i_1}+\left(\frac{1}{\lvert E_{i_2}^{Bs}\rvert+1}\right)a_{i_2}$ (e.g., E, Q3)	$\sum_{k=1}^{n}\left(\frac{1}{\lvert E_{i_k}^{Bs}\rvert+1}\right)a_{i_k}$
Interior and Boundary	none	$\left(\frac{1}{\lvert E_{i_1}^{Bs}\rvert+1}\right)a_{i_1}+a_{i_2}$ (e.g., F, Q1)	$\sum_{m=1}^{X} a_{i_m}+\sum_{k=1}^{Y}\left(\frac{1}{\lvert E_{i_k}^{Bs}\rvert+1}\right)a_{i_k}$, where X is the number of interior and Y is the number of boundary ($X+Y$=n) (e.g., G: X=1, Y=3)

point is fully captured (e.g., a demand point right hand-side of F) and the other is partially captured (e.g., a demand point left hand-side of F). Table 1 describes this geometric relationship formally with exampled cases in Figure 3. Note that a point on the boundary of circles centered at unserved demand points can capture full potential demands (e.g., Q1, Q3).

Now we relate the concept of the WBED to the allocation coefficient in the formulation (Ψ_{ij}^{A}) as follows.

$$\Psi_{ij}^{A}=\begin{cases} 1 & \text{if } D_{mw}(i,j)<B_i^{SI} \\ \dfrac{1}{|E_i^{Bs}|+1} & \text{if } D_{mw}(i,j)<B_i^{SI} \quad \forall i\in AL_{Bs}\cup AL_{ABs}\cup AL_{NONE},\ j\in L_{EMPTY} \\ 0 & \text{otherwise} \end{cases}$$

When the weighted distance from a demand point i to a new facility at j ($D_{mw}(i,j)$) is smaller than the weighted break-even distance (B_i^{SI}) (interior of the CC), a demand point i is fully captured by a new facility. When $D_{mw}(i,j)=B_i^{SI}$ (on the boundary of the CC), the portion to be captured by a new facility is shared with existing competing facilities.

Based on the geometric concepts discussed above, we can effectively reduce the original potential sites before implementing the problem. Consider a set of the capturing circles, $P=\{C_1,\ldots,C_n\}$, where n is the number of demand points and a fixed number of potential sites predefined. By definition of the CCs, any potential site exterior of $\bigcup_n C_n$ is infeasible. That is, any site exterior of circle union cannot capture any customers because of its locational inferiority to existing facility. Reversely, any new facility established at potential sites in the union of the capturing circles can capture customers at a demand point i by at least $\left(\frac{1}{|E_i^{Bs}|+1}\right)a_i$4). Consequently, a set of points in the union of the capturing circles, named as Points in Union (PCU), is regarded as an alternative feasible set. As illustrated in Figure 4, the PCU allows us to efficiently reduce the size of the competitive location problem, instead of using the entire set of original potential sites.

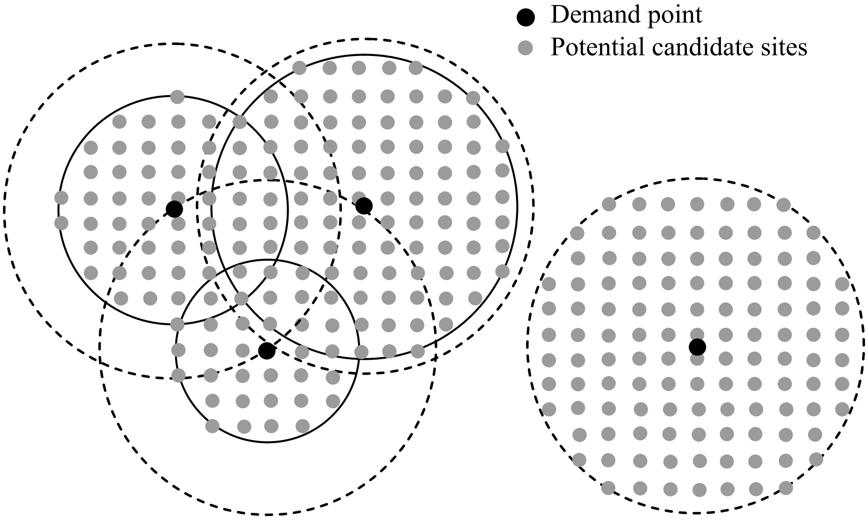

Figure 4. Reduced feasible set (PCU)

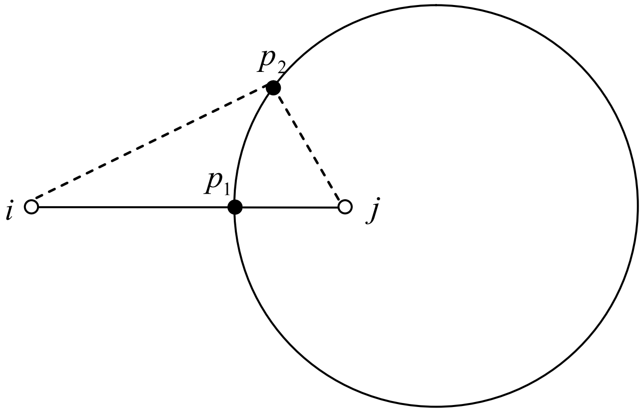

Figure 5. The concept of Apollonius circle

Since it is often difficult to acquire the actual market share in the broadband service market due to the commercial confidentiality, alternatively we can utilize Voronoi diagrams (or Thiessen polygon) to estimate initial market share given locations of existing facilities. The Voronoi diagrams have been used widely to delimit trade areas of competing facilities, providing reasonable approximations of actual trade areas quickly and inexpensively (Boots and South, 1997). Given the locations of existing competing facilities, each facility has a dominant geographic area, called a *trade area*, on the basis of its attraction to customers. In a trade area, the serving facility is a spatial monopoly. On the boundary of trade areas customers are evenly shared with the involved facilities. Regarding the spatial interaction based allocation, the multiplicatively weighted Voronoi diagram (MWVD) can be used effectively. The MWVD is a set of all Voronoi polygons of n facilities. For more formal definitions of various types of Voronoi diagrams, see Okabe *et al.*, 2000; Okabe and Suzuki, 1997. Most commercial GIS software provides the capability of generating the ordinary Voronoi diagram based on the Euclidean distance metric as a standard function. However, there is no commercial GIS software employing the MWVD as a standard function. To compute the MWVD, we utilize the topological overlay approach which is based on the Apollonius circle (Figure 5) which is the locus of a point (P_1 and P_2) with constant ratio of distances to two fixed points (e.g., i and j) (Aurenhammer and Edelsbrunner, 1984; Mu, 2004).

2) A GIS-based geometric algorithm

By adopting geometric concepts above, this section proposes a GIS-based geometric algorithm for a more efficient solution for the deterministic competitive broadband location problem. The algorithm consists of three main phases. For the first phase, the initial market share is estimated using the Voronoi diagram. In the second phase, the WBED is identified. Finally, the PCU is identified as a set of reduced candidate sites for new facilities. The detail of the algorithm is described as follows:

Phase I: Delimitating market share

Step 1. Generate the MWVD from existing facilities

Step 2. Generate the Coverage Constrained Multiplicatively Weighted Voronoi Diagram: CC-MWVD

Step 3. Assign each demand point to the corresponding CC-MWVD

Phase II: Identifying the WBED

Step 4. Compute the B_i^{SI} for each of n demand points as the multiplicatively weighted distance to the center of the corresponding polygon of the CC-MWVD

Phase III: Identifying the PCU

Step 5. Generate a capturing circle C_i, centered on the demand point i with a radius B_i^{SI} for all demand point i

Step 6. Compute $\bigcup_i C_i$ for all demand point i

Step 7. Find potential sites intersecting the area of $\bigcup_i C_i$ and generate the PCU

Step 8. Run the competitive location model with the PCU

More recently, location analysis has benefited from the advances in GIS for practical implementation. Most commercial GIS software provide the capabilities of storing, retrieving, analyzing, and visualizing spatial data (Church, 2002). Since GIS is geometrically based, useful geometric techniques for location analysis can be readily performed on a GIS platform. In competitive location modeling, for example, GIS helps compute various Voronoi diagrams quickly. Also, several spatial tasks, such as the buffering to generate service coverage and filter out infeasible potential sites, the computation of the spatial separation between customers and facilities, generating covering circles or capturing circles, can be efficiently achieved using GIS.

4. Application

1) Assumption and details

In this section, the proposed location model and geometric algorithms are applied to DSL broadband market of Columbus MSA (Metropolitan Statistical Area), Ohio. DSL technique utilizes existing telephone infrastructure, using copper wire as a physical medium and central office as a concentrator where digital traffic is exchanged. The digital signal transmitted from the central office tends to be attenuated through the physical medium (e.g. copper wires) as the distance from the central office increases. Thus, the availability of DSL service is geographically limited. A coverage limit of DSL service with reasonable quality is 12,000 feet from the central office (Grubersic and Murray, 2002). It is worth noting, however, that not all central offices can provide DSL services to customers within this service coverage. Special equipment, DSLAM (DSL Access Multiplexer) is required to be built in the central office. However, identifying DSL capable central offices is challenging because the majority of providers do not want to release their commercially confidential information to the public.

In reality the structure of competition among DSL service providers is very complicated. While major providers actually own and operate the critical facilities for DSL service, a number of small companies are offering the same kind of DSL service to some customers by co-locating equipment in the facility or purchasing wholesale services without any physical infrastructure in the space (Lee and O'Kelly, 2009). Therefore, it is quite difficult to distinguish the trade area geographically. For simplicity of analysis, this paper assumes that all central offices are capable of providing DSL service[5)] and only facility-based companies are concerned in the market share of DSL broadband service.

The Columbus MSA, currently 66 central offices are in operation by four major Internet Service Providers (ISPs), including AT&T, Verizon Communications, Embarq Corporation, and Windstream Communications. A total of 1,209 blockgroup centroids represent demand points to be served. A total of 3,537 regularly spacing points (1,000 feet) are generated as a representation of potential sites for facility placement. The analysis is implemented in a commercial GIS platform (ArcGIS 9.2, ESRI). Also, optimization software (CPLEX 10.0, ILOG) is used for solving integer linear program on a machine of Intel Xeon 3 GHz CPU with 3 GB memory. The geometric concepts and techniques as previously described are implemented by using both standard GIS functions embedded in ArcGIS and more advanced spatial analytical functions coded with a built-in programming language, Visual Basic Applications with ArcObjects.

2) Implementation

To initialize the market share of each firm, the MWVD based on the spatial interaction is utilized to approximate trade areas of service providers given the locations of existing central offices. Figure 6 presents allocation of demand points on the basis of the MWVD. The whole trade area of a particular company is extracted by merging a number of Voronoi polygons of its own central offices because broadband service is a kind of franchise business pursuing the system-wide profit.

Due to the physical limit of service (i.e., 12,000 feet), allocation depicted in Figure 6 must be refined as Figure 7.

Based on the configuration above, some demand points in the trade area are kept unserved. The initial market share of each firm is now estimated as Table 2, given locations of existing central offices. Windstream Communications, which has the second lowest market share, is randomly chosen as the entering firm for model implementation.

Given an initial market configuration, now we implement the optimization model with different potential sites: an original set of potential sites over the entire study area and a reduced set of potential sites (PCU) derived by geometrical exploration.

Table 3 shows the comparison of the problem reduction of different models when three facilities

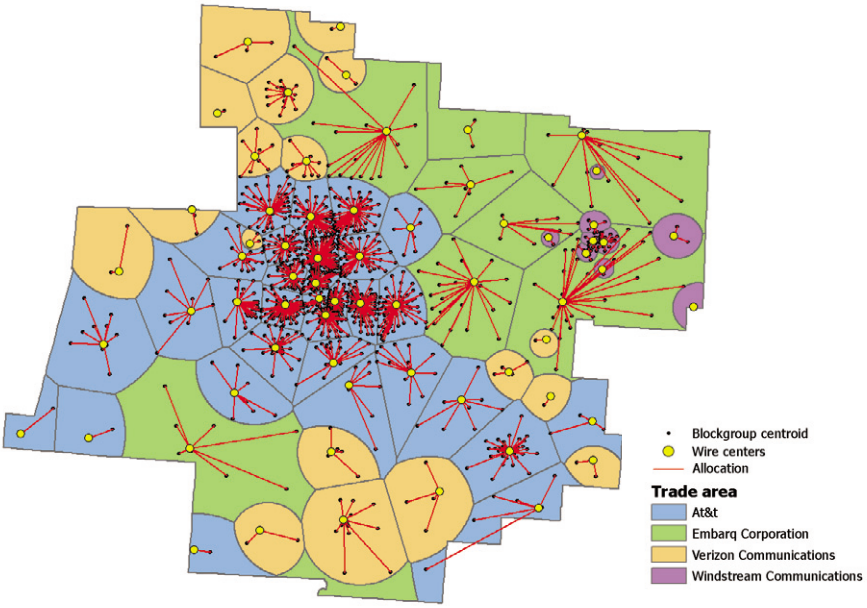

Figure 6. Initial trade areas and allocation

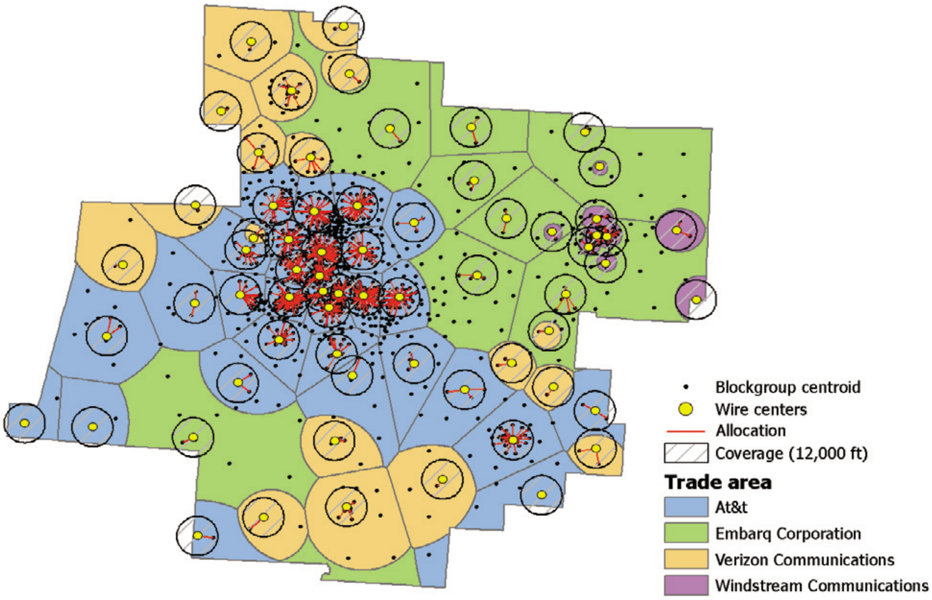

Figure 7. Coverage constrained trade areas and allocation

Table 2. Estimated initial market share

ISPs	# of facilities	# of blockgroups	# of people captured	% of market share
AT&T	31	733	809,249	52.54
Verizon	18	52	70,614	4.58
Embarq	8	21	23,716	1.54
Windstream	9	43	48,797	3.17
Not served	None	360	587,781	38.16
Total	66	1,209	1540,157	100.00

Table 3. Computational results for different models

Sets	# of points	Reading time (sec.)	Solution time (sec.)	Iteration	Objective
Original set of potential sites	3,537	47.22	5.00	5,866	111,654
PCU	2,459	21.61	3.98	7,200	111,654

Table 4. Market share change with p=3.

ISPs	Initial market share			Resulting market share		
	# of facilities	*# of blockgroups*	*% of market share*	*# of facilities*	*# of blockgroups*	*% of market share*
AT&T	31	733	52.54	31	725	51.73
Verizon	18	52	4.58	18	52	4.58
Embarq	8	21	1.54	8	21	1.54
Windstream	9	43	3.17	12	113	10.42
Not served	None	360	38.16	None	298	31.73
Total	66	1,209	100.00	69	1,209	100.00

are selected (p=3) on the basis of the spatial interaction based attraction.

As a result, the PCU model can reduce the problem size by 30% and solving time by 20%, compared to the original set of potential sites. The reduction of running time will be particularly significant for a large problem. Regarding the objective values, it turns out that all reduced models enable to solve the problem optimally. The change of market share by newly located facilities of Windstream is detailed in Table 4. AT&T, which is the most dominating service provider of Columbus MSA , has lost customers by 1%, while Windstream has increased its market share about 7% by locating new facilities in highly populated Franklin County, where most areas are being served by AT&T. These new facilities also cover a large number of unserved customers. Figure 8 visualizes the new spatial configuration of the market showing the change in trade areas and allocation, when new entries of Windstream are introduced.

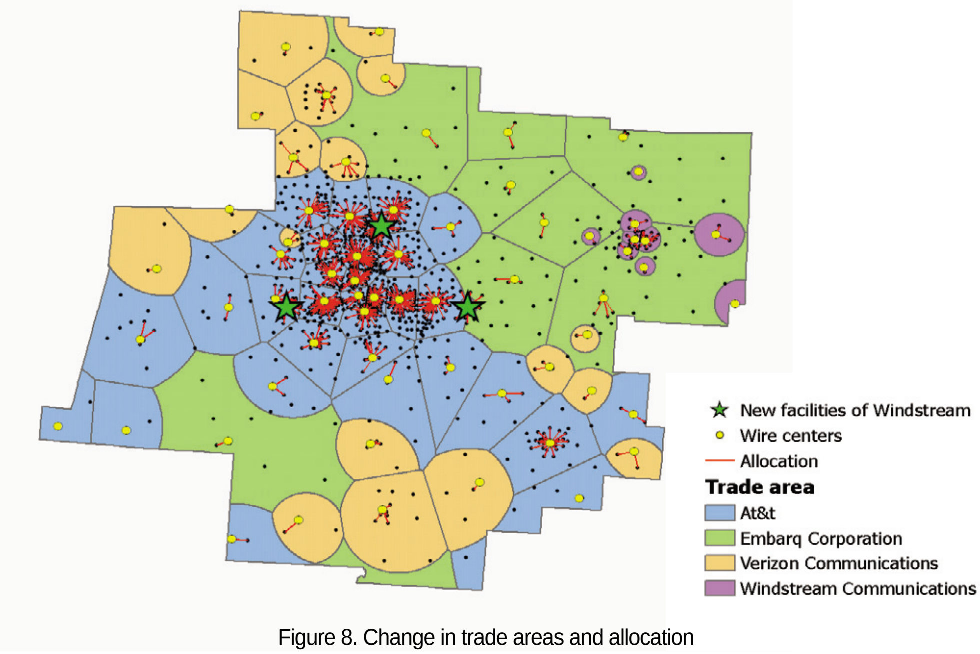

Figure 8. Change in trade areas and allocation

5. Conclusion

In many covering location models, competitive location models in discrete space generally require high computational efforts to solve because of combinatorial feature. In this paper, we propose an alternative method for solving competitive location problem in discrete space, particularly employing deterministic allocation. The suggested solution method is a kind of pre-reduction of eligible potential sites. Specifically, we explore geometric properties of customer capture in terms of facility locations in discrete decision space and solve the competitive location model exactly for examining computational efficiency.

As an empirical example, we apply the method to solve the competitive location problem in existing broadband marketplace. In particular, we compare computational results and spatial configurations of two cases: the classical case with original potential sites covering study area and the alternative case with reduce potential sites. As expected, the optimally sited facilities capture customers maximally and the optimal locations for both cases are exactly equivalent, indicating that an alternative subset geometrically derived contains optimal locations indeed. More importantly, our method makes discrete decision space simplistic and solves the competitive location problem with less computational effort.

Meanwhile, the competitive location model is successfully implemented by optimization tool in a GIS environment. GIS is broadly utilized for visualizing the market configuration and optimal solution, and model implementation, for example, generating potential sites, geometric tasks such as computing break-even distances, creating capturing circles, identifying points in merged capturing circles, and creating a text file for optimization tool.

This paper makes the positive contribution that a deterministic competitive location problem in discrete space can be simplified and be effectively solved by a geometrical exploration. The approach of this paper will be more significant to a larger application and be well-suited to location problems conducted in a GIS environment. Moreover, introducing the geometric concepts and practical ways of incorporating geometric properties of customer capture into the model will provide an insightful guideline to a competitive location problem in continuous space as well.

Notes

1) As an example, DSL (Digital Subscriber Line) is only available within 18,000 feet from the central office location (Newton, 2005).
2) Analogous to the distance effect in traditional retail activity of customers, digital signals in broadband service are significantly attenuated as the increase of distance from transmitter and customer's premise, making bandwidth less usable. In other words, this signal attenuation will impact actual network performance and ultimately user's choice of service.
3) In fact, the weighted distance is equivalent to the reverse of the interaction (see the equation 1). Therefore, minimizing the weighted distance is identical to maximizing the interaction.
4) As shown in the boundary of a single CC (Table 1), a new facility in $\bigcup_n C_n$ can capture at least $\left(\frac{1}{|E_i^{Bs}|+1}\right)a_i$.
5) It is true that all central offices in Ohio have capability of DSL service (Grubesic, 2008).

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교신: 이건학, 500-757, 광주광역시 북구 용봉동 300 전남대학교 사범대학 지리교육과(이메일: gunhlee@jnu.ac.kr, 전화: 062-530-2382, 팩스: 062-530-2389)

Correspondence: Gunhak Lee, Department of Geography Education, Chonnam National University, 300 Yongbong-dong, Buk-gu, Gwangju, 500-757, Korea (e-mail: gunhlee@jnu.ac.kr, phone: +82-62-530-2382, fax: +82-62-530-2382)

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