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SOME COMMON FIXED POINT THEOREMS FOR GENERALIZED f-WEAKLY CONTRACTIVE MAPPINGS

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Abstract. In this paper, we first prove a common fixed point theorem for generalized nonlinear contraction mappings in complete metric spaces there by generalizing and extending some known results. Then we present this result in the context of ordered metric spaces by using monotone non-decreasing mapping.

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1. Introduction and Preliminaries

It is well known that Banach's fixed point theorem for contraction mappings is one of the pivotal result of analysis. Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be *contraction* if there exists $0 < k < 1$ such that for all $x, y \in X$,

$$d(Tx, Ty) \leq kd(x, y). \tag{1.1}$$

If the metric space (X, d) is complete, then the mapping satisfying (1.1) has a unique fixed point.

A natural question is that whether we can find contractive conditions which will imply existence of fixed point in a complete metric space but will not imply continuity.

Kannan [7], [8] proved the following result, giving an affirmative answer to above question.

Theorem 1. *If $T : X \to X$, where (X, d) is a complete metric space, satisfies*

$$d(Tx, Ty) \leq k[d(x, Tx) + d(y, Ty)], \tag{1.2}$$

where $0 < k < \frac{1}{2}$ and $x, y \in X$, then T has a unique fixed point.

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The mappings satisfying (1.2) are called Kannan type mappings. A similar type of contractive condition has been studied by Chatterjee [3] and he proved the following result.

Theorem 2. *If $T : X \to X$, where (X, d) is a complete metric space, satisfies*

$$d(Tx, Ty) \leq k[d(x, Ty) + d(y, Tx)], \tag{1.3}$$

where $0 < k < \frac{1}{2}$ and $x, y \in X$, then T has a unique fixed point.

In Theorems 1 and 2 there is no requirement of continuity of T.

A map $T : X \to X$ is called a *weakly contractive* (see [1], [4], [11]) if for each $x, y \in X$,

$$d(Tx, Ty) \leq d(x, y) - \psi(d(x, y)) \tag{1.4}$$

where $\psi : [0, \infty) \to [0, \infty)$ is continuous and nondecreasing, $\psi(x) = 0$ if and only if $x = 0$ and $\lim \psi(x) = \infty$.

If we take $\psi(x) = kx$, $0 < k < 1$, then a weakly contractive mapping is called contraction.

A map $T : X \to X$ is called *f-weakly contractive* (see [5]) if for each $x, y \in X$,

$$d(Tx, Ty) \leq d(fx, fy) - \psi(d(fx, fy)) \tag{1.5}$$

where $f : X \to X$ is a self-mapping, $\psi : [0, \infty) \to [0, \infty)$ is continuous and nondecreasing, $\psi(x) = 0$ if and only if $x = 0$ and $\lim \psi(x) = \infty$.

If we take $\psi(x) = (1 - k)x$, $0 < k < 1$, then a f-weakly contractive mapping is called f-contraction. Further, if f = identity mapping and $\psi(x) = (1 - k)x$, $0 < k < 1$, then a f-weakly contractive mapping is called contraction.

A map $T : X \to X$ is called a *generalized weakly contractive* (see [4]) if for each $x, y \in X$,

$$d(Tx, Ty) \leq \frac{1}{2}[d(x, Ty) + d(y, Tx)] - \psi(d(x, Ty), d(y, Tx)) \tag{1.6}$$

where $\psi : [0, \infty)^2 \to [0, \infty)$ is continuous and nondecreasing, $\psi(x, y) = 0$ if and only if $x = y = 0$.

If we take $\psi(x, y) = k(x + y)$, $0 < k < \frac{1}{2}$, then inequality (1.6) reduces to (1.3). Choudhury [4] shows that generalized weakly contractive mapping are generalizations of contractive mappings given by Chatterjee (1.3) and it constitute a strictly larger class of mappings than given by Chatterjee.

A map $T : X \to X$ is called a *generalized f-weakly contractive* if for each $x, y \in X$,

$$d(Tx, Ty) \leq \frac{1}{2}[d(fx, Ty) + d(fy, Tx)] - \psi(d(fx, Ty), d(fy, Tx)) \tag{1.7}$$

where $f : X \to X$ is a self-mapping, $\psi : [0, \infty)^2 \to [0, \infty)$ is continuous and nondecreasing, $\psi(x, y) = 0$ if and only if $x = y = 0$.

If f =identity mapping, then generalized f-weakly contractive mapping is generalized weakly contractive.

Alber and Guerre-Delabriere [1] introduced the concept of weakly contractive mappings and proved the existence of fixed points for single-valued weakly contractive mappings in Hilbert spaces. Thereafter, in 2001, Rhoades [11] proved the fixed point theorem which is one of the generalizations of Banach's Contraction Mapping Principle, because the weakly contractions contains contractions as the special case and he also showed that some results of [1] are true for any Banach space. In fact, weakly contractive mappings are closely related to maps of Boyd and Wong [2] and of Reich types [10].

A point $x \in X$ is a coincidence point (common fixed point) of f and T if $fx = Tx$ ($x = fx = Tx$). The set of coincidence points (fixed points) of f and T is denoted by $C(f,T)$ ($F(f,T)$). The pair (f,T) is called *commuting* if $Tfx = fTx$ for all $x \in X$.

In this paper, we first prove a common fixed point theorem for generalized f-weakly contractive mappings in complete metric spaces and then present this result in the context of ordered metric spaces by using monotone f-nondecreasing mapping. The proved results generalize and extend some of the known results of the subject.

2. Main Results

2.1.Common fixed point. In this section, we proved a common fixed point theorem for generalized f-weakly contractive mappings in complete metric spaces.

Theorem 1. *Let f and T are self-mappings of X, where (X,d) is a complete metric space and $T(X) \subseteq f(X)$. If f and T satisfy*

$$d(Tx,Ty) \leq \frac{1}{2}[d(fx,Ty) + d(fy,Tx)] - \psi(d(fx,Ty), d(fy,Tx)), \tag{2.1}$$

where $\psi : [0,\infty)^2 \to [0,\infty)$ is a continuous mapping such that $\psi(x,y) = 0$ if and only if $x = y = 0$, for all $x, y \in X$, then T and f have a unique coincidence point in M. Further, if T, f commute at their coincidence points, then T and f have a common fixed point.

Proof. Let $x_0 \in X$. Since $T(X) \subseteq f(x)$, we can choose $x_1 \in X$ so that $fx_1 = Tx_0$. Since $Tx_1 \in f(X)$, there exists $x_2 \in X$ such that $fx_2 = Tx_1$. By induction, we construct a sequence $\{x_n\}$ in X such that $fx_{n+1} = Tx_n$, for every $n \geq 0$. Consider

$$\begin{aligned}
d(Tx_{n+1}, Tx_n) &\leq \frac{1}{2}[d(fx_{n+1}, Tx_n) + d(fx_n, Tx_{n+1})] - \\
&\quad \psi(d(fx_{n+1}, Tx_n), d(fx_n, Tx_{n+1})) \\
&= \frac{1}{2}d(Tx_{n-1}, Tx_{n+1}) - \psi(0, d(Tx_{n-1}, Tx_{n+1})) \qquad (*) \\
&\leq \frac{1}{2}d(Tx_{n-1}, Tx_{n+1}) \\
&\leq \frac{1}{2}[d(Tx_{n-1}, Tx_n) + d(Tx_n, Tx_{n+1})]
\end{aligned}$$

Hence for all $n = 1, 2 \ldots,$ we have $d(Tx_{n+1}, Tx_n) \leq d(Tx_n, Tx_{n-1})$. Thus $\{d(Tx_{n+1}, Tx_n)\}$ is a monotone decreasing sequence of non-negative real numbers and hence is convergent.

Let $d(Tx_{n+1}, Tx_n) \to r$. From inequality $(*)$, we have

$$\begin{aligned} d(Tx_{n+1}, Tx_n) &\leq \frac{1}{2} d(Tx_{n-1}, Tx_{n+1}) \\ &\leq \frac{1}{2}[d(Tx_{n-1}, Tx_n) + d(Tx_n, Tx_{n+1})] \end{aligned}$$

Taking $n \to \infty$, we have

$$r \leq \lim \frac{1}{2} d(Tx_{n-1}, Tx_{n+1}) \leq \frac{1}{2} r + \frac{1}{2} r,$$

So $\lim d(Tx_{n-1}, Tx_{n+1}) = 2r$. Using the continuity of ψ and inequality $(*)$, we have $r \leq r - \psi(0, 2r)$, and consequently, $\psi(0, 2r) \leq 0$. Thus $r = 0$. Hence $d(Tx_{n+1}, Tx_n) \to 0$.

Now, we show that $\{Tx_n\}$ is a cauchy sequence. If otherwise, then there exist $\epsilon > 0$ for which we can find subsequences $\{Tx_{m(k)}\}$ and $\{Tx_{n(k)}\}$ of $\{Tx_n\}$ with $n(k) > m(k) > k$ such that for every k, $d(Tx_{m(k)}, Tx_{n(k)}) \geq \epsilon$, $d(Tx_{m(k)}, Tx_{n(k)-1}) < \epsilon$. So, we have

$$\begin{aligned} \epsilon &\leq d(Tx_{m(k)}, Tx_{n(k)}) \\ &\leq d(Tx_{m(k)}, Tx_{n(k)-1}) + d(Tx_{n(k)-1}, Tx_{n(k)}) \\ &\leq \epsilon + d(Tx_{n(k)-1}, Tx_{n(k)}). \end{aligned}$$

Letting $n \to \infty$ and using $d(Tx_{n-1}, Tx_n) \to 0$, we have

$$\lim d(Tx_{m(k)}, Tx_{n(k)}) = \epsilon = \lim d(Tx_{m(k)}, Tx_{n(k)-1}). \tag{2.2}$$

Again,

$$\begin{aligned} d(Tx_{m(k)}, Tx_{n(k)-1}) \leq\ & d(Tx_{m(k)}, Tx_{m(k)-1}) + d(Tx_{m(k)-1}, Tx_{n(k)}) + \\ & d(Tx_{n(k)}, Tx_{n(k)-1}), \end{aligned}$$

and

$$d(Tx_{m(k)-1}, Tx_{n(k)}) \leq d(Tx_{m(k)-1}, Tx_{m(k)}) + d(Tx_{m(k)}, Tx_{n(k)}).$$

Taking $n \to \infty$ in the above two inequalities and using (2.2) we get,

$$\lim d(Tx_{m(k)-1}, Tx_{n(k)}) = \epsilon.$$

Also, we have

$$\begin{aligned} \epsilon &\leq d(Tx_{m(k)}, Tx_{n(k)}) \\ &\leq \frac{1}{2}[d(fx_{m(k)}, Tx_{n(k)}) + d(fx_{n(k)}, Tx_{m(k)})] - \\ &\quad \psi(d(fx_{m(k)}, Tx_{n(k)}), d(fx_{n(k)}, Tx_{m(k)})) \\ &= \frac{1}{2}[d(Tx_{m(k)-1}, Tx_{n(k)}) + d(Tx_{n(k)-1}, Tx_{m(k)})] - \\ &\quad \psi(d(Tx_{m(k)-1}, Tx_{n(k)}), d(Tx_{n(k)-1}, Tx_{m(k)})). \end{aligned}$$

Taking $n \to \infty$, we have $\epsilon \leq \frac{1}{2}[\epsilon+\epsilon]-\psi(\epsilon,\epsilon)$ and consequently $\psi(\epsilon,\epsilon) \leq 0$, which is contradiction since $\epsilon > 0$. Hence $\{Tx_n\}$ is a cauchy sequence and therefore is convergent in the complete metric space (X, d). Let $Tx_n \to Tz$. Consider

$$d(Tz, fz) \quad \leq \quad d(Tz, Tx_n) + d(Tx_n, fz).$$

Letting $n \to \infty$, we obtain $d(Tz, fz) \leq d(Tz, fz)$. This implies that $d(Tz, fz) = 0$. Hence $Tz = fz$. Thus z is a coincidence point of T and f.

Now suppose that T and f commute at z. Let $w = T(z) = f(z)$. Then $T(w) = T(f(z)) = f(T(z)) = f(w)$. Consider

$$\begin{aligned} d(T(z), T(w)) &\leq \frac{1}{2}[d(fz, Tw) + d(fw, Tz)] - \psi(d(fz, Tw), d(fw, Tz)) \\ &= \frac{1}{2}[d(Tz, Tw) + d(Tw, Tz)] - \psi(d(Tz, Tw), d(Tw, Tz)) \\ &= d(Tw, Tz) - \psi(d(Tz, Tw), d(Tw, Tz)). \end{aligned}$$

This implies that $d(Tz, Tw) = 0$, by the property of ψ. Therefore, $T(w) = f(w) = w$. Hence the result. □

If f =identity mapping, then we have:

Corollary 1. *(see* [4]*) Let T a self-mapping of X, where (X, d) is a complete metric space. If T satisfy*

$$d(Tx, Ty) \leq \frac{1}{2}[d(x, Ty) + d(y, Tx)] - \psi(d(x, Ty), d(y, Tx)), \tag{2.3}$$

where $\psi : [0, \infty)^2 \to [0, \infty)$ is a continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$, for all $x, y \in X$, then T has a unique fixed point.

Remark 1. *Corollary 1 extends Theorem 2, if $\psi(x, y) = (\frac{1}{2} - k)(x + y)$, $0 < k < \frac{1}{2}$ in Corollary 1.*

Example 1. *Let $X = \{p, q, r\}$ and d is a metric defined on X. Let T and f are self-mappings of X such that $Tp = fq$, $Tq = fq$, $Tr = fp$, $d(fp, fq) = 1$, $d(fq, fr) = 2$, $d(fr, fp) = 1.5$ and $\psi(a, b) = \frac{1}{2}\min\{a, b\}$. Then T is generalized f-weakly contraction and q is the coincidence point of T and f.*

If f =identity mapping, this example given in [4].

2.2.Common fixed theorem in ordered metric spaces. In this section, we extend the Theorem 1 and proved a common fixed point theorem for f-nondecreasing generalized nonlinear contraction mappings in the context of ordered metric spaces.

Definition 1. *Suppose $(X, \leq)$ is a partially ordered set and $T, f : X \to X$. T is said to be monotone f-nondecreasing if for all $x, y \in X$,*

$$fx \leq fy \text{ implies } Tx \leq Ty. \tag{2.4}$$

If f=identity mapping in Definition 1, then T is *monotone nondecreasing.*

Theorem 2. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let f and T are self-mappings of X and $T(X) \subseteq f(X)$, T is f-nondecreasing mapping. If f and T satisfy*

$$d(Tx, Ty) \leq \frac{1}{2}[d(fx, Ty) + d(fy, Tx)] - \psi(d(fx, Ty), d(fy, Tx)) \text{ for } x \geq y, \tag{2.5}$$

where $\psi : [0, \infty)^2 \to [0, \infty)$ is a continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$, for all $x, y \in X$, for which $f(x) \leq f(y)$.

If $\{f(x_n)\} \subset X$ is a nondecreasing sequence with $f(x_n) \to f(z)$ in $f(X)$, then $f(x_n) \leq f(z)$, $f(z) \leq f(f(z))$ for every n.

Also suppose that $f(X)$ is closed. If there exists an $x_0 \in X$ with $f(x_0) \leq T(x_0)$, then T and f have a coincidence point. Further, if T and f commute at their coincidence points, then T and f have a common fixed point.

Proof. Let $x_0 \in X$ such that $f(x_0) \leq T(x_0)$. Since $T(X) \subseteq f(x)$, we can choose $x_1 \in X$ so that $fx_1 = Tx_0$. Since $Tx_1 \in f(X)$, there exists $x_2 \in X$ such that $fx_2 = Tx_1$. By induction, we construct a sequence $\{x_n\}$ in X such that $fx_{n+1} = Tx_n$, for every $n \geq 0$. Since $f(x_0) \leq T(x_0)$, $T(x_0) = f(x_1)$, $f(x_0) \leq f(x_1)$, T is f-nondecreasing mapping,$T(x_0) \leq T(x_1)$. Similarly $f(x_1) \leq f(x_2)$, $Tx_1) \leq T(x_2)$, $f(x_2) \leq f(x_3)$. Continuing, we obtain $T(x_0) \leq T(x_1) \leq T(x_2) \leq \ldots \leq T(x_n) \leq T(x_{n+1}) \leq \ldots$.

In what follows we will suppose that $d(T(x_n), T(x_{n+1})) > 0$ for all n, since if $T(x_{n+1}) = T(x_n)$ for some n, then $T(x_{n+1}) = f(x_{n+1})$, i.e. T and f have a coincidence point x_{n+1}, and so we have the result. We will show that $d(T(x_n), T(x_{n+1})) < d(T(x_{n-1}), T(x_n))$, for every $n \geq 1$. Consider

$$\begin{aligned} d(Tx_{n+1}, Tx_n) &\leq \frac{1}{2}[d(fx_{n+1}, Tx_n) + d(fx_n, Tx_{n+1})] - \\ &\quad \psi(d(fx_{n+1}, Tx_n), d(fx_n, Tx_{n+1})) \\ &= \frac{1}{2}d(Tx_{n-1}, Tx_{n+1}) - \psi(0, d(Tx_{n-1}, Tx_{n+1})) \qquad (**) \\ &\leq \frac{1}{2}d(Tx_{n-1}, Tx_{n+1}) \\ &\leq \frac{1}{2}[d(Tx_{n-1}, Tx_n) + d(Tx_n, Tx_{n+1})]. \end{aligned}$$

Hence for all $n = 1, 2 \ldots$, we have $d(Tx_{n+1}, Tx_n) \leq d(Tx_n, Tx_{n-1})$. Thus $\{d(Tx_{n+1}, Tx_n)\}$ is a monotone decreasing sequence of non-negative real numbers and hence is convergent.

Now, proceeding as in Theorem 1, we can prove that $\{Tx_n\}$ is a cauchy sequence. Also, $\{T(x_n)\} = \{f(x_{n+1})\} \subseteq X$ and $f(X)$ is closed, there exists

$z \in X$ such that $fx_n \to fz$. Since $fx_n \to fz$, $fx_n \leq fz$, we have

$$\begin{aligned} d(Tz, fx_{n+1}) &= d(Tz, Tx_n) \\ &\leq \frac{1}{2}[d(fz, Tx_n) + d(fx_n, Tz)] - \psi(d(fz, Tx_n), d(fx_n, Tz)) \\ &= \frac{1}{2}[d(fz, fx_{n+1}) + d(fx_n, Tz)] - \psi(d(fz, fx_{n+1}), d(fx_n, Tz)) \end{aligned}$$

Letting $n \to \infty$, and using continuity of ψ, we have

$$\begin{aligned} d(Tz, fz) &\leq \frac{1}{2}d(fz, Tz) - \psi(0, d(fz, Tz)) \\ &\leq \frac{1}{2}d(fz, Tz) \end{aligned}$$

This implies that $d(Tz, fz) = 0$. Hence $Tz = fz$. Thus z is a coincidence point of T and f.

Now suppose that T and f commute at z. Let $w = T(z) = f(z)$. Then $T(w) = T(f(z)) = f(T(z)) = f(w)$. Consider

$$\begin{aligned} d(T(z), T(w)) &\leq \frac{1}{2}[d(fz, Tw) + d(fw, Tz)] - \psi(d(fz, Tw), d(fw, Tz)) \\ &= \frac{1}{2}[d(Tz, Tw) + d(Tw, Tz)] - \psi(d(Tz, Tw), d(Tw, Tz)) \\ &= d(Tw, Tz) - \psi(d(Tz, Tw), d(Tw, Tz)). \end{aligned}$$

This implies that $d(Tz, Tw) = 0$, by the property of ψ. Therefore, $T(w) = f(w) = w$. Hence the result. □

Corollary 2. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T is a nondecreasing self-mapping of X and T satisfy*

$$d(Tx, Ty) \leq \frac{1}{2}[d(x, Ty) + d(y, Tx)] - \psi(d(x, Ty), d(y, Tx)), \text{ for } x \geq y, \tag{2.6}$$

where $\psi : [0, \infty)^2 \to [0, \infty)$ is a continuous mapping such that $\psi(x, y) = 0$ if and only if $x = y = 0$, for all $x, y \in X$. Also suppose either

(i) if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \leq z$ for every n.

or

(ii) T is continuous.

If there exists an $x_0 \in X$ with $x_0 \leq T(x_0)$, then T has a fixed point.

Proof. If (i) holds, then taking f =identity mapping in Theorem 2 we get the result.

If (ii) holds then proceeding as in Theorem 2 with f =identity mapping, we can prove that $\{Tx_n\}$ is a cauchy sequence, $z = \lim x_{n+1} = \lim T(x_n) = T(\lim x_n) = T(z)$. Hence the result. □

If $\psi(x,y) = (\frac{1}{2} - k)(x+y)$, $0 < k < \frac{1}{2}$, we have

Corollary 3. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T is a nondecreasing self-mapping of X and T satisfy*

$$d(Tx, Ty) \leq k[d(x, Ty) + d(y, Tx)], \text{ for } x \geq y, \tag{2.7}$$

where $0 < k < \frac{1}{2}$, for all $x, y \in X$. Also suppose either

(i) if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \leq z$ for every n.

or

(ii) T is continuous.

If there exists an $x_0 \in X$ with $x_0 \leq T(x_0)$, then T has a fixed point.

Remark 2. *Theorem 2 extends the corresponding results of* [4], [6], [9] *and* [11] *for generalized f-weakly contractive mappings in the context of ordered metric spaces.*

For weakly contractive mappings, the following result proved in [6] in the context of ordered metric spaces which is an extension of the corresponding result in [11].

Corollary 4. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T is a nondecreasing self-mapping of X and T satisfy*

$$d(Tx, Ty) \leq d(x, y) - \psi(d(x, y)), \text{ for } x \geq y, \tag{2.8}$$

where $\psi : [0, \infty) \to [0, \infty)$ is a continuous mapping such that $\psi(x) = 0$ if and only if $x = 0$ and $\lim \psi(x) = \infty$, for all $x, y \in X$. Also suppose either

(i) if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \leq z$ for every n.

or

(ii) T is continuous.

If there exists an $x_0 \in X$ with $x_0 \leq T(x_0)$, then T has a fixed point.

If $\psi(x) = kx$, $0 < k < 1$, we have

Corollary 5. *(see* [9]*) Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T is a nondecreasing self-mapping of X and T satisfy*

$$d(Tx, Ty) \leq kd(x, y), \text{ for } x \geq y, \tag{2.9}$$

where $0 < k < 1$, for all $x, y \in X$. Also suppose either

(i) if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to z$ in X, then $x_n \leq z$ for every n.

or

(ii) T is continuous.

If there exists an $x_0 \in X$ with $x_0 \leq T(x_0)$, then T has a fixed point.

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