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GLOBAL EXPONENTIAL STABILITY OF BAM NEURAL NETWORKS WITH IMPULSES AND DISTRIBUTED DELAYS

YUANFU SHAO* AND ZHENGUO LUO

Abstract. By using an important lemma, some analysis techniques and Lyapunov functional method, we establish the sufficient conditions of the existence of equilibrium solution of a class of BAM neural network with impulses and distributed delays. Finally, applications and an example are given to illustrate the effectiveness of the main results.

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1. Introduction

Recently, BAM neural networks have attracted the attention of many researchers due to its applications in many fields such as pattern recognition, automatic control and optimization, and many results for BAM neural networks have been derived [1-8]. Further, the theory of impulsive differential equations is now being recognized to be not only richer than the corresponding theory of differential equations without impulse, but also represents a more natural framework for mathematical modelling of many real world phenomena, such as population dynamics and neural networks, hence, the impulsive differential equations have been extensively studied recently [5,6,9-19]. On the other hand, in practice, it is preferable and desirable that neural networks not only converge to equilibrium points but also admit a convergence rate which is as fast as possible. Since the exponential stability gives a fast convergence rate to the equilibrium point, it is necessary to study the exponential stability and to estimate the exponential convergence rate, see [9,10,20-27].

Therefore, it is necessary and important for scholars to study the existence and exponential stability of equilibrium points for impulsive neural networks

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with delays [9,10,23-26]. For example, Zhou [10] investigated the following BAM neural networks:

$$\begin{cases} x_i'(t) & = -a_i x_i(t) + \sum\limits_{j=1}^{m} h_{ji} g_j(y_j(t)) \\ & \quad + \sum\limits_{j=1}^{m} l_{ji} \int_0^{\infty} k_{ji}(s) f_j(y_j(t-\tau_{ji}-s))ds + b_i,\ t \neq t_k \\ \Delta x_i(t) & = I_{ik}(x_i(t)) = B_{ik} x_i(t_k) + \int_{t_k-1}^{t_k} C_{ik}(s) x_i(s) ds + \alpha_{ik}, t = t_k \\ y_j'(t) & = -\bar{a}_j y_j(t) + \sum\limits_{i=1}^{n} \bar{h}_{ij} \bar{g}_i(x_i(t)) \\ & \quad + \sum\limits_{i=1}^{n} \bar{l}_{ij} \int_0^{\infty} \bar{k}_{ij}(s) \bar{f}_i(x_i(t-\sigma_{ij}-s))ds + \bar{b}_j,\ t \neq t_k \\ \Delta y_j(t) & = J_{jk}(y_j(t)) = \bar{B}_{jk} y_j(t_k) + \int_{t_k-1}^{t_k} \bar{C}_{jk}(s) y_j(s) ds + \bar{\alpha}_{jk}, t = t_k \end{cases} \quad (1.1)$$

By using the contraction mapping principle and Lyapunov functional, the sufficient conditions ensuring global exponential stability of the equilibrium points of (1.1) are established.

Motivated by above discussion, in this paper, we shall establish a class of impulsive BAM neural network with distributed delays as follows:

$$\begin{cases} x_i'(t) & = -a_i e_i(x_i(t)) + \sum\limits_{j=1}^{m} b_{ji} f_j(y_j(t)) \\ & \quad + \sum\limits_{j=1}^{m} l_{ji} \int_0^{\tau} k_{ji}(s) g_j(y_j(t-\tau_{ji}-s))ds + I_i,\ t \neq t_k \\ \Delta x_i(t) & = x_i(t^+) - x_i(t^-) = \tilde{I}_{ik}(x_i(t)), t = t_k \\ y_j'(t) & = -c_j h_j(y_j(t)) + \sum\limits_{i=1}^{n} d_{ij} p_i(x_i(t)) \\ & \quad + \sum\limits_{i=1}^{n} \tilde{l}_{ij}(t) \int_0^{\sigma} \tilde{k}_{ij}(s) q_i(x_i(t-\sigma_{ij}-s))ds + J_j,\ t \neq t_k \\ \Delta y_j(t) & = y_j(t^+) - y_j(t^-) = \tilde{J}_{jk}(y_j(t)), t = t_k \end{cases} \quad (1.2)$$

with initial values

$$x_i(s) = \phi_{x_i}(s), -h \leq s \leq 0, y_j(s) = \phi_{y_j}(s), -\tilde{h} \leq s \leq 0, h = \sigma + \max_{1\leq i\leq n, 1\leq j\leq m}\{\sigma_{ij}\},$$

$$\tilde{h} = \tau + \max_{1\leq i\leq n, 1\leq j\leq m}\{\tau_{ji}\},\ \phi_{x_i} \in C([-h,0],R),\ \phi_{y_j} \in C([-\tilde{h},0],R).$$

where $x_i(t)$ and $y_j(t)$ are the states of the ith neuron and the jth neuron at time $t, t \in R^+ = [0,+\infty)$, respectively. a_i, c_j denote the neuron charging times. b_{ji}, l_{ji}, d_{ij} and $\tilde{l}_{ij}(t)$ are the weights of the neuron interconnections. I_i and J_j are the external inputs on the neurons. $\Delta x_i(t)$ and $\Delta y_j(t)$ are the impulses at moments $t = t_k$ and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = \infty, i = 1,2,\cdots,n, j = 1,2,\cdots,m$. $\tau > 0, \sigma > 0$ are constants. As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $z(t) = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_m(t))^T$, we assume that $z(t_k^+)$ exists, and $z(t_k^-) = z(t_k)$. It is clear that there exist the limits $z'(t_k^-), z'(t_k^+)$ such that $z'(t_k^-) = z'(t_k)$.

Our aim is, under the generalized r-norm ($r > 1$), by using an important lemma and constructing suitable Lyapunov functional, to obtain the sufficient conditions ensuring the existence and globally exponential stability of equilibrium solution of (1.2).

The rest of this paper is organized as follows. In section 2, definitions and lemmas are introduced. In section 3, by using Forti and Tesi's theorem, the sufficient conditions of the existence of equilibrium solution are established. In section 4, the conditions ensuring the globally exponential stability of the equilibrium point are derived. Finally in section 5, applications and an illustrative example are given to show the usefulness of the main results.

2. Preliminaries

First we make some preparation and introduce some elementary definitions and lemmas.

Let PC be a class of function $\phi = (\phi_x, \phi_y)^T : ([-h, 0], [-\tilde{h}, 0])^T \to (R^n, R^m)^T$ satisfying:

(i) ϕ is piecewise continuous with first kind discontinuity at point t_k, and is left-continuous at $t_k, k = 1, 2, \cdots, p$.

(ii) $\Delta x_i(t_k) = \tilde{I}_{ik}(x_i(t_k)),\ \Delta y_j(t_k) = \tilde{J}_{jk}(y_j(t_k))$ for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m, k = 1, 2, \cdots$.

For each $\phi = (\phi_x^T, \phi_y^T)^T \in PC, z(t) \in R^{n+m}$, we define

$$\|\phi\| = \left(\sum_{i=1}^{n} \sup_{s\in[-h,0]} |\phi_{x_i}(s)|^r + \sum_{j=1}^{m} \sup_{s\in[-\tilde{h},0]} |\phi_{y_j}(s)|^r \right)^{\frac{1}{r}},$$

$$\|z(t)\| = \left(\sum_{i=1}^{n} |x_i(t)|^r + \sum_{j=1}^{m} |y_j(t)|^r \right)^{\frac{1}{r}},$$

where $r > 1$ is a constant, $z(t) = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$, $\phi_x = (\phi_{x_1}, \phi_{x_2}, \cdots, \phi_{x_n})^T$ and $\phi_y = (\phi_{y_1}, \phi_{y_2}, \cdots, \phi_{y_m})^T$.

Definition 2.1. A constant vector $z^* = (x_1^*, x_2^*, \cdots, x_n^*, y_1^*, \cdots, y_m^*)^T$ is said to be an equilibrium solution of impulsive system (1.2) if

$$\text{(i)} \begin{cases} a_i e_i(x_i^*) = \sum_{j=1}^{m} b_{ji} f_j(y_j^*) + \sum_{j=1}^{m} l_{ji} g_j(y_j^*) \int_0^{\tau} k_{ji}(s)ds + I_i \\ c_j h_j(y_j^*) = \sum_{i=1}^{n} d_{ij} p_i(x_i^*) + \sum_{i=1}^{n} \tilde{l}_{ij} q_i(x_i^*) \int_0^{\sigma} \tilde{k}_{ij}(s)ds + J_j \end{cases} \tag{2.1}$$

(ii) $\tilde{I}_{ik}(x_i^*) = 0,\ \tilde{J}_{jk}(y_j^*) = 0$.

Definition 2.2. The unique equilibrium $z^* = (x_1^*, x_2^*, \cdots, x_n^*, y_1^*, \cdots, y_m^*)^T$ of system (1.2) is said to be globally exponentially stable if there exists constant

$\alpha > 0, M \geq 1$ such that for all $t > 0$,

$$\left\{\sum_{i=1}^{n} |x_i(t) - x_i^*|^r + \sum_{j=1}^{m} |y_j(t) - y_j^*|^r\right\}^{\frac{1}{r}} \leq Me^{-\alpha t}\|\phi - z^*\|,$$

where
$\|\phi-z^*\| = \left\{\sum_{i=1}^{n} \sup_{s\in[-h,0]} |\phi_{x_i}(s) - x_i^*|^r + \sum_{j=1}^{m} \sup_{s\in[-\tilde{h},0]} |\phi_{y_j}(s) - y_j^*|^r\right\}^{\frac{1}{r}}$.

Definition 2.3 [28]. A real matrix $A = (a_{ij})_{n\times n}$ is said to be an M-matrix if $a_{ii} > 0, a_{ij} \leq 0(i, j = 1, 2, \cdots, n, i \neq j)$ and successive principle minors of A are positive.

Lemma 2.1 [29]. Let Q be an $n \times n$ matrix with non-positive off-diagonal elements. Then Q is an M-matrix if and only if one of the following conditions holds:
(i) There exists a vector $\xi > 0$ such that $Q\xi > 0$;
(ii) There exists a vector $\xi > 0$ such that $\xi^T Q > 0$.

Lemma 2.2 [30]. (Young inequality) Assume that $a, b, p, q > 0, p + q = 1$, then $a^p b^q \leq pa + qb$.

Lemma 2.3 [31]. (Forti and Tesi' theorem) If $H(x) \in C^0$ satisfies the following conditions:
(i) $H(x)$ is injective on R^{n+m},
(ii) $\|H(x)\| \to +\infty$ as $\|x\| \to +\infty$,
then $H(x)$ is homeomorphism of R^{n+m} onto itself.

Throughout this paper, we always assume that:
(A_1) $a_i > 0, c_j > 0, b_{ji}, d_{ij}, l_{ji}, \tilde{l}_{ij}, I_i$ and J_j are constants for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$.
(A_2) $e_i, h_j : R \to R$ are differentiable function satisfying $0 < \varrho_i \leq e_i'(u), e_i(0) = 0$ and $0 < \tilde{\varrho}_j \leq h_j'(v), h_j(0) = 0$ for any $u, v \in R, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.
(A_3) Functions $f_j(u), g_j(u), p_i(u), q_i(u)$ satisfy the Lipschitz conditions, i.e., there exist positive constants F_j, G_j, P_i, Q_i such that

$$|f_j(u) - f_j(v)| \leq F_j|u - v|, \quad |g_j(u) - g_j(v)| \leq G_j|u - v|,$$
$$|p_i(u) - p_i(v)| \leq P_i|u - v|, \quad |q_i(u) - q_i(v)| \leq Q_i|u - v|$$

with $f_j(0) = g_j(0) = 0, p_i(0) = q_i(0) = 0$ for any $u, v \in R, i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$.
(A_4) Functions $k_{ji}(t)$ and $\tilde{k}_{ij}(t)$ are positive piecewise continuous and satisfy

$$\int_0^{\tau} e^{\eta t}k_{ji}(t)dt = \psi(\eta, \tau), \qquad \int_0^{\sigma} e^{\eta t}\tilde{k}_{ij}(t)dt = \tilde{\psi}(\eta, \sigma),$$

where $\psi(\eta, \tau)$ and $\tilde{\psi}(\eta, \sigma)$ are continuous in η. When $\tau = \infty, \sigma = \infty$, $\psi(\eta, \tau) \equiv \varphi(\eta)$, $\tilde{\psi}(\eta, \sigma) \equiv \varphi(\eta)$ with $\varphi(0) = \tilde{\varphi}(0) = 1$.

3. Existence of equilibrium solution

In this section, employing the Forti and Tesi's theorem, we will establish the sufficient conditions of the existence of equilibrium solution of system (1.2).

Theorem 3.1. Assume that $(A_1)-(A_4)$ hold. Further, if there exists a constant $r>1$ such that the following condition holds.
(A_5) $\Gamma=\begin{pmatrix} rA-(r-1)\tilde{G} & -\tilde{P} \\ -\tilde{F} & rC-(r-1)\tilde{Q}\end{pmatrix}$ is a nonsingular M-matrix,
where $A=diag(a_1\varrho_1,a_2\varrho_2,\cdots,a_n\varrho_n)$, $C=diag(c_1\tilde{\varrho}_1,c_2\tilde{\varrho}_2,\cdots,c_m\tilde{\varrho}_m)$, $\tilde{G}=diag(\tilde{G}_1,\tilde{G}_2,\cdots,\tilde{G}_n)$, $\tilde{Q}=diag(\tilde{Q}_1,\tilde{Q}_2,\cdots,\tilde{Q}_m)$, $\tilde{P}=(\tilde{p}_{ij})_{n\times m}$, $\tilde{F}=(\tilde{f}_{ji})_{m\times n}$, $\tilde{G}_i=\sum_{j=1}^m|b_{ji}|F_j+|l_{ji}|G_j\int_0^\tau k_{ji}(s)ds$, $\tilde{Q}_j=\sum_{i=1}^n|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^\sigma\tilde{k}_{ij}(s)ds$, $\tilde{p}_{ij}=P_i|d_{ij}|+Q_i|\tilde{l}_{ij}|\int_0^\sigma\tilde{k}_{ij}(s)ds$, $\tilde{f}_{ji}=|b_{ji}|F_j+|l_{ji}|G_j\int_0^\tau k_{ji}(s)ds$. Then system (1.2) admits exactly one equilibrium solution $z^*=(x_1^*,\cdots,x_n^*,y_1^*,\cdots,y_m^*)^T$.

Proof. For $z=(x_1,x_2,\cdots,x_n,y_1,\cdots,y_m)\in R^{n+m}$, define a mapping $\psi: R^{n+m}\to R^{n+m}$ as follows:

$$\begin{cases} \psi_i(z)=a_ie_i(x_i)-\sum\limits_{j=1}^m b_{ji}f_j(y_j)-\sum\limits_{j=1}^m\int_0^\tau k_{ji}(s)g_j(y_j)ds-I_i \\ \psi_{n+j}(z)=c_jh_j(y_j)-\sum\limits_{i=1}^n d_{ij}p_i(x_i)-\sum\limits_{i=1}^n\int_0^\sigma k_{ij}(s)q_i(x_i)ds-J_j, \end{cases} \tag{3.1}$$

where $\psi(z)=(\psi_1(z),\psi_2(z),\cdots,\psi_n(z),\psi_{n+1}(z),\cdots,\psi_{n+m}(z))^T\in R^{n+m}$.

Firstly, we demonstrate that the mapping ψ is injective, i.e., $\psi(z)=\psi(\tilde{z})$ implies that $z=\tilde{z}$ for any $z,\tilde{z}\in R^{n+m}$. It is clear that $\psi(z)=\psi(\tilde{z})$ means:

$$\begin{cases} a_i(e_i(x_i)-e_i(\tilde{x}_i))-\sum_{j=1}^m b_{ji}(f_j(y_j)-f_j(\tilde{y}_j)) \\ \qquad -\sum_{j=1}^m l_{ji}\int_0^\tau k_{ji}(s)(g_j(y_j)-g_j(\tilde{y}_j))ds=0 \\ c_j(h_j(y_j)-h_j(\tilde{y}_j))-\sum_{i=1}^n d_{ij}(p_i(x_i)-p_i(\tilde{x}_i)) \\ \qquad -\sum_{i=1}^n\tilde{l}_{ji}\int_0^\sigma k_{ij}(s)(q_i(x_i)-q_i(\tilde{x}_i))ds=0 \end{cases} \tag{3.2}$$

Then from $(A_2)-(A_4)$ and (3.2), we derive that

$$\begin{cases} a_i\varrho_i|x_i-\tilde{x}_i|\le\sum_{j=1}^m\left(|b_{ji}|F_j+|l_{ji}|G_j\int_0^\tau k_{ji}(s)ds\right)|y_j-\tilde{y}_j|, \\ c_j\tilde{\varrho}_j|y_j-\tilde{y}_j|\le\sum_{i=1}^n\left(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^\sigma\tilde{k}_{ij}(s)ds\right)|x_i-\tilde{x}_i|. \end{cases} \tag{3.3}$$

On the other hand, we obtain from (A_5) and Lemma 2.1 that, there exists $\xi=(\xi_1,\xi_2,\cdots,\xi_n,\xi_{n+1},\cdots,\xi_{n+m})^T>0$ such that

$$\begin{cases} r\xi_ia_i\varrho_i-\xi_i(r-1)\sum\limits_{j=1}^m(|b_{ji}|F_j+|l_{ji}|G_j\int_0^\tau k_{ji}(s)ds) \\ \qquad -\sum\limits_{j=1}^m\xi_{n+j}(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^\sigma\tilde{k}_{ij}(s)ds))>0 \\ r\xi_{n+j}c_j\tilde{\varrho}_j-\xi_{n+j}(r-1)\sum\limits_{i=1}^n(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^\sigma\tilde{k}_{ij}(s)ds) \\ \qquad -\sum\limits_{i=1}^n\xi_i(|b_{ji}|F_j+|l_{ji}|G_j\int_0^\tau k_{ji}(s)ds)>0. \end{cases} \tag{3.4}$$

Further, by Lemma 2.2, it follows from (3.3) that

$$\begin{aligned}&\sum_{i=1}^{n}\xi_i a_i\varrho_i|x_i-\tilde{x}_i|^r\\&\le\sum_{i=1}^{n}\xi_i\sum_{j=1}^{m}\left(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau}k_{ji}(s)ds\right)|y_j-\tilde{y}_j||x_i-\tilde{x}_i|^{r-1}\\&\le\sum_{i=1}^{n}\xi_i\sum_{j=1}^{m}\left(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau}k_{ji}(s)ds\right)\times\left(\frac{r-1}{r}|x_i-\tilde{x}_i|^r+\frac{1}{r}|y_j-\tilde{y}_j|^r\right),\end{aligned}\tag{3.5}$$

and

$$\begin{aligned}&\sum_{j=1}^{m}\xi_{n+j}c_j\tilde{\varrho}_j|y_j-\tilde{y}_j|^r\\&\le\sum_{j=1}^{m}\xi_{n+j}\sum_{i=1}^{n}\left(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^{\tau}\tilde{k}_{ij}(s)ds\right)|y_j-\tilde{y}_j|^{r-1}|x_i-\tilde{x}_i|\\&\le\sum_{j=1}^{m}\xi_{n+j}\sum_{i=1}^{n}\left(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^{\sigma}\tilde{k}_{ij}(s)ds\right)\times\left(\frac{r-1}{r}|y_j-\tilde{y}_j|^r+\frac{1}{r}|x_i-\tilde{x}_i|^r\right).\end{aligned}\tag{3.6}$$

(3.5) plus (3.6) lead to

$$\begin{aligned}&\sum_{i=1}^{n}\Bigg(\xi_i a_i\tilde{\varrho}_i-\frac{\xi_i(r-1)}{r}\sum_{j=1}^{m}(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau}k_{ji}(s)ds)-\sum_{j=1}^{m}\frac{\xi_{n+j}}{r}(|d_{ij}|P_i\\&+|\tilde{l}_{ij}|Q_i\int_0^{\sigma}\tilde{k}_{ij}(s)ds)\Bigg)|x_i-\tilde{x}_i|^r+\sum_{j=1}^{m}\Bigg(\xi_{n+j}c_j\tilde{\varrho}_j-\frac{\xi_{n+j}(r-1)}{r}\sum_{i=1}^{n}(|d_{ij}|P_i\\&+|\tilde{l}_{ij}|Q_i\int_0^{\sigma}\tilde{k}_{ij}(s)ds)-\sum_{i=1}^{n}\frac{\xi_i}{r}(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau}k_{ji}(s)ds)\Bigg)|y_j-\tilde{y}_j|^r\le 0\end{aligned}\tag{3.7}$$

Substituting (3.4) into (3.7), we have $|x_i-\tilde{x}_i|^r-0, |y_j-\tilde{y}_j|^r=0$. That is, $x_i=\tilde{x}_i, y_j=\tilde{y}_j$ for $i=1,2,\cdots,n, j=1,2,\cdots,m$, namely, $z=\tilde{z}$, which means $\psi\in C^0$ is injective on R^{n+m}.

Next we demonstrate the property $\|\psi(z)\|\to\infty$ as $\|z\|\to\infty$. Consider mapping $\widetilde{\psi}(z)=\psi(z)-\psi(0)$, i.e.,

$$\tilde{\psi}_i(z)=a_ie_i(x_i)-\sum_{j=1}^{m}b_{ji}f_j(y_j)-\sum_{j=1}^{m}l_{ji}\int_0^{\tau}k_{ji}(s)g_j(y_j)ds,$$

$$\widetilde{\psi}_{n+j}(z)=c_jh_j(y_j)-\sum_{i=1}^{n}d_{ij}p_i(x_i)-\sum_{i=1}^{n}\tilde{l}_{ij}\int_0^{\sigma}\tilde{k}_{ij}(s)q_i(x_i)ds$$

for $z=(x_1,x_2,\cdots,x_n,y_1,\cdots,y_m)^T\in R^{n+m}, i=1,2,\cdots,n, j=1,2,\cdots,m$. It is enough to show that $\|\widetilde{\psi}(z)\|\to\infty$ as $\|z\|\to\infty$. Using the Young inequality, we have

$$\begin{aligned}
&\sum_{i=1}^{n} r\xi_i|x_i|^{r-1}sgn(x_i)(a_ie_i(x_i)-\widetilde{\psi}_i(z))\\
&=\sum_{i=1}^{n} r\xi_i|x_i|^{r-1}sgn(x_i)\left(\sum_{j=1}^{m} b_{ji}f_j(y_j)+\sum_{j=1}^{m} l_{ji}\int_0^{\tau} g_j(y_j)k_{ji}(s)ds\right)\\
&\leq \sum_{i=1}^{n}\sum_{j=1}^{m} r\xi_i|x_i|^{r-1}\left(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau} k_{ji}(s)ds\right)|y_j| \qquad (3.8)\\
&\leq \sum_{i=1}^{n}\sum_{j=1}^{m} \xi_i\left(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau} k_{ji}(s)ds\right)((r-1)|x_i|^r+|y_j|^r)
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{j=1}^{m} r\xi_{n+j}|y_j|^{r-1}sgn(y_j)(c_jh_j(y_j)-\widetilde{\psi}_j(z))\\
&=\sum_{j=1}^{m} r\xi_{n+j}|y_j|^{r-1}sgn(y_j)\left(\sum_{i=1}^{n} d_{ij}p_i(x_i)+\sum_{i=1}^{n} \tilde{l}_{ij}\int_0^{\tau} q_i(x_i)\tilde{k}_{ij}(s)ds\right)\\
&\leq \sum_{j=1}^{m}\sum_{i=1}^{n} r\xi_{n+j}|y_j|^{r-1}\left(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^{\sigma} \tilde{k}_{ij}(s)ds\right)|x_i| \qquad (3.9)\\
&\leq \sum_{j=1}^{m}\sum_{i=1}^{n} \xi_{n+j}\left(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^{\sigma} \tilde{k}_{ij}(s)ds\right)((r-1)|y_j|^r+|x_i|^r)
\end{aligned}$$

(3.8) plus (3.9), then

$$\begin{aligned}
&\sum_{i=1}^{n} r\xi_i|x_i|^{r-1}sgn(x_i)(a_ie_i(x_i)-\widetilde{\psi}_i(z))+\sum_{j=1}^{m} r\xi_{n+j}|y_j|^{r-1}sgn(y_j)(c_jh_j(y_j)-\widetilde{\psi}_j(z))\\
&\leq \sum_{i=1}^{n}\sum_{j=1}^{m}\left(\xi_i(r-1)\left(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau} k_{ji}(s)ds\right)\right.\\
&\left.+\xi_{n+j}\left(|d_{ij}|P_i+|\tilde{l}|_{ij}Q_i\int_0^{\sigma} \tilde{k}_{ij}(s)ds\right)\right)|x_i|^r\\
&+\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\xi_{n+j}(r-1)\left(|d_{ij}|P_i+|\tilde{l}_{ij}|Q_i\int_0^{\sigma} \tilde{k}_{ij}(s)ds\right)\right.\\
&\left.+\xi_i\left(|b_{ji}|F_j+|l_{ji}|G_j\int_0^{\tau} k_{ji}(s)ds\right)\right)|y_j|^r.
\end{aligned}$$

That is,

$$\sum_{i=1}^{n}\left\{r\xi_i a_i \varrho_i - \sum_{j=1}^{m}\left(\xi_i(r-1)\left(|b_{ji}|F_j + |l_{ji}|G_j\int_0^{\tau} k_{ji}(s)ds\right)\right.\right.$$
$$\left.\left.+\xi_{n+j}\left(|d_{ij}|P_i + |\tilde{l}_{ij}|Q_i\int_0^{\sigma}\tilde{k}_{ij}(s)ds\right)\right)\right\}|x_i|^r$$
$$+\sum_{j=1}^{m}\left\{r\xi_{n+j}c_j\tilde{\varrho}_j - \sum_{i=1}^{n}\left(\xi_{n+j}(r-1)\left(|d_{ij}|P_i + |\tilde{l}_{ij}|Q_i\int_0^{\sigma}\tilde{k}_{ij}(s)ds\right)\right.\right.$$
$$\left.\left.+\xi_i\left(|b_{ji}|F_j + |l_{ji}|G_j\int_0^{\tau}k_{ji}(s)ds\right)\right)\right\}|y_j|^r$$
$$\leq \sum_{i=1}^{n}\xi_i r\widetilde{\psi}_i(z)|x_i|^{r-1} + \sum_{j=1}^{m}\xi_{n+j}r\widetilde{\psi}_j(z)|y_j|^{r-1}.$$

Therefore,

$$\vartheta\left(\sum_{i=1}^{n}|x_i|^r + \sum_{j=1}^{m}|y_j|^r\right) \leq r\xi^+\left(\sum_{i=1}^{n}\widetilde{\psi}_i(z)|x_i|^{r-1} + \sum_{j=1}^{m}\widetilde{\psi}_j(z)|y_j|^{r-1}\right)$$

where

$$\vartheta = \min\left\{\min_{1\leq i\leq n}\left(r\xi_i a_i\varrho_i - \sum_{j=1}^{m}((r-1)\xi_i(|b_{ji}|F_j + |l_{ji}|G_j\int_0^{\tau}k_{ji}(s)ds)\right.\right.$$
$$\left. +\xi_{n+j}(|d_{ij}|P_i + |\tilde{l}_{ij}Q_i\int_0^{\sigma}\tilde{k}_{ij}(s)ds))\right), \min_{1\leq j\leq m}\left(r\xi_{n+j}c_j\tilde{\varrho}_j - \sum_{i=1}^{n}(\xi_{n+j}(|d_{ij}|P_i\right.$$
$$\left.\left. +|\tilde{l}_{ij}|Q_i\int_0^{\sigma}\tilde{k}_{ij}(s)ds)(r-1) + \xi_i(|bji|F_j + |l_{ji}|G_j\int_0^{\tau}k_{ji}(s)ds))\right)\right\} > 0,$$
$$\xi^+ = \max\{\xi_1, \xi_2, \cdots, \xi_n, \xi_{n+1}, \cdots, \xi_{n+m}\}.$$

By applying Hölder inequality, we have

$$\sum_{i=1}^{n}|x_i|^r + \sum_{j=1}^{m}|y_j|^r \leq \frac{r\xi^+}{\vartheta}\left(\sum_{i=1}^{n}|x_i|^r + \sum_{j=1}^{m}|y_j|^r\right)^{\frac{1}{s}}\left(\sum_{i=1}^{n}|\widetilde{\psi}_i(z)|^r + \sum_{j=1}^{m}|\widetilde{\psi}_j(z)|^r\right)^{\frac{1}{r}}$$

where $s > 0, r > 0$ such that $\frac{1}{s} + \frac{1}{r} = 1$. That is,

$$\left(\sum_{i=1}^{n}|x_i|^r + \sum_{j=1}^{m}|y_j|^r\right)^{\frac{1}{r}} \leq \frac{r\xi^+}{\vartheta}\left(\sum_{i=1}^{n}|\widetilde{\psi}_i(z)|^r + \sum_{j=1}^{m}|\widetilde{\psi}_j(z)|^r\right)^{\frac{1}{r}},$$

i.e., $\|z\| \leq \frac{r\xi^+}{\vartheta}\|\widetilde{\psi}(z)\|$, from which we assert that $\|\widetilde{\psi}(z)\| \to \infty$ as $\|z\| \to \infty$. By Lemma 2.3, we conclude that $\psi \in C^0$ is a homeomorphism on R^{n+m}, which guarantees the existence of a unique solution $z^* \in R^{n+m}$ of the algebraic system (2.1) which defines the unique equilibrium state of the impulsive network (1.2). This completes the proof.

Remark 3.1. The proof of the existence of equilibrium point of (1.2) is different from those [8-10], and by applications in section 5, one can see that the results here improve or extend the corresponding results [8-10, 20].

In Theorem 3.1, if $r \to 1$, then we have

Corollary 3.1. Assume that $(A_1) - (A_4)$ hold. Further,
(A_6) $\Gamma' = \begin{pmatrix} A & -\tilde{P} \\ -\tilde{F} & C \end{pmatrix}$ is a nonsingular M-matrix,
where $A = diag(a_1^- \varrho_1, a_2^- \varrho_2, \cdots, a_n^- \varrho_n)$, $C = diag(c_1^- \tilde{\varrho}_1, c_2^- \tilde{\varrho}_2, \cdots, c_m^- \tilde{\varrho}_m)$,
$\tilde{P} = (\tilde{p}_{ij})_{n\times m}$, $\tilde{F} = (\tilde{f}_{ji})_{m\times n}$, $\tilde{p}_{ij} = P_i|d_{ij}| + Q_i|\tilde{l}_{ij}| \int_0^\sigma \tilde{k}_{ij}(s)ds$,
$\tilde{f}_{ji} = |b_{ji}|F_j + |l_{ji}|G_j \int_0^\tau k_{ji}(s)ds$. Then system (1.2) has at least one equilibrium.

4. Globally exponential stability

Theorem 4.1. Assume that $(A_1) - (A_5)$ hold. Further,
(A_7) $\tilde{I}_{ik}(x_i(t_k)) = -\beta_{ik}(x_i(t_k) - x_i^*)$, $\tilde{J}_{jk}(y_j(t_k)) = -\gamma_{jk}(y_j(t_k) - y_j^*)$,
$|1-\beta_{ik}|^r - 1 \le 0$, $|1-\gamma_{jk}|^r - 1 \le 0$ for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$,
$k = 1, 2, \cdots$. Then the equilibrium solution z^* of (1.2) is globally exponentially stable.

Proof. By Theorem 3.1, there exists a unique equilibrium solution
$z^* = (x_1^*, x_2^*, \cdots, x_n^*, y_1^*, \cdots, y_m^*)^T$ of (1.2).
Let $z(t) = (x^T(t), y^T(t))^T = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ be an arbitrary solution of (1.2), then we have

$$\begin{cases} \frac{d|x_i(t)-x_i^*|}{dt} & \le -a_i|e_i(x_i(t)) - e_i(x_i^*)| + \sum_{i=1}^n |b_{ji}||f_j(y_j(t)) - f_j(y_j^*)| \\ & \quad + \sum_{j=1}^m |l_{ji}| \int_0^\tau k_{ji}(s)|g_j(t-\tau_{ji}-s) - g_j(y_j^*)|ds \\ & \le -a_i\varrho_i|x_i(t) - x_i^*| + \sum_{i=1}^n |b_{ji}|F_j|y_j(t) - y_j^*| \\ & \quad + \sum_{i=1}^n |l_{ji}|G_j \int_0^\tau k_{ji}(s)|y_j(t-\tau_{ji}-s) - y_j^*|ds \\ \frac{d|y_j(t)-y_j^*|}{dt} & \le -c_j|h_j(y_j(t)) - h_j(y_j^*)| + \sum_{j=1}^m |d_{ij}||p_i(x_i(t)) - p_i(x_i^*)| \\ & \quad + \sum_{j=1}^m |\tilde{l}_{ij}| \int_0^\tau \tilde{k}_{ij}(s)|q_i(x_i(t-\sigma_{ij}-s)) - q_i(x_i^*)|ds \\ & \le -c_j\tilde{\varrho}_j|y_j(t) - y_j^*| + \sum_{j=1}^m P_i|d_{ij}||x_i(t) - x_i^*| \\ & \quad + \sum_{j=1}^m |\tilde{l}_{ij}| \int_0^\tau \tilde{k}_{ij}(s)Q_i|x_i(t-\sigma_{ij}-s) - x_i^*|ds \end{cases} \tag{4.1}$$

for $t > 0, t \ne t_k, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

On the other hand, according to condition (A_5) and Lemma 2.1, there exist a vector $(\xi_1, \xi_2, \cdots, \xi_n, \xi_{n+1}, \cdots, \xi_{n+m})^T$ such that

$$\xi_i \Big(ra_i\varrho_i - (r-1)\sum_{j=1}^m (|b_{ji}|F_j + |l_{ji}|G_j \int_0^\tau k_{ji}(s)ds) \\ - \sum_{j=1}^m \xi_{n+j}(P_i|d_{ij}| + Q_i|\tilde{l}_{ij}| \int_0^\sigma \tilde{k}_{ij}(s)ds)\Big) > 0,$$
$$\xi_{n+j} \Big(rc_j\tilde{\varrho}_j - (r-1)\sum_{i=1}^n (|d_{ij}|P_i + |\tilde{l}_{ij}|Q_i \int_0^\sigma \tilde{k}_{ij}(s)ds) \\ - \sum_{i=1}^n \xi_i(F_j|b_{ji}| + G_j|l_{ji}| \int_0^\tau k_{ji}(s)ds)\Big) > 0.$$

Let

$$\begin{cases} \chi_i(\varepsilon) = \xi_i \Big(\varepsilon - r a_i \varrho_i + (r-1)\sum_{j=1}^m (|b_{ji}|F_j + |l_{ji}|G_j \int_0^\tau k_{ji}(s)ds)\Big) \\ \qquad + \sum_{j=1}^m \xi_{n+j} \left(P_i|d_{ij}| + Q_i e^{\varepsilon\sigma_{ij}}|\tilde{l}_{ij}| \int_0^\sigma \tilde{k}_{ij}(s)e^{\varepsilon s}ds\right) \\ \kappa_j(\varepsilon) = \xi_{n+j}\left(\varepsilon - r c_j \tilde{\varrho}_j + (r-1)\sum_{i=1}^n (d_{ij}P_i + \tilde{l}_{ij}Q_i \int_0^\sigma \tilde{k}_{ij}(s)ds)\right) \\ \qquad + \sum_{i=1}^n \xi_i \left(F_j b_{ji} + G_j e^{\varepsilon\tau_{ji}} l_{ji}\int_0^\tau k_{ji}(s)ds\right) \end{cases}$$

It is clear that $\chi_i(0) < 0, \kappa_j(0) < 0$. Since $\chi_i(\varepsilon), \kappa_j(\varepsilon)$ are continuous on $[0,\infty)$ and $\chi_i(\varepsilon), \kappa_j(\varepsilon) \to +\infty$ as $\varepsilon \to +\infty$, and $\frac{d\chi_i(\varepsilon)}{d\varepsilon} > 0, \frac{d\kappa_j(\varepsilon)}{d\varepsilon} > 0$, then there exist constant ξ_i^*, η_j^* such that

$$\begin{cases} \chi_i(\xi_i^*) = \xi_i \Big(\xi_i^* - r a_i \varrho_i + (r-1)\sum_{j=1}^m \left(|b_{ji}|F_j + |l_{ji}|G_j \int_0^\tau k_{ji}(s)ds\right)\Big) \\ \qquad + \sum_{j=1}^m \xi_{n+j}\left(P_i|d_{ij}| + Q_i e^{\xi_i^*\sigma_{ij}}|\tilde{l}_{ij}| \int_0^\sigma \tilde{k}_{ij}(s)e^{\xi_i^* s}ds\right) = 0 \\ \kappa_j(\eta_j^*) = \xi_{n+j}\left(\eta_j^* - r c_j \tilde{\varrho}_j + (r-1)\sum_{i=1}^n \left(|d_{ij}|P_i + |\tilde{l}_{ij}|Q_i \int_0^\sigma \tilde{k}_{ij}(s)ds\right)\right) \\ \qquad + \sum_{i=1}^n \xi_i \Big(F_j|b_{ji}| + G_j e^{\eta_j^*\tau_{ji}}|l_{ji}| \int_0^\tau k_{ji}(s)e^{\eta_j^* s}ds\Big) = 0 \end{cases} \tag{4.2}$$

By choosing $0 < \lambda < \min\{\xi_1^*, \xi_2^*, \cdots, \xi_n^*, \eta_1^*, \cdots, \eta_m^*\}$ for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, we have

$$\begin{cases} \chi_i(\lambda) = \xi_i \Big(\lambda - r a_i \varrho_i + (r-1)\sum_{j=1}^m \left(|b_{ji}|F_j + |l_{ji}|G_j \int_0^\tau k_{ji}(s)ds\right)\Big) \\ \qquad + \sum_{j=1}^m \xi_{n+j}\left(P_i|d_{ij}| + e^{\lambda\sigma_{ij}}Q_i|\tilde{l}_{ij}| \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s}ds\right) < 0 \\ \kappa_j(\lambda) = \xi_{n+j}\left(\lambda - r c_j \tilde{\varrho}_j + (r-1)\sum_{i=1}^n \left(|d_{ij}|P_i + |\tilde{l}_{ij}|Q_i \int_0^\sigma \tilde{k}_{ij}(s)ds\right)\right) \\ \qquad + \sum_{i=1}^n \xi_i \left(F_j|b_{ji}| + e^{\lambda\tau_{ji}}G_j|l_{ji}| \int_0^\tau k_{ji}(s)e^{\lambda s}ds\right) < 0. \end{cases} \tag{4.3}$$

Let $u_i(t) = e^{\lambda t}|x_i(t) - x_i^*|^r, v_j(t) = e^{\lambda t}|y_j(t) - y_j^*|^r$, from (4.1), we derive that

$$\begin{cases} \frac{d^+u_i(t)}{dt} \le \lambda e^{\lambda t}|x_i(t) - x_i^*|^r + r e^{\lambda t}|x_i(t) - x_i^*|^{r-1} sgn(x_i(t) - x_i^*) \\ \qquad (-a_i\varrho_i|x_i(t) - x_i^*| + \sum_{i=1}^n |b_{ji}|F_j|y_j(t) - y_j^*| \\ \qquad + \sum_{i=1}^n |l_{ji}|G_j \int_0^\tau k_{ji}(s)|y_j(t - \tau_{ji} - s) - y_j^*|ds) \\ \frac{d^+v_j(t)}{dt} \le \lambda e^{\lambda t}|y_j(t) - y_j^*|^r + r e^{\lambda t}|y_j(t) - y_j^*|^{r-1} sgn(y_j(t) - y_j^*) \\ \qquad \big(-c_j\tilde{\varrho}_j|y_j(t) - y_j^*| + \sum_{j=1}^m P_i|d_{ij}||x_i(t) - x_i^*| \\ \qquad + \sum_{j=1}^m |\tilde{l}_{ij}| \int_0^\tau \tilde{k}_{ij}(s)Q_i|x_i(t - \sigma_{ij} - s) - x_i^*|ds) \end{cases} \tag{4.4}$$

for $t > 0, t \ne t_k$. When $t = t_k$, for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, it follows from (A_7) that

$$u_i(t_k^+) = |1 - \alpha_{ik}|^r u_i(t_k) \le u(t_k), \;\; v_j(t_k^+) = |1 - \beta_{jk}|^r v_j(t_k) \le v_j(t_k) \tag{4.5}$$

Define a Lyapunov functional as follows:

$$\begin{aligned} V(t) = &\sum_{i=1}^n \xi_i \Big(u_i(t) + \sum_{j=1}^m |l_{ji}|e^{\lambda\tau_{ji}}G_j \int_0^\tau k_{ji}(s)e^{\lambda s}\int_{t-\tau_{ji}-s}^t v_j(z)dzds\Big) \\ &+ \sum_{i=1}^n \xi_{n+j}\Big(v_j(t) + \sum_{i=1}^n |\tilde{l}_{ij}|e^{\lambda\sigma_{ij}}Q_i \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s}\int_{t-\sigma_{ij}-s}^t u_i(z)dzds\Big) \end{aligned}$$

By calculating the derivative of $V(t)$ along the solution of (1.2) and from (4.3), (4.4) and Lemma 2.2, we have

$$
\begin{aligned}
&\tfrac{d^+V(t)}{dt}\\
&= \sum_{i=1}^n \xi_i \Big(\tfrac{d^+u_i(t)}{dt} + \sum_{j=1}^m |l_{ji}| e^{\lambda\tau_{ji}} G_j \int_0^\tau k_{ji}(s)e^{\lambda s} v_j(t)ds \\
&\quad - \sum_{j=1}^m |l_{ji}| G_j e^{\lambda\tau_{ji}} \int_0^\tau k_{ji}(s)e^{\lambda s} v_j(t-\tau_{ji}-s)ds \Big)\\
&\quad + \sum_{j=1}^m \xi_{n+j} \Big(\tfrac{d^+v_j(t)}{dt} + \sum_{i=1}^n |\tilde{l}_{ij}| Q_i e^{\lambda\sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s} u_i(t)ds \\
&\quad - \sum_{i=1}^n |\tilde{l}_{ij}| Q_i e^{\lambda\sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s} u_i(t-\sigma_{ij}-s)ds \Big)\\
&\le \sum_{i=1}^n \xi_i \Big((\lambda - r a_i \varrho_i)|u_i(t)|^r + r e^{\lambda t} \sum_{j=1}^m |b_{ji}| F_j |y_j(t)-y_j^*||x_i(t)-x_i^*|^{r-1}\\
&\quad + r e^{\lambda t} \sum_{j=1}^m |l_{ji}| G_j \int_0^\tau k_{ji}(s)|x_i(t)-x_i^*|^{r-1}|y_j(t-\tau_{ji}-s)-y_j^*|ds\\
&\quad + \sum_{j=1}^m e^{\lambda\tau_{ji}} |l_{ji}| G_j \int_0^\tau k_{ji}(s)e^{\lambda s} ds v_j(t)\\
&\quad - \sum_{j=1}^m |l_{ji}| G_j e^{\lambda\tau_{ji}} \int_0^\tau k_{ji}(s)e^{\lambda s} v_j(t-\tau_{ji}-s)ds \Big)\\
&\quad + \sum_{j=1}^m \xi_{n+j} \big((\lambda - r c_j \tilde{\varrho}_j)|v_j(t)|^r + r e^{\lambda t} \sum_{i=1}^n |d_{ij}| P_i |x_i(t)-x_i^*||y_j(t)-y_j^*|^{r-1}\\
&\quad + r e^{\lambda t} \sum_{i=1}^n |\tilde{l}_{ij}| Q_i \int_0^\sigma \tilde{k}_{ij}(s)|y_j(t)-y_j^*|^{r-1}|x_i(t-\sigma_{ij}-s)-x_i^*|ds\\
&\quad + \sum_{i=1}^n |\tilde{l}_{ij}| Q_i e^{\lambda\sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s}(u_i(t)-u_i(t-\sigma_{ij}-s))ds \big)\\
&\le \sum_{i=1}^n \xi_i \Big((\lambda - r a_i \varrho_i)|u_i(t)|^r + r e^{\lambda t} \sum_{j=1}^m |b_{ji}| F_j \big(\tfrac{1}{r}|y_j(t)-y_j^*|^r\\
&\quad + \tfrac{r-1}{r}|x_i(t)-x_i^*|^r \big) + r e^{\lambda t} \sum_{j=1}^m |l_{ji}| G_j \int_0^\tau k_{ji}(s) \big(\tfrac{r-1}{r}|x_i(t)-x_i^*|^r\\
&\quad + \tfrac{1}{r}|y_j(t-\tau_{ji}-s) y_j^*|^r \big) ds + \sum_{j=1}^m e^{\lambda\tau_{ji}} |l_{ji}| \int_0^\tau k_{ji}(s)e^{\lambda s} ds |v_j(t)|\\
&\quad - \sum_{j=1}^m |l_{ji}| G_j e^{\lambda\tau_{ji}} \int_0^\tau k_{ji}(s)e^{\lambda s} v_j(t-\tau_{ji}-s)ds \Big) + \sum_{j=1}^m \xi_{n+j}\\
&\quad \big((\lambda - r c_j \tilde{\varrho}_j)|v_j(t)|^r + r e^{\lambda t} \sum_{i=1}^n |d_{ij}| P_i \big(\tfrac{1}{r}|x_i(t)-x_i^*|^r + \tfrac{r-1}{r}|y_j(t)-y_j^*|^r \big)\\
&\quad + r e^{\lambda t} \sum_{i=1}^n |\tilde{l}_{ij}| Q_i \int_0^\sigma \tilde{k}_{ij}(s) \big(\tfrac{r-1}{r}|y_j(t)-y_j^*|^r + \tfrac{1}{r}|x_i(t-\sigma_{ij}-s)-x_i^*|^r \big) ds\\
&\quad + \sum_{i=1}^n |\tilde{l}_{ij}| Q_i e^{\lambda\sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s}(u_i(t)-u_i(t-\sigma_{ij}-s))ds \big)\\
&\le \sum_{i=1}^n \Big(\xi_i (\lambda - r a_i \varrho_i + (r-1) \sum_{j=1}^m \big(|b_{ji}| F_j + |l_{ji}| G_j \int_0^\tau k_{ji}(s)ds \big)\\
&\quad + \sum_{j=1}^m \xi_{n+j} \big(P_i |d_{ij}| + Q_i |\tilde{l}_{ij}| e^{\lambda\sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s} ds \big) \Big) u_i(t)\\
&\quad + \sum_{j=1}^m \big(\xi_{n+j} (\lambda - r c_j \tilde{\varrho}_j + (r-1) \sum_{i=1}^n \big(|d_{ij}| P_i + |\tilde{l}_{ij}| Q_i \int_0^\sigma \tilde{k}_{ij}(s)ds \big)\\
&\quad + \sum_{i=1}^n \xi_i \big(F_j |b_{ji}| + G_j |l_{ji}| e^{\lambda\tau_{ji}} \int_0^\tau k_{ji}(s)e^{\lambda s} ds \big) \big) v_j(t)\\
&< 0
\end{aligned}
\tag{4.6}
$$

for $t > 0, t \neq t_k, k = 1, 2, \cdots$. When $t = t_k$, we obtain from (4.5) that

$$
\begin{aligned}
&V(t_k^+)\\
&= \sum_{i=1}^n \xi_i \Big(u_i(t_k^+) + \sum_{j=1}^m |l_{ji}| G_j \int_0^\tau k_{ji}(s)e^{\lambda s} \int_{t_k^+-\tau_{ji}-s}^{t_k^+} v_j(z)dzds \Big)\\
&\quad + \sum_{i=1}^n \xi_{n+j} \Big(v_j(t_k^+) + \sum_{j=1}^m |\tilde{l}_{ij}| Q_i \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s} \int_{t_k^+-\sigma_{ij}-s}^{t_k^+} u_i(z)dzds \Big)\\
&= \sum_{i=1}^n \xi_i \Big(u_i(t_k^+) + \sum_{j=1}^m |l_{ji}| G_j \int_0^\tau k_{ji}(s)e^{\lambda s} \int_{t_k-\tau_{ji}-s}^{t_k} v_j(z)dzds \Big)\\
&\quad + \sum_{j=1}^m \xi_{n+j} \Big(v_j(t_k^+) + \sum_{i=1}^n |\tilde{l}_{ij}| Q_i \int_0^\sigma \tilde{k}_{ij}(s)e^{\lambda s} \int_{t_k-\sigma_{ij}-s}^{t_k} u_i(z)dzds \Big)\\
&\le V(t_k), \quad k = 1, 2,
\end{aligned}
\tag{4.7}
$$

It follows from (4.6) and (4.7) that

$$V(t) \le V(0) \text{ for all } t > 0. \tag{4.8}$$

By the definition of $V(t)$ and (4.8), we have

$$\begin{aligned}
&\xi_i^- \left(\sum_{i=1}^n u_i(t) + \sum_{j=1}^m v_j(t) \right) \\
&\le \sum_{i=1}^n \xi_i \left(u_i(0) + \sum_{j=1}^m G_j |l_{ji}| e^{\lambda \tau_{ji}} \int_0^\tau k_{ji}(s) e^{\lambda s} \int_{-\tau_{ji}-s}^0 v_j(z) dz ds \right) \\
&+ \sum_{j=1}^m \xi_{n+j} \left(v_j(0) + \sum_{i=1}^n Q_i |\tilde{l}_{ij}| e^{\lambda \sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s) e^{\lambda s} \int_{-\sigma_{ij}-s}^0 u_i(z) dz ds \right) \\
&\le \xi^+ \sum_{i=1}^n \left(1 + \sum_{j=1}^m Q_i |\tilde{l}_{ij}| e^{\lambda \sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s) e^{\lambda s} (\sigma_{ij} + s) ds \right) \sup_{-h < t \le 0} u_i(t) \\
&+ \xi^+ \sum_{j=1}^m \left(1 + \sum_{j=1}^m G_j |l_{ji}| e^{\lambda \tau_{ji}} \int_0^\tau k_{ji}(s) e^{\lambda s} (\tau_{ji} + s) ds \right) \sup_{-\tilde{h} < t \le 0} v_j(t) \\
&\le \xi^+ \iota \left(\sum_{i=1}^n \sup_{-h < t \le 0} u_i(t) + \sum_{j=1}^m \sup_{-\tilde{h} < t \le 0} v_j(t) \right)
\end{aligned}$$

where $\xi^- = \min\{\xi_1, \xi_2, \cdots, \xi_{n+m}\}$, $\xi^+ = \max\{\xi_1, \xi_2, \cdots, \xi_{n+m}\}$,

$$\iota = \max \left\{ \max_{1 \le i \le n} (1 + \textstyle\sum_{j=1}^m Q_i |\tilde{l}_{ij}| e^{\lambda \sigma_{ij}} \int_0^\sigma \tilde{k}_{ij}(s) e^{\lambda s} (\sigma_{ij} + s) ds), \right.$$
$$\left. \max_{1 \le j \le m} (1 + \textstyle\sum_{j=1}^m G_j |l_{ji}| e^{\lambda \tau_{ji}} \int_0^\tau k_{ji}(s) e^{\lambda s} (\tau_{ji} + s) ds) \right\} \ge 1.$$

It leads to

$$\begin{aligned}
&\left\{ \sum_{i=1}^n |x_i(t) - x_i^*|^r + \sum_{j=1}^m |y_j(t) - y_j^*|^r \right\}^{\frac{1}{r}} \\
&\le \left(\frac{\xi^+ \iota}{\xi^-} \right)^{\frac{1}{r}} e^{-\frac{\lambda}{r} t} \left\{ \sum_{i=1}^n \sup_{-h < s \le 0} |\phi_{x_i}(s) - x_i^*|^r + \sum_{j=1}^m \sup_{-\tilde{h} < s \le 0} |\phi_{y_j}(s) - y_j^*|^r \right\}^{\frac{1}{r}} \\
&= M e^{-\alpha t} \left\{ \sum_{i=1}^n \sup_{-h < t \le 0} |\phi_{x_i}(s) - x_i^*|^r + \sum_{j=1}^m \sup_{-\tilde{h} < s \le 0} |\phi_{y_j}(s) - y_j^*|^r \right\}^{\frac{1}{r}}
\end{aligned}$$

where $M = \left(\frac{\iota \xi^+}{\xi^-} \right)^{\frac{1}{r}} \ge 1, \alpha = \frac{\lambda}{r} > 0$. Therefore, the equilibrium z^* of system (1.2) is globally exponentially stable. This completes the proof.

Remark 4.1. The method and analysis techniques employed here are different from [8-10, 27], and the conditions ensuring the stability of the equilibrium point are simpler and easier to verified than [27].

Let $r \to 1$ in Theorem 4.1, we get the corollary immediately.

Corollary 4.1. Assume that $(A_1)-(A_4)$ and (A_6) hold. Further, (A_8) $\tilde{I}_{ik}(x_i(t_k) = -\beta_{ik}x_i(t_k),\ \tilde{J}_{jk}(y_j(t_k)) = -\gamma_{jk}y_j(t_k),\ |1-\alpha_{ik}|-1 \le 0,$ $|1-\beta_{jk}|-1 \le 0$. Then system (1.2) admits one equilibrium which is globally exponential stable.

5. Applications and an illustrative example

For (1.2), let $\tau \to \infty, \sigma \to \infty$ and $\int_0^\infty k_{ji}(s)ds = 1, \int_0^\infty \bar{k}_{ij}(s)ds = 1$, by Corollary 3.1, one can obtain Theorem 3.1 in [10], i.e.,

Corollary 5.1. Suppose conditions $(A_1)-(A_2)$ in [10] hold. Further, (A_9) $a_i > \sum_{j=1}^m(\bar{G}_i|\bar{h}_{ij}| + \bar{F}_i|\bar{l}_{ij}|),\ \bar{a}_j > \sum_{i=1}^n G_j|h_{ji}| + F_j|l_{ji}|),\ i = 1,\cdots,n,$ $j = 1,\cdots,m$. Then (1.1) has a unique equilibrium point.

Similarly, one can obtain the result of existence of equilibrium point of the models in [9,20]. It is in this sense that we extend the previously known results.

Considering the following system studied by Wu [8]:

$$\begin{cases} u_i(t) &= -a_i(t)e_i(u_i) + \sum_{j=1}^n b_{ji}(t)f_j(v_j) \\ & \quad + \sum_{j=1}^n l_{ji}(t)\int_0^\tau k_{ji}(s)g_j(v_j(t-\tau_{ji}-s))ds + I_i(t) \\ v_j(t) &= -c_j(t)h_j(v_j) + \sum_{i=1}^m d_{ij}(t)p_i(u_i) \\ & \quad + \sum_{i=1}^m \tilde{l}_{ij}(t)\int_0^\sigma k_{ij}(s)q_i(u_i(t-\sigma_{ij}-s))ds + J_j(t) \end{cases} \tag{5.1}$$

By similar proof of Theorem 3.1, we can derive the sufficient conditions ensuring the existence of a unique equilibrium point of (5.1). For function $f(t)$, denote $f^+ = \sup_{0\le t<\infty}|f(t)|$, then we have

Corollary 5.2. Suppose $(A_1)-(A_4)$ hold. Further, (A_9) $\Gamma' = \begin{pmatrix} A & -\check{P} \\ -\check{F} & C \end{pmatrix}$ is a nonsingular M-matrix, where $\check{P} = (\check{p}_{ij})_{n\times m}, \check{F} = (\check{f}_{ji})_{m\times n},\ \check{p}_{ij} = P_i d_{ij}^+ + Q_i\tilde{l}_{ij}^+\int_0^\sigma \tilde{k}_{ij}(s)ds,\ \check{f}_{ji} = b_{ji}^+F_j + l_{ji}^+G_j\int_0^\tau k_{ji}(s)ds, A, C$ are defined as Corollary 3.1. Then system (5.1) has at least one equilibrium.

Remark 5.1. The conditions of the existence of a unique equilibrium point of (5.1) are simpler and easier to verified than Theorem 4.2 in [8]. Particularly, it shows that the condition (2) of Theorem 4.2 in [8] is unnecessary, hence, we improve the main results [8].

Example. Let

$$\begin{cases} x_1'(t) = -a_1e_1(x_1(t)) + b_{11}f_1(y_1(t)) + l_{11}\int_0^\tau k_{11}(s)g_1(y_1(t-\tau_{11}-s))ds + I_1,\ t \ne t_k, \\ x_2'(t) = -a_2e_2(x_2(t)) + b_{12}f_1(y_1(t)) + l_{12}\int_0^\tau k_{12}(s)g_1(y_1(t-\tau_{12}-s))ds + I_2,\ t \ne t_k, \\ \Delta x_1(t) = x_1(t^+) - x_1(t^-) = -\beta_{1k}(x_1(t)),\ t = t_k, \\ \Delta x_2(t) = x_2(t^+) - x_2(t^-) = -\beta_{2k}(x_2(t)),\ t = t_k, \\ y_1'(t) = -c_1h_1(y_1(t)) + d_{11}p_1(x_1(t)) + d_{21}p_2(x_2(t))\tilde{l}_{11}\int_0^\sigma \tilde{k}_{11}(s) \\ \qquad q_1(x_1(t-\sigma_{11}-s))ds + \tilde{l}_{21}\int_0^\sigma \tilde{k}_{21}(s)q_2(x_2(t-\sigma_{21}-s))ds + J_1, \\ \Delta y_1(t) = y_1(t^+) - y_1(t^-) = -\gamma_{1k}(y_1(t)),\ t = t_k, \end{cases} \tag{5.2}$$

where $e_i(u) = \frac{u}{2},\ h_1(u) = u,\ f_1(u) = g_1(u) = |u|,\ p_i(u) = q_i(u) = \frac{|u|}{4}$, for $i = 1, 2,\ u \in R$. $a_1 = 2,\ a_2 = 4,\ c_1 = 7, b_{11} = -\frac{1}{3},\ l_{11} = \frac{2}{3},\ k_{11}(s) = k_{12}(s) = \frac{1}{\tau},\ b_{12} = -2,\ l_{12} = 1,\ d_{11} = 7,\ \tilde{l}_{11} = -1,\ d_{21} = -10,\ \tilde{l}_{21} = -2,\ \tilde{k}_{11}(s) = \tilde{k}_{21}(s) = \frac{1}{\sigma},\ \beta_{1k} = \beta_{2k} = \frac{1}{2},\ \gamma_{1k} = \frac{1}{8}$. Then $\varrho_1 = \varrho_2 = \frac{1}{2}, \tilde{\varrho}_1 = 1, F_1 = G_1 = 1, P_1 = P_2 = Q_1 = Q_2 = \frac{1}{4}$.

By simple calculation, we have $\tilde{f}_{11} = 31, \tilde{f}_{12} = 43, \tilde{p}_{11} = 2, \tilde{p}_{21} = 3$, and the corresponding matrix $\Gamma' = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & -3 \\ -1 & -3 & 7 \end{pmatrix}$. It is easy to show that there exists a constant vector $\xi = (\frac{25}{6}, \frac{28}{9}, 2)^T > 0$ such that $\Gamma'\xi > 0$. Using Lemma 2.1, one obtains that Γ' is a nonsingular M-matrix and (A_6) holds. By easy verification, (A_8) holds too. Therefore, by Corollary 4.1, one concludes that (5.2) admits an equilibrium which is globally exponentially stable.

References

1 Y.H. Xia, J.D. Cao and M.R. Lin, *New results on the existence and uniqueness of almost periodic solutions for BAM neural networks with continuously distributed delays*, Chaos Solit. Fract. **314** (2007) 928-936.

2 J.D. Cao, *Global asymptotic stability of delayed bi-directional associative memory neural networks*, Appl. Math. Comput. **142** (2003) 333-339.

3 J.D. Cao, *Exponential stability of delayed bi-directional associative memory neural networks*, Appl. Math. Comput. **135** (2003) 105-112.

4 A.P. Chen, L.H. Huang, Z.G. Liu and J.D. Cao, *Periodic bi-directional associative memory neural networks with distributed delays*, J. Math. Anal. Appl. **317** (2006) 80-102.

5 H. Wang, X.F. Liao and C.D. Li, *Existence and exponential stability of periodic solution of BAM neural networks with impulse and time varying delays*, Chaos Solit. Fract. **33** (2007) 1028-1039.

6 W.C.daniel, J.L. Liang and L.James, *Global exponential stability of impulsive high-order BAM neural networks with time varying delays*, Neural Networks, **19** (2006) 1581-1590.

7 Arik Sabri and T.Vedat, *Global asymptotic stability analysis of bidirectional associative memory neural networks with constant time delays*, Neucomputing **68** (2005) 161-176.

8 R.C. Wu, *Exponential convergence of BAM neural networks with time-varying coefficients and distributed delays*, doi:10.1016/j.nonrwa.2009.02.003.

9 J. Chen and B.T. Cui, *Impulsive effects on global asymptotic stability of delay BAM neural networks*, Chaos Solit. Fract. **38** (2008) 1115-1125.

10 Q.H. Zhou, *Global exponential stability of BAM neural networks with distributed delays and impulses*, Nonlinear Anal. RWA **10** (2009) 144-153.

11 Z.K. Huang and Y.H. Xia, *Exponential periodic attractor of impulsive BAM networks with finite distributed delays*, Chaos Solit. Fract. **39** (2009) 373-384.

12 H.Q. Wu and C.H. Shan, *Stability analysis for periodic solution of BAM neural networks with discontinuous neuron activations and impulses*, Appl. Math. Model. **33** (2009) 2564-2574.

13 Z. Gui and W. Ge, *Existence and uniqueness of periodic solutions of nonautonomous cellular neural networks with impulses*, Phys. Lett. A **354** (2006) 84-94.

14 C.Z. Bai, *Global exponential stability and existence of periodic solution of Cohen-Grossberg type neural networks with delays and impulses*, Nonlinear Anal. RWA **9** (2008) 747-761.

15 X.S. Yang, *Existence and global exponential stability of periodic solution for Cohen-Grossberg shunting inhibitory cellular neural networks with delays and impulses*, doi: 10.1016/j.neucom. 2009.01.003.

16 H.B. Gu, H.J. Jiang and Z.D. Teng, *Existence and globally exponential stability of periodic solution of BAM neural networks with impulses and recent-history distributed delays*, Neurocomputing, **71** (2008) 813-822.
17 Y.T. Li and J.Y. Wang, *An analysis on the global exponential stability and the existence of periodic solutions for non-autonomous hybrid BAM neural networks with distributed delays and impulses*, Comput. Math. Appl. **56** (2008) 2256-2267.
18 Z. Gui and W. Ge, *Periodic solutions of nonautonomous cellular neural networks with impulses*, Chaos Solit. Fract. **32** (2007) 1760-1771.
19 J. Zhang and Z. Gui, *Periodic solutions of nonautonomous cellular neural networks with impulses and delays*, Nonlinear Anal. **10** (2009) 1891-1903.
20 Y.K. Li, *Global exponential stability of BAM neural networks with delays and impulses*, Chaos Solit. Fract. **24** (2005) 279-285.
21 H.Y. Zhao, *Global stability of bidirectional associative memory neural networks with distributed delays*, Phys. Lett. A **297** (2002) 182-190.
22 B. Liu and L. Huang, *Global exponential stability of BAM neural networks with recent-history distributed delays and impulses*, Neucomputing **69** (2006) 2090-2096.
23 Y.H. Xia, Z.K. Huang and M.A. Han, *Existence and globally exponential stability of equilibrium for BAM neural networks with impulses*, Chaos Solit. Fract. **37** (2008) 588-597.
24 S. Mohamad, K. Gopalsamy and H. Acka, *Exponential stability of artificial neural networks with distributed delays and large impulses*, Nonl. Anal. RWA **9** (2008) 872-888.
25 Z.K. Huang and Y.H. Xia, *Global exponential stability of BAM neural networks with transmission delays and nonlinear impulses*, Chaos Solit. Frac. **38** (2008) 489-498.
26 Y.H. Xia and Patr.Y. Wong, *Global exponential stability of a class of retarded impulsive differential equations with applications*, Chaos Solit. Fract. bf39 (2009) 440-453.
27 R. Samidurai, R. Sakthivel and S.M. Anthoni, *Global asymptotic stability of BAM neural networks with mixed delays and impulses*, doi:10.1016/j.amc.2009.02.002.
28 Richard and s. Varga, *Matrix Iterative Analysis*, Springer Press, 2004.
29 B. Fang, J. Zhou and Y. Li, *Matrix Theory*, Tinghua University Press and Springer Press, 2004.
30 Q. Zhang, X. Wei and J. Xu, *Delay-dependent exponential stability criteria for non-autonomous cellular neural networks with time-varying delays*, Chaos Solit. Fract. **36** (2008) 985-990.
31. M. Forti and A. Tesi, *New conditions for global stability of neural networks with application to linear and quadratic programming problems*, IEEE Trans. Circuit. syst. I **42** (1995) 354-366.

Yuanfu Shao is a Ph. D. student in School of Mathematical Sciences and Computing Technology, Central South University, Changsha, China, and he is also an associate professor in school of Mathematics and Computer Science, Guizhou Normal University, Guiyang, China. He received his M. S. from Yunnan University, Kunming, in 2005. His research interests focus on the stability theory of impulsive differential equation and neural network.

1. School of Mathematics and Computer Science, Guizhou Normal University, Guiyang, Guizhou 550001, P. R. China.
2. School of Mathematical Sciences and Computing Technology, Central South University, Changsha, Hunan 410075, P. R. China.
e-mail: `yfshao@163.com`

Zhenguo Luo is a Ph. D. student. His research interests focus on the theory of differential and difference equation.

School of Mathematical Sciences and Computing Technology, Central South University, Changsha, Hunan 410083, P. R. China.
e-mail: `zhenguoluo@163.com`