

Path Following Control of Mobile Robot Using Lyapunov Techniques and PID Controller

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Abstract

Path following of the mobile robot is one research hot for the mobile robot navigation. For the control system of the wheeled mobile robot(WMR) being in nonholonomic system and the complex relations among the control parameters, it is difficult to solve the problem based on traditional mathematics model. In this paper, we presents a simple and effective way of implementing an adaptive following controller based on the PID for mobile robot path following. The method uses a non-linear model of mobile robot kinematics and thus allows an accurate prediction of the future trajectories. The proposed controller has a parallel structure that consists of PID controller with a fixed gain. The control law is constructed on the basis of Lyapunov stability theory. Computer simulation for a differentially driven nonholonomic mobile robot is carried out in the velocity and orientation tracking control of the nonholonomic WMR. The simulation results of wheel type mobile robot platform are given to show the effectiveness of the proposed algorithm.

Key Words : Path following, wheeled mobile robot, Lyapunov, Nonholonomic, Kinematics.

1. Introduction

In the last decade years, mobile robot has been developed as industrial Automated Guided Vehicles (AGV) with high nonlinearities that are often unknown and time-varying. Therefore, if we want to design a controller for mobile robot, we should consider that the exact path-following(PF) performance, which is concerned with the ability to drive a mobile robot autonomously as close as possible to a previously defined reference path[1].

Wheeled mobile robots have many application fields in industrial and service robotics, particularly when flexible motion capabilities are required on reasonably smooth grounds and surfaces [2]. They are especially necessary for tasks that are difficult and dangerous for men to perform. Many researchers have shown interest in mobile robots. Most of them have focused on trajectory tracking—to control the robot to follow a desired trajectory, and point stabilization—to stabilize a robot at a desired point. [2-3].

Among the many control strategies that have been proposed for various nonholonomic systems, research results can generally be classified into two classes. The first class is kinematic control, which provides the solutions only at the pure kinematic level, where the systems are represented by their kinematic models and velocity acts as the control input. Based on exact system kinematic, different control strategies have

been proposed [4,5,7]. Recently, a few research works have been carried out to design controllers against possible existence of modeling uncertainties and external disturbance[8-10]. Robust exponential regulation is proposed in [8] by assuming known bounds of the nonlinear drifts. It is also required that the x_0 -subsystem is Lipschitz. To relax this condition, adaptive state feedback control is proposed in [9] for system with strong nonlinear drifts.

However, the control input of the controller for the kinematic model is generally velocity, but it is more realistic that the input is torque. In [6], a kinematic controller is designed first so that the tracking error between a real robot and a reference robot converges to zero, and secondly a torque controller is designed by using backstepping so that the velocities of a mobile robot converge to the desired velocities, which are given by the kinematic controller designed at the first step.

The conventional feedback controllers, including proportional integral derivative (PID) controller, are commonly used in the field of industries because their control architectures are very simple and easy to implement. But when these conventional feedback controllers are directly applied to nonlinear systems, they suffer from the poor performance and robustness due to the unknown nonlinearities and the external disturbances. During decades, various control strategies to deal with the unknown nonlinearities and the external disturbances are proposed such as automatic tuning of PID control, variable structure control known as nonlinear robust control, feedback linearization, model reference adaptive control, direct adaptive control, and intelligent control approaches, etc [11-12].

An adaptive control strategy has found many applications in such areas as robot manipulators, ship steering, aircraft control, and process control, because it can continuously adjust parameters of a controller to accommodate changes in system

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dynamics and disturbances [12]. In these applications, an on-line adaptation law is usually used to estimate the unknown parameters of the system and then, an appropriate controller is designed to control the plant to satisfy a desired performance. To apply an adaptive control method to robot manipulators as a main controller, the a priori knowledge about the robot manipulator is required: the linearity about unknown parameters and skew-symmetry features, etc. Additionally it takes a long time to calculate the regression matrix used in the algorithm.

In this paper, the adaptive following control is used as a following controller of mobile robot [9-11]. And also, a classical PID approach is applied for the path-following controller, which is the control strategy most frequently used in the industry. A very simple model of the mobile robot kinematics is used and thus a robust PID tuning is necessary. PID's advantages include simplicity, robustness and their familiarity in the control community. Because of this, a great deal of effort has been spent to find the best choice of PID parameters for different process models. The structure of the newly proposed control algorithm of nonlinear PID-based adaptive control is shown in Fig. 3. This control algorithm has the characteristics such as simple structure, little computation time, and continuous auto-tuning method of the PID controller.

The paper is organized as follows: In Section 2, we discuss the model and model transformation of the systems including the kinematics of the mobile robot. Section 3 deals with the problem of designing controller based on Lyapunov techniques and PID, and Section 4 deals with several results to show that the proposed method is effective. Finally, some conclusions are drawn in Section 5.

2. Kinematics Analysis of Mobile Robot

A two-wheel differential drive mobile robot was chosen as the object in this paper. Its wheel rotation is limited to one axis. Therefore, the navigation is controlled by the speed a change on either side of the robot. This kind of robot has nonholonomic constraints. The kinematics scheme of a two-wheel differential drive mobile robot is as shown in Fig. 1, where $\{O, X, Y\}$ is the global coordinate, v_m is the velocity of the robot centroid, ω is the angular velocity of the robot centroid, v_l is the velocity of the left driving wheel, v_r is the velocity of the right driving wheel, L is the distance between two driving wheels, R is the radius of each driving wheel, x_i and y_i are the position of the robot, θ_i is the orientation of the robot, and ω_m is the angular velocity. According to the motion principle of rigid body kinematics, the motion of a two-wheel differential drive mobile robot can be described using equations (1) and (2), where $\omega_{m,l}$ and $\omega_{m,r}$ are angular velocities of the left and right driving wheels respectively.

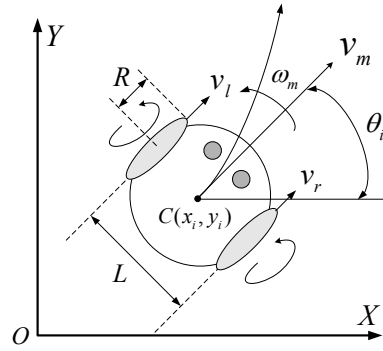


Fig. 1 Kinematics scheme of differential drive mobile robot.

$$\begin{aligned} v_l &= \omega_{m,l} R \\ v_r &= \omega_{m,r} R \end{aligned} \quad (1)$$

$$\begin{aligned} v_m &= (v_r + v_l) / 2 \\ \omega_m &= (v_r - v_l) / L \end{aligned} \quad (2)$$

The nonholonomic constraint equation of the robot is as following:

$$\dot{x}_i \sin \theta - \dot{y}_i \cos \theta = 0 \quad (3)$$

Deriving (4) from (1) and (2):

$$\begin{aligned} v_m &= \frac{\omega_{m,r} + \omega_{m,l}}{2} R \\ \omega_m &= \frac{\omega_{m,r} - \omega_{m,l}}{L} R \end{aligned} \quad (4)$$

Moreover, we can define the dynamic function of the robot as formula (5).

$$\begin{aligned} \dot{x}_i &= v_m \cos \theta_i = \frac{R}{2} (\omega_{m,r} + \omega_{m,l}) \cos \theta_i \\ \dot{y}_i &= v_m \sin \theta_i = \frac{R}{2} (\omega_{m,r} + \omega_{m,l}) \sin \theta_i \\ \dot{\theta}_i &= \omega_m = \frac{R}{L} (\omega_{m,r} - \omega_{m,l}) \end{aligned} \quad (5)$$

Rewriting the equation (5), we can express the Jacobian matrix as (6)

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \frac{R}{2} \cos \theta_i & \frac{R}{2} \cos \theta_i \\ \frac{R}{2} \sin \theta_i & \frac{R}{2} \sin \theta_i \\ -\frac{R}{L} & \frac{R}{L} \end{bmatrix} \begin{bmatrix} \omega_{m,l} \\ \omega_{m,r} \end{bmatrix} \quad (6)$$

All variables are interrelated in equation (6), which causes the controller design to be more complex. Therefore, equation (6) should be decoupled. For θ_i is only related to ω_m , x_i and y_i are only related to v_m .

3. Designing Controller

Contrary to the usual situation, tracking is easier than regulation for a nonholonomic WMR. An intuitive explanation of this can be given in terms of a comparison between the number of controlled variables (outputs) and the number of control inputs. For the two-wheel differential drive mobile robot of Section 2, two input commands are available while three variables (x_i, y_i and the orientation θ_i) are needed to determine its configuration. Thus, regulation of the WMR posture to a desired configuration implies zeroing three independent configuration errors. When tracking a trajectory, instead, the output p_e has the same dimension as the input and the control problem is square.

Let us denote the current position of the mobile robot as p_m , the velocity as \dot{p}_m . The Cartesian velocity \dot{p}_m , is represented in terms of joint variables as (7). So, the kinematics model of the robot is as following:

$$\dot{p}_m = J(p_m)\dot{q}_m \quad (7)$$

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_m \\ \omega_m \end{bmatrix} = J(p_m)\dot{q}_m \quad (8)$$

Therefore, in order to obtain the real-time position of the robot, we can control the control law as $\dot{q}_m = [v_m, \omega_m]^T$.

The following is the way to get the desired inputs of the mobile robot system mathematically. The issue of mobile robot path following can generally be transformed into following one reference mobile robot. To assume the robot current position to be $p_m = [x_i, y_i, \theta_i]^T$, and the speed to be $\dot{q}_m = [v_m, \omega_m]^T$, the reference mobile robot position is $p_{m,r} = [x_r, y_r, \theta_r]^T$, and the speed to be $\dot{q}_{m,r} = [v_r, \omega_r]^T$, as shown in Fig. 2.

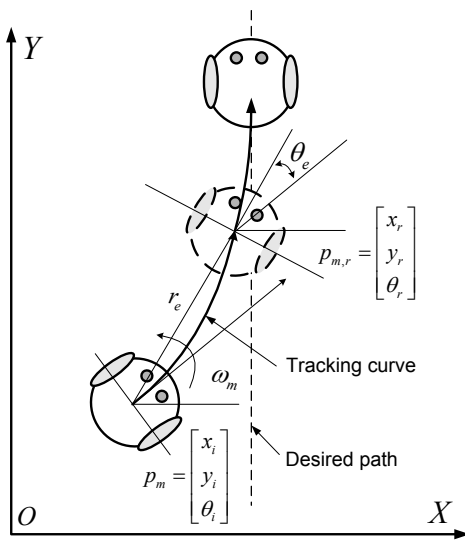


Fig. 2 Position error of mobile robot under Cartesian coordinate.

To order $p_e = [x_e, y_e, \theta_e]^T = [x_r - x_i, y_r - y_i, \theta_r - \theta_i]^T$, and the mobile robot position error equation (9) is obtained from the geometric relationship shown in Fig. 2. The reference state in the robot coordinate is expressed as follow by the transformation from the world coordinate to the robot coordinate.

$$\begin{aligned} p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} &= \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (p_{m,r} - p_m) \\ &= T_e (p_{m,r} - p_m) \end{aligned} \quad (9)$$

Then the position error differential equation is as following:

$$\dot{p}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} y_e \omega_m - v + v_r \cos \theta_e \\ -x_e \omega_m + v_r \sin \theta_e \\ \omega_r - \omega_m \end{bmatrix} \quad (10)$$

Path following of mobile robot based on kinematics model is to search the bounded input as $\dot{q} = [v_m, \omega_m]^T$, that causes position error vector, $p_e = [x_e, y_e, \theta_e]^T$ to be bounded, and $\lim_{t \rightarrow \infty} \| [x_e, y_e, \theta_e]^T \| = 0$, with arbitrary initial error and the equation (10) controlled by the control law.

By using the error vector $p_e = (x_e, y_e, \theta_e)$, the following curve in the robot coordinate can be calculated as follow;

$$\dot{r}_e = -v_m \cos \theta_e \quad (27)$$

$$\dot{\theta}_e = -\omega_m + \frac{v_m \sin \theta_e}{r_e} \quad (28)$$

$$\dot{\theta}_r = \frac{v_m \sin \theta_e}{r_e} \quad (29)$$

A Lyapunov candidate function is defined as in equation (30).

$$V = V_1 + V_2 = \frac{1}{2} \lambda r_e^2 + \frac{1}{2} (\theta_e^2 + h\theta_r^2) \quad (30)$$

where V_1 means the error energy to the distance and V_2 means the error energy in the direction. After differentiating both sides in equation (32) in terms of time, we can acquire the result as in equation (31).

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \lambda r_e \dot{r}_e + (\theta_e \dot{\theta}_e + h\theta_r \dot{\theta}_r) \quad (31)$$

Let us substitute equation (31) into the corresponding part in equation (31), it results in equation (32).

$$\begin{aligned} \dot{V} &= -\lambda r_e v_m \cos \theta_e \\ &+ \theta_e \left[-\omega_m + \frac{v_m \sin \theta_e}{\theta_e} \cdot \frac{(\theta_e + h\theta_r)}{r_e} \right] \end{aligned} \quad (32)$$

Note that $\dot{V} < 0$ is required for a given V to be a stable system. On this basis, we can design the nonlinear controller of the mobile robot as in equation (33),(34).

$$v_m = \gamma(r_e \cos \theta_e), (\gamma > 0) \quad (33)$$

$$\omega_m = k\theta_e + \gamma \frac{\cos \theta_e \cdot \sin \theta_e}{\theta_e} (\theta_e + h\theta_r), (k, h > 0) \quad (34)$$

Therefore, using this controller for the mobile robot, \dot{V} approaches to zero as $t \rightarrow \infty$; r_e and θ_e also approach almost to zero as shown in (35).

$$\dot{V} = -\lambda(\gamma \cos^2 \theta_e)r_e^2 - k\theta_e^2 \leq 0 \quad (35)$$

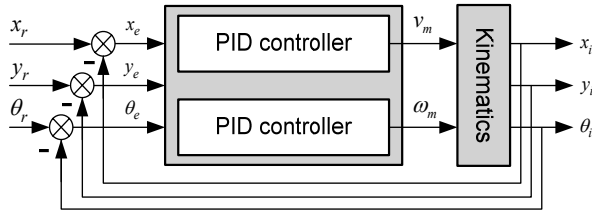


Fig. 3 Control system diagram.

In this paper, we use a PID algorithm for controlling position and velocity of the mobile robot platform. Using the PID algorithm, we can control the important system characteristics, let say us, rising time, steady state error, system stability, etc. Each term in the control algorithm has a different effect on the system characteristics [15]. In the PID control, the input for control of a standard PID controller in continuous time is stated as in equation (36).

$$m(t) = K \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right] \quad (36)$$

$$= K_p e(t) + K_i \int_0^t e(\tau) d\tau + K \frac{de(t)}{dt}$$

where $e(t)$ represents error signal that is the difference between desired input and output signal. For the digital control PID equation in discrete time is expressed as equation (37).

$$\Delta m(k) = K_p \Delta e(k) + K_i e(k) + K_d \Delta^2 e(k) \quad (37)$$

Also the above equation (37) can be rewritten as equation (38).

$$\Delta m(k) = v_0 e(k) + v_1 e(k-1) + v_2 e(k-2) \quad (38)$$

where $v_0 = K_p + K_i + K_d$, $v_1 = -K_p - 2K_d$, and $v_2 = 2K_d$.

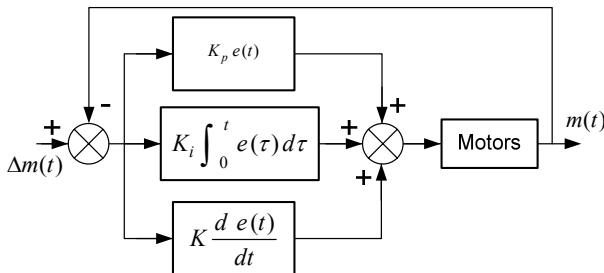


Fig. 4 PID controller diagram.

4. Experiments

In order to test the method of this paper, the controller is simulated with the circular track and the line track as reference trajectories, respectively.

The simulation: the reference track is as $x^2 + y^2 = 1.25^2$, the reference velocity is as $v_r = 0.2$, and the reference angular velocity is as $\omega_r = 0.2$. While the initial velocity of the mobile robot is as $v_m = 0.4$, and the initial angular velocity of the mobile robot is as $\omega_m = 0.3$, the initial position for the mobile robot is as $p_m = (-1, -1.2, 2\pi/3)^T$. The simulation results are shown in Fig. 6. It is important to note the good performance of the controller in spite of the very simple non-linear model used for the computation of the control law. This type of model simplification is very important when a simple and low-time consuming control law is needed because of processor limitations.

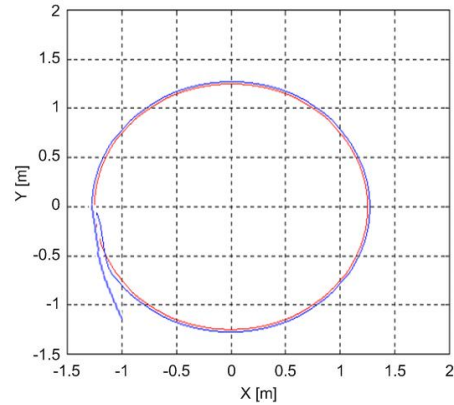


Fig. 5 Path following for the mobile robot.

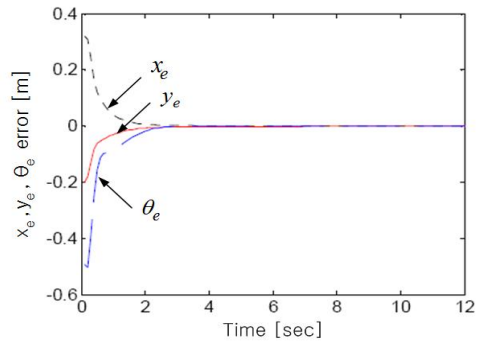


Fig. 6 Following error: x_e, y_e , and θ_e .

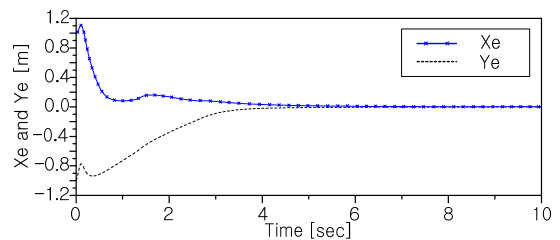


Fig. 7 Trajectory errors of mobile robot.

Figs. 7 shows the outputs and control inputs that make the mobile robot follow desired path. It should be noted that the horizontal and vertical displacements are brought to zero in 5 [sec] and 6 [sec] in Fig. 7, respectively.

5. Conclusions

This paper presents an adaptive following controller for mobile robot based on the PID. The proposed controller has a parallel structure that consists of PID controller with a fixed gain. The control law is constructed on the basis of Lyapunov stability theory. And also, this paper shows that the following errors are bounded uniformly and ultimately under the existence of the disturbances and modeling error, mathematically.

The wheeled type mobile robot is employed as a test-bed to apply the proposed controller. The experimental results show that the proposed controller is adaptable to the environment changes and is implemented to the real system and the experimental results demonstrate the effectiveness of the methods. For the future work, robust tuning and position estimation should be considered to realize more accurate motions of mobile robots.

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