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# Interval-Valued Fuzzy $m\beta$-continuous mappings on Interval-Valued Fuzzy Minimal Spaces

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**Abstract**

We introduce the concepts of interval-valued fuzzy $m\beta$-open sets and interval-valued fuzzy $m\beta$-continuous mappings. And we study some characterizations and properties of such concepts.

**Key words** : interval-valued fuzzy minimal spaces, interval-valued fuzzy $m\beta$-open sets, interval-valued fuzzy $m\beta$-continuous mappings

## 1. Introduction

Zadeh [9] introduced the concept of fuzzy set and several researchers were concerned about the generalizations of the concept of fuzzy sets, intuitionistic fuzzy sets [1] and interval-valued fuzzy sets [3]. In [2], Alimohammady and Roohi introduced fuzzy minimal structures and fuzzy minimal spaces and gave some results. In [5], the author introduced the concept of interval-valued fuzzy minimal space (simply, IVF minimal space) as a generalization of interval-valued fuzzy topology introduced by Mondal and Samanta [8]. The concept of interval-valued fuzzy $m$-continuous mappings on between interval-valued fuzzy minimal spaces, which are generalizations of interval-valued fuzzy continuous mappings. The concepts of interval-valued fuzzy $m\alpha$-open set, interval-valued fuzzy $m$-semiopen set and interval-valued fuzzy $m$-preopen set are introduced and investigated. In this paper, we introduce the concepts of interval-valued fuzzy $m\beta$-open sets and interval-valued fuzzy $m\beta$-continuous mappings defined on interval-valued fuzzy minimal spaces. These concepts are generalizations of interval-valued fuzzy $m$-open sets and interval-valued fuzzy $m$-continuity, respectively. We also study characterizations and some basic properties of such concepts.

## 2. Preliminaries

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M = [M^L, M^U]$, where $M^L$ and $M^U$ are the lower and the upper end points of $M$, respectively. Especially, we denote $\mathbf{0} = [0,0], \mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for $a \in (0,1)$. We also note that

(1) $(M, N \in D[0,1])(M = N \Leftrightarrow M^L = N^L, M^U = N^U)$.

(2) $(M, N \in D[0,1])(M \leq N \Leftrightarrow M^L \leq N^L, M^U \leq N^U)$.

For every $M \in D[0,1]$, the complement of $M$, denoted by $M^c$, is defined by $M^c = 1 - M = [1 - M^U, 1 - M^L]$.

Let $X$ be a nonempty set. A mapping $A : X \to D[0,1]$ is called an interval-valued fuzzy set(simply, IVF set) in $X$. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at $p$ and **0** elsewhere in $X$ is called an interval-valued fuzzy point (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by $a_p$. Denoted by IVF(X) the set of all IVF sets in $X$. An IVF point $M_x$, where $M \in D[0,1]$, is said to belong to an IVF set $A$ in $X$, denoted by $M_x \widetilde{\in} A$, if $A(x)^L \geq M^L$ and $A(x)^U \geq M^U$. In [7], it has been shown that $A = \cup\{M_x : M_x \widetilde{\in} A\}$.

For every $A, B \in IVF(X)$, we define

$A = B \Leftrightarrow x \in X, ([A(x)]^L = [B(x)]^L$ and $[A(x)]^U = [B(x)]^U)$,

$A \subseteq B \Leftrightarrow x \in X, ([A(x)]^L \leq [B(x)]^L$ and $[A(x)]^U \leq [B(x)]^U)$.

The complement $A^c$ of $A$ is defined by, for all $x \in X$, $[A^c(x)]^L = 1 - [A(x)]^U$and $[A^c(x)]^U = 1 - [A(x)]^L$.

For a family of IVF sets $\{A_i : i \in J\}$ where $J$ is an index set, the union $G = \cup_{i \in J} A_i$ and the meet $F = \cap_{i \in J} A_i$ are defined by

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$x \in X$, $([G(x)]^L = \sup_{i\in J}[A_i(x)]^L$, $[G(x)]^U = \sup_{i\in J}[A_i(x)]^U)$,

$x \in X$, $([F(x)]^L = \inf_{i\in J}[A_i(x)]^L$, $[F(x)]^U = \inf_{i\in J}[A_i(x)]^U)$, respectively.

Let $f : X \to Y$ be a mapping and let $A$ be an IVF set in $X$. Then the image of $A$ under $f$, denoted by $f(A)$, is defined as follows

$$[f(A)(y)]^L = \begin{cases} \sup_{f(x)=y}[A(x)]^L, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise ,} \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} \sup_{f(x)=y}[A(x)]^U, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise ,} \end{cases}$$

for all $y \in Y$.

Let $B$ be an IVF set in $Y$. Then the inverse image of $B$ under $f$, denoted by $f^{-1}(B)$, is defined by

$([f^{-1}(B)(x)]^L = [B(f(x))]^L$, $[f^{-1}(B)(x)]^U = [B(f(x))]^U)$ for all $x \in X$.

**Definition 2.1 ([5]).** A family $\mathcal{M}$ of interval-valued fuzzy sets in $X$ is called an *interval-valued fuzzy minimal structure* on $X$ if

$$\mathbf{0}, \mathbf{1} \in \mathcal{M}.$$

In this case, $(X, \mathcal{M})$ is called an *interval-valued fuzzy minimal space* (simply, *IVF minimal space*). Every member of $\mathcal{M}$ is called an interval-valued fuzzy $m$-open set (simply IVF $m$-open set). An IVF set $A$ is called an IVF $m$-closed set if the complement of $A$ (simply, $A^c$) is an IVF $m$-open set.

Let $(X, \mathcal{M})$ be an IVF minimal space and $A$ in IVF(X). The IVF minimal-closure and the IVF minimal-interior of $A$ [5], denoted by $mC(A)$ and $mI(A)$, respectively, are defined as

$$mC(A) = \cap\{B \in IVF(X) : B^c \in \mathcal{M} \text{ and } A \subseteq B\},$$

$$mI(A) = \cup\{B \in IVF(X) : B \in \mathcal{M} \text{ and } B \subseteq A\}.$$

**Theorem 2.2 ([5]).** Let $(X, \mathcal{M})$ be an IVF minimal space and $A, B$ in IVF($X$).

(1) $mI(A) \subseteq A$ and if $A$ is an IVF $m$-open set, then $mI(A) = A$.

(2) $A \subseteq mC(A)$ and if $A$ is an IVF $m$-closed set, then $mC(A) = A$.

(3) If $A \subseteq B$, then $mI(A) \subseteq mI(B)$ and $mC(A) \subseteq mC(B)$.

(4) $mI(A) \cap mI(B) \supseteq mI(A \cap B)$ and $mC(A) \cup mC(B) \subseteq mC(A \cup B)$.

(5) $mI(mI(A)) = mI(A)$ and $mC(mC(A)) = mC(A)$.

(6) $\mathbf{1} - mC(A) = mI(\mathbf{1} - A)$ and $\mathbf{1} - mI(A) = mC(\mathbf{1} - A)$.

# 3. Interval-valued fuzzy $m\beta$-open sets and Interval-valued fuzzy $m\beta$-continuous mappings

**Definition 3.1.** Let $(X, \mathcal{M})$ be an IVF minimal space and $A$ in IVF($X$). Then an IVF set $A$ is called an *interval-valued fuzzy $m\beta$-open* (simply *IVF $m\beta$-open) set* in $X$ if

$$A \subseteq mC(mI(mC(A))).$$

And an IVF set $A$ is called an *interval-valued fuzzy $m\beta$-closed* (simply *IVF $m\beta$-closed) set* if the complement of $A$ is an IVF $m\beta$-open set.

**Remark 3.2.** Let $(X, \mathcal{M})$ be an IVF minimal space and $A$ in IVF($X$). If $\mathcal{M}$ is an IVF topology on $X$, then an IVF $m\beta$-open set is IVF semi-preopen [4]

Let $(X, \mathcal{M})$ be an IVF minimal space and $A$ in IVF(X). Then an IVF set $A$ is called an IVF $m\alpha$-open [7] (resp. *IVF $m$-semiopen*, *IVF $m$-preopen*) set [6] in $X$ if $A \subseteq mI(mC(mI(A)))$ (resp. $A \subseteq mC(mI(A))$, $A \subseteq mI(mC(A))$).

From the definitions of several types of IVF $m$-open sets, obviously the following are obtained:

**Lemma 3.3.** Let $(X, \mathcal{M})$ be an IVF minimal space. Then the statements are hold.

(1) Every IVF $m$-semiopen set is IVF $m\beta$-open.

(2) Every IVF $m$-preopen set is IVF $m\beta$-open.

(3) $A$ is an IVF $m\beta$-closed set if and only if $mI(mC(mI(A))) \subseteq A$.

**Example 3.4.** Let $X = [0, 1]$ and let $A$, $B$ and $C$ be IVF sets defined as follows

$A(x) = [\frac{1}{2}, \frac{1}{2}(x+1)]$ for all $x \in X$;

$B(x) = [\frac{1}{2}, -\frac{1}{2}(x-2)]$ for all $x \in X$;

$C(x) = \widetilde{[\frac{1}{4}, \frac{1}{2}]}$; $D(x) = \widetilde{[\frac{3}{8}, \frac{1}{2}]}$;

$E(x) = \widetilde{[\frac{1}{2}, \frac{5}{8}]}$; $F(x) = \widetilde{[0, \frac{1}{2}]}$.

Consider $\mathcal{M} = \{\mathbf{0}, A, B, C, D, \mathbf{1}\}$ as an IVF minimal structure on $X$. Then IVF sets $E, F$ are IVF $m\beta$-open. But $E$ is not IVF $m$-preopen and $F$ is not IVF $m$-semiopen.

IVF $m$-semiopen<br>
↗ ↘<br>
IVF $m$-open → IVF $m\alpha$-open IVF $m\beta$-open<br>
↘ ↗<br>
IVF $m$-preopen

**Theorem 3.5.** Let $(X, \mathcal{M})$ be an IVF minimal space. Any union of IVF $m\beta$-open sets is IVF $m\beta$-open.

*Proof.* Let $A_i$ be an IVF $m\beta$-open set for $i \in J$. Then

$$A_i \subseteq mC(mI(mC((A_i)))) \subseteq mC(mI(mC(\cup A_i))).$$

Thus $\cup A_i \subseteq mC(mI(mC(\cup A_i)))$. □

**Remark 3.6.** Let $(X, \mathcal{M})$ be an IVF minimal space. The intersection of any two IVF $m\beta$-open sets may not be IVF $m\beta$-open set as seen in the next example.

**Example 3.7.** In Example 3.4, the IVF sets $A, B$ are IVF $m\beta$-open but $A \cap B$ is not IVF $m\beta$-open.

**Definition 3.8.** Let $(X, \mathcal{M})$ be an IVF minimal space. For $A \in IVF(X)$, the *$\beta$-closure* and the *$\beta$-interior*, denoted by $\beta mC(A)$ and $\beta mI(A)$ in $(X, \mathcal{M})$, respectively, are defined as the following:

$\beta mC(A) = \cap\{F \in IVF(X) : A \subseteq F,$ $F$ is IVF $m\beta$-closed in $X\}$,

$\beta mI(A) = \cup\{U \in IVF(X) : U \subseteq A,$ $U$ is IVF $m\beta$-open in $X\}$.

**Theorem 3.9.** Let $(X, \mathcal{M})$ be an IVF minimal space and $A, B \in IVF(X)$. Then

(1) $\beta mI(A) \subseteq A$.

(2) If $A \subseteq B$, then $\beta mI(A) \subseteq \beta mI(B)$.

(3) $A$ is IVF $m\beta$-open iff $\beta mI(A) = A$.

(4) $\beta mI(\beta mI(A)) = \beta mI(A)$.

(5) $\beta mC(\mathbf{1} - A) = \mathbf{1} - \beta mI(A)$ and $\beta mI(\mathbf{1} - A) = \mathbf{1} - \beta mC(A)$.

*Proof.* (1), (2) Obvious.

(3) It follows from Theorem 3.5.

(4) It follows from (3).

(5) For $A \in IVF(X)$,

$$\begin{aligned} \mathbf{1} - \beta mI(A) &= \mathbf{1} - \cup\{U : U \subseteq A, \\ &\qquad U \text{ is IVF } m\beta\text{-open}\} \\ &= \cap\{\mathbf{1} - U : U \subseteq A, \\ &\qquad U \text{ is IVF } m\beta\text{-open}\} \\ &= \cap\{\mathbf{1} - U : X - A \subseteq \mathbf{1} - U, \\ &\qquad U \text{ is IVF } m\beta\text{-open}\} \\ &= \beta mC(\mathbf{1} - A). \end{aligned}$$

Similarly, it is proved $\beta mI(\mathbf{1}-A) = \mathbf{1}-\beta mC(A)$. □

**Theorem 3.10.** Let $(X, \mathcal{M})$ be an IVF minimal space and $A, B, F \in IVF(X)$. Then

(1) $A \subseteq \beta mC(A)$.

(2) If $A \subseteq B$, then $\beta mC(A) \subseteq \beta mC(B)$.

(3) $F$ is IVF $m\beta$-closed iff $\beta mC(F) = F$.

(4) $\beta mC(\beta mC(A)) = \beta mC(A)$.

*Proof.* It is similar to the proof of Theorem 3.9. □

**Lemma 3.11.** Let $(X, \mathcal{M})$ be an IVF minimal space and $A, U, V \in IVF(X)$. Then

(1) $M_x \widetilde{\in} \beta mC(A)$ if and only if $A \cap V \neq \mathbf{0}$ for every IVF $m\beta$-open set $V$ containing $M_x$.

(2) $M_x \widetilde{\in} \beta mI(A)$ if and only if there exists an IVF $m\beta$-open set $U$ such that $U \subseteq A$.

*Proof.* (1) Suppose there is an IVF $m\beta$-open set $V$ containing $M_x$ such that $A \cap V = \mathbf{0}$. Then $X - V$ is an IVF $m\beta$-closed set such that $A \subseteq \mathbf{1} - V$ and $M_x \widetilde{\notin} \mathbf{1} - V$. Thus $M_x \widetilde{\notin} \beta mC(A)$.

The converse is proved obviously.

(2) Obvious. □

**Definition 3.12.** Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : (X, \mathcal{M}_X) \to (Y, \mathcal{M}_Y)$ is said to be *interval-valued fuzzy $m\beta$-continuous* (simply, IVF $m\beta$-continuous) if for each IVF point $M_x$ and each IVF $m$-open set $V$ containing $f(M_x)$, there exists an IVF $m\beta$-open set $U$ containing $M_x$ such that $f(U) \subseteq V$.

Let $f : X \to Y$ be a mapping on two IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then

(1) $f$ is said to be *interval-valued fuzzy minimal continuous* (simply, IVF $m$-continuous) [5] if for every $A \in \mathcal{M}_Y$, $f^{-1}(A)$ is in $\mathcal{M}_X$.

(2) $f$ is said to be *interval-valued fuzzy $m$-semicontinuous* (simply, IVF $m$-semicontinuous) [6] if for each IVF point $M_x$ and each IVF $m$-open set $V$ containing $f(M_x)$, there exists an $m$-semiopen set $U$ containing $M_x$ such that $f(U) \subseteq V$.

(3) $f$ is said to be *interval-valued fuzzy $m$-precontinuous* (simply, IVF $m$-precontinuous) [6] if for each IVF point $M_x$ and each IVF $m$-open set $V$ containing $f(M_x)$, there exists an IVF $m$-preopen set $U$ containing $M_x$ such that $f(U) \subseteq V$.

(4) $f$ is said to be *interval-valued fuzzy $m\alpha$-continuous* (simply, IVF $m\alpha$-continuous) [7] if for each IVF point $M_x$ and each IVF $m$-open set $V$ containing $f(M_x)$, there exists an IVF $m\alpha$-open set $U$ containing $M_x$ such that $f(U) \subseteq V$.

We have the following implications but the converses are not always true as shown in the next example.

IVF $m$-semiconti.
↗ ↘
IVF $m$-conti. → IVF $m\alpha$-conti. IVF $m\beta$-conti.
↘ ↗
IVF $m$-preconti.

**Example 3.13.** In Example 3.4, consider $\mathcal{N} = \{\mathbf{0}, E, F, \mathbf{1}\}$ and the identity mapping $f : (X, \mathcal{M}) \to$

$(X, \mathcal{N})$. Then $f$ is IVF $m\beta$-continuous but neither IVF $m$-semicontinuous nor IVF $m$-precontinuous.

**Theorem 3.14.** Let $f : X \to Y$ be a mapping on IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following are equivalent:

(1) $f$ is IVF $m\beta$-continuous.

(2) $f^{-1}(V)$ is an IVF $m\beta$-open set for each IVF $m$-open set $V$ in $Y$.

(3) $f^{-1}(B)$ is an IVF $m\beta$-closed set for each IVF $m$-closed set $B$ in $Y$.

(4) $f(\beta mC(A)) \subseteq mC(f(A))$ for $A \in IVF(X)$.

(5) $\beta mC(f^{-1}(B)) \subseteq f^{-1}(mC(B))$ for $B \in IVF(Y)$.

(6) $f^{-1}(mI(B)) \subseteq \beta mI(f^{-1}(B))$ for $B \in IVF(Y)$.

*Proof.* $(1) \Rightarrow (2)$ Let $V$ be an IVF $m$-open set in $Y$ and $M_x \widetilde{\in} f^{-1}(V)$. Then there exists an IVF $m\beta$-open set $U_{M_x}$ containing $M_x$ such that $f(U_{M_x}) \subseteq V$. So $M_x \widetilde{\in} U_{M_x} \subseteq f^{-1}(V)$ for every $M_x \widetilde{\in} f^{-1}(V)$. Hence $f^{-1}(V)$ is IVF $m\beta$-open.

$(2) \Rightarrow (3)$ Obvious.

$(3) \Rightarrow (4)$ For any IVF set $A$ in $X$,

$$\begin{aligned} f^{-1}(mC(f(A))) &= f^{-1}(\cap\{F \in IVF(Y) : f(A) \subseteq F \\ &\qquad \text{and } F \text{ is IVF } m\text{-closed}\}) \\ &= \cap\{f^{-1}(F) : A \subseteq f^{-1}(F) \\ &\qquad \text{and } f^{-1}(F) \text{ is IVF } m\beta\text{-closed}\} \\ &\supseteq \cap\{K \in IVF(X) : A \subseteq K \text{ and} \\ &\qquad K \text{ is IVF } m\beta\text{-closed}\} \\ &= \beta mC(A). \end{aligned}$$

Hence $f(\beta mC(A)) \subseteq mC(f(A))$.

$(4) \Rightarrow (5)$ Obvious.

$(5) \Rightarrow (6)$ For $B \in IVF(Y)$, from $mI(B) = \mathbf{1} - mC(\mathbf{1} - B)$ and Theorem 3.9, it follows

$$\begin{aligned} f^{-1}(mI(B)) &= f^{-1}(\mathbf{1} - mC(\mathbf{1} - B)) \\ &= \mathbf{1} - f^{-1}(mC(\mathbf{1} - B)) \\ &\subseteq \mathbf{1} - \beta mC(f^{-1}(\mathbf{1} - B)) \\ &= \beta mI(f^{-1}(B)). \end{aligned}$$

Hence (6) is obtained.

(6) $\Rightarrow$ (1) Let $M_x$ be an IVF point in $X$ and $V$ an IVF $m$-open set containing $f(M_x)$. Since $V = mI(V)$,

$$M_x \widetilde{\in} f^{-1}(V) = f^{-1}(mI(V)) \subseteq \beta mI(f^{-1}(V)).$$

Thus from Lemma 3.11, there exists an IVF $m\beta$-open set $U$ containing $M_x$ such that $M_x \widetilde{\in} U \subseteq f^{-1}(V)$. Hence $f$ is IVF $m\beta$-continuous. □

**Lemma 3.15.** Let $(X, \mathcal{M}_X)$ be an IVF minimal space and $A \in IVF(X)$. Then

(1) $mI(mC(mI(A))) \subseteq mI(mC(mI(\beta mC(A)))) \subseteq \beta mC(A)$.

(2) $\beta mI(A) \subseteq mC(mI(mC(\beta mI(A)))) \subseteq mC(mI(mC(A)))$.

*Proof.* The proof is obvious from Theorem 3.9 and Theorem 3.10.

□

**Theorem 3.16.** Let $f : X \to Y$ be a function on IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following are equivalent:

(1) $f$ is IVF $m\beta$-continuous.

(2) $f^{-1}(V) \subseteq mC(mI(mC(f^{-1}(V))))$ for each IVF $m$-open set $V$ in $Y$.

(3) $mI(mC(mI(f^{-1}(F)))) \subseteq f^{-1}(F)$ for each IVF $m$-closed set $F$ in $Y$.

(4) $f(mI(mC(mI(A)))) \subseteq mC(f(A))$ for $A \in IVF(X)$.

(5) $mI(mC(mI(f^{-1}(B)))) \subseteq f^{-1}(mC(B))$ for $B \in IVF(Y)$.

(6) $f^{-1}(mI(B)) \subseteq mC(mI(mC(f^{-1}(B))))$ for $B \in IVF(Y)$.

*Proof.* $(1) \Leftrightarrow (2)$ Obvious.

$(1) \Leftrightarrow (3)$ Obvious.

$(3) \Rightarrow (4)$ Let $A \in IVF(X)$. Then from Theorem 3.14 and Lemma 3.15, it follows $mI(mC(mI(A))) \subseteq \beta mC(A) \subseteq f^{-1}(f(\beta mC(A))) \subseteq f^{-1}(mC(f(A)))$, and so $f(mI(mC(mI(A)))) \subseteq mC(f(A))$.

$(4) \Rightarrow (5)$ Obvious.

$(5) \Rightarrow (6)$ From (5), it follows

$$\begin{aligned} f^{-1}(mI(B)) &= f^{-1}(\mathbf{1} - mC(\mathbf{1} - B)) \\ &= \mathbf{1} - (f^{-1}(mC(\mathbf{1} - B))) \\ &\subseteq \mathbf{1} - mI(mC(mI(f^{-1}(\mathbf{1} - B)))) \\ &= mC(mI(mC(f^{-1}(B)))). \end{aligned}$$

Hence, (6) is obtained.

$(6) \Rightarrow (1)$ Let $V$ an IVF $m$-open set in $Y$. Then $f^{-1}(V) = f^{-1}(mI(V)) \subseteq mC(mI(mC(f^{-1}(V))))$. This implies $f^{-1}(V)$ is an IVF $m\beta$-open set. Hence $f$ is IVF $m\beta$-continuous. □

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