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Noninformative priors for stress-strength reliability in the Pareto distributions

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Abstract

In this paper, we develop the noninformative priors for stress-strength reliability from the Pareto distributions. We develop the matching priors and the reference priors. It turns out that the second order matching prior does not match the alternative coverage probabilities, and is not a highest posterior density matching or a cumelative distribution function matching priors. Also we reveal that the one-at-a-time reference prior and Jeffreys' prior are the second order matching prior. We show that the proposed reference prior matches the target coverage probabilities in a frequentist sense through simulation study, and an example is given.

Keywords: Matching prior, Pareto distribution, reference prior, stress-strength reliability.

1. Introduction

The Pareto distribution provides a statistical model which has an extensive variety of applications. It has been found in describing distributions of studies of income, property values, insurance risk, stock prices fluctuations, migration, size of cities and firms, word frequencies, occurrences of natural resources, business failures, service time in queuing systems, error clustering in communications circuits and lifetime data, etc (Arnold and Press, 1983; Fernández, 2008). In a Bayesian point of view, many authors have studied statistical inferences on Pareto distribution (e.g., Arnold and Press, 1983, 1989; Geisser, 1984, 1985; Lwin, 1972; Nigm and Hamdy, 1987; Tiwari, Yang and Zalkikar, 1996; Ko and Kim, 1999; Fernández, 2008; Kim, Kang and Lee, 2009; Kang, 2010).

In reliability contexts, inferences about reliability

$$R = P(Y < X),$$

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where X and Y have independent distributions, are a subject of interest. The problem of making inference about R have been received a considerable attention in statistical literature. For example, an item is able to perform its intended function if its strength is greater than the stress imposed upon it. The probability that an item is strong enough to overcome the stress is the measure of confidence of the item. So, to make statistical inference about this probability is very important.

The present paper focuses on developing noninformative priors for the reliability R in Pareto distributions. Subjective priors are ideal when sufficient information from past experience, expert opinion or previously collected data exist. However, often even without adequate prior information, one can use Bayesian analysis efficiently with some noninformative or default priors.

The notion of a noninformative prior has attracted much attention in recent years. There are different notions of noninformative prior. One is a probability matching prior introduced by Welch and Peers (1963) which matches the posterior and frequentist probabilities of confidence intervals. Interest in such priors revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may cite the work of Mukerjee and Dey (1993), DiCiccio and Stern (1994), Datta and Ghosh (1995a, b, 1996) and Mukerjee and Ghosh (1997).

The other is the reference prior introduced by Bernardo (1979) which maximizes the Kullback-Leibler divergence between the prior and the posterior. Ghosh and Mukerjee (1992), and Berger and Bernardo (1989, 1992) give a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. This approach is very successful in various practical problems. Quite often reference priors satisfy the matching criterion described earlier.

The problem of estimating R for Pareto distributions with the common scale parameter and the different scale parameters was considered by Rezaei et al. (2010). When the common scale parameter is known, Rezaei et al. (2010) derived the maximum likelihood estimator, uniformly minimum variance unbiased estimator and Bayes estimator based on gamma priors, and revealed that their performance are quite similar in nature. Also they derived the confidence interval based on the exact distribution of the maximum likelihood estimator, and showed that the coverage probabilities of the confidence intervals reach the nominal level, 95% when the sample size is increase.

This situation with known common scale parameter and different shape parameters in Pareto distribution is somewhat realistic. If one knows the information of an item very well, one will give stronger stress to the item for the purpose of proving the strength of an item. Then he will try higher stress X than strength Y. Usually stress changes shape of distribution rather than scale of distribution in many cases. So, we postulate with known common scale parameter and different shape parameters in Pareto distributions (Pandey and Upadhyay, 1986).

The outline of the remaining sections is as follows. In Section 2, we develop first order and second order probability matching priors for R. We revealed that the second order matching prior is not a highest posterior density (HPD) matching prior and a cumulative distribution function (CDF) matching prior, and does not match the alternative coverage probabilities up to the second order. Also we derive the reference priors for R. It turns out that the one-at-a-time reference prior and Jeffreys' prior are the second order matching prior. We provide the propriety of the posterior distribution for the general prior including the reference and matching priors. In Section 4, we will find the frequentist coverage probabilities under the proposed prior. And a real data example will be given.

2. The noninformative priors

Let X and Y be two independent the Pareto distributions with the common scale parameter λ, and shape parameters η and β, respectively. Typically, X represents the amount of the stress subjected to an item and Y represents the strength of an item. The Pareto probability density functions of X and Y are given by

$$f(x|\eta) = \eta\lambda(1+\lambda x)^{-(\eta+1)}, x > 0, \eta > 0, \tag{2.1}$$

and

$$f(y|\beta) = \beta\lambda(1+\lambda y)^{-(\beta+1)}, y > 0, \beta > 0, \tag{2.2}$$

respectively. The reliability R can be easily calculated by,

$$R = P(Y < X) = \frac{\beta}{\eta+\beta}.$$

This reliability R is parameter of interest. Now we develop the noninformative priors for R when λ is known. Without of generality, we can assume that $\lambda = 1$.

Suppose that $X_1, \cdots, X_n$ is a random sample of stress X and $Y_1, \cdots, Y_m$ is a random sample of strength Y. Then the likelihood function of η and β is given by

$$L(\eta, \beta) = \eta^n \beta^m \prod_{i=1}^{n}(1+x_i)^{-(\eta+1)} \prod_{j=1}^{m}(1+y_j)^{-(\beta+1)}. \tag{2.3}$$

2.1. The probability matching priors

For a prior π, let $\theta_1^{1-\alpha}(\pi; \mathbf{X})$ denote the $(1-\alpha)$th posterior quantile of θ_1, that is,

$$P^{\pi}[\theta_1 \le \theta_1^{1-\alpha}(\pi; \mathbf{X})|\mathbf{X}] = 1-\alpha, \tag{2.4}$$

where $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_t)^T$ and θ_1 is the parameter of interest. We want to find priors π for which

$$P[\theta_1 \le \theta_1^{1-\alpha}(\pi; \mathbf{X})|\boldsymbol{\theta}] = 1-\alpha + o(n^{-r}). \tag{2.5}$$

for some $r > 0$, as n goes to infinity. Priors π satisfying (2.5) are called matching priors. If $r = 1/2$, then π is referred to as a first order matching prior, while if $r = 1$, π is referred to as a second order matching prior.

In order to find such matching priors π, it is convenient to consider the orthogonal reparametrization which a parameter of interest is orthogonal to the rest of parameters in information matrix. To achieve orthogonality, let

$$\theta_1 = \frac{\beta}{\eta+\beta} \text{ and } \theta_2 = \eta^n\beta^m.$$

With this parametrization, the likelihood function of parameters (θ_1, θ_2) for the model (2.3) is given by

$$L(\theta_1,\theta_2) \propto \theta_2 \prod_{i=1}^{n} (1+x_i)^{-\theta_1^{\frac{-m}{n+m}}(1-\theta_1)^{\frac{m}{n+m}}\theta_2^{\frac{1}{n+m}}-1} \prod_{j=1}^{m} (1+y_j)^{-\theta_1^{\frac{n}{n+m}}(1-\theta_1)^{\frac{-n}{n+m}}\theta_2^{\frac{1}{n+m}}-1}. \quad (2.6)$$

Based on (2.6), the information matrix is given by

$$\mathbf{I}(\theta_1,\theta_2) = \begin{pmatrix} \frac{nm}{n+m}\theta_1^{-2}(1-\theta_1)^{-2} & 0 \\ 0 & \frac{1}{n+m}\theta_2^{-2} \end{pmatrix}. \quad (2.7)$$

From the above Fisher information matrix $\mathbf{I}$, θ_1 is orthogonal to θ_2 in the sense of Cox and Reid(1987). Following Tibshirani(1989), the class of first order probability matching prior is characterized by

$$\pi_m^{(1)}(\theta_1,\theta_2) \propto \theta_1^{-1}(1-\theta_1)^{-1}d(\theta_2), \quad (2.8)$$

where $d(\theta_2) > 0$ is an arbitrary function differentiable in its argument.

The class of prior given in (2.8) can be narrowed down to the second order probability matching priors as given in Mukerjee and Ghosh (1997). A second order probability matching prior is of the form (2.8), and also d must satisfy an additional differential equation (2.10) of Mukerjee and Ghosh (1997), that is

$$\frac{1}{6}d(\theta_2)\frac{\partial}{\partial\theta_1}\{I_{11}^{-\frac{3}{2}}L_{1,1,1}\} + \frac{\partial}{\partial\theta_2}\{I_{11}^{-\frac{1}{2}}L_{112}I^{22}d(\theta_2)\} = 0, \quad (2.9)$$

where

$$\begin{aligned} L_{1,1,1} &= E\left[\left(\frac{\partial \log L}{\partial\theta_1}\right)^3\right] = \frac{2nm(n-m)}{(n+m)^2}\theta_1^{-3}(1-\theta_1)^{-3}, \\ L_{112} &= E\left[\frac{\partial^3 \log L}{\partial\theta_1^2\partial\theta_2}\right] = -\frac{nm}{(n+m)^2}\theta_1^{-2}(1-\theta_1)^{-2}\theta_2^{-1}, \\ I_{11} &= \frac{nm}{n+m}\theta_1^{-2}(1-\theta_1)^{-2}, I^{22} = (n+m)\theta_2^2. \end{aligned}$$

Then (2.9) simplifies to

$$\frac{\partial}{\partial\theta_2}\left\{-\frac{(nm)^{\frac{1}{2}}}{(n+m)^{\frac{1}{2}}}\theta_1^{-1}(1-\theta_1)^{-1}\theta_2 d(\theta_2)\right\} = 0. \quad (2.10)$$

Hence the set of solution of (2.10) is of the form $d(\theta_2) = \theta_2^{-1}$. Thus the resulting second order probability matching prior is

$$\pi_m^{(2)}(\theta_1,\theta_2) \propto \theta_1^{-1}(1-\theta_1)^{-1}\theta_2^{-1}. \quad (2.11)$$

There are alternative ways through which matching can be accomplished. Datta, Ghosh and Mukerjee (2000) provided a theorem which establishes the equivalence of second order matching priors and HPD matching priors (DiCiccio and Stern, 1994; Ghosh and Mukerjee, 1995) within the class of first order matching priors. The equivalence condition is that $I_{11}^{-3/2}L_{111}$ dose not depend on θ_1. Since

$$L_{111} = E\left[\frac{\partial^3 \log L}{\partial\theta_1^3}\right] = -\frac{2nm}{(n+m)^2}\frac{3(n+m)\theta_1 - n - 2m}{\theta_1^3(1-\theta_1)^3},$$

$I_{11}^{-3/2}L_{111}$ depends on θ_1. Therefore the second order probability matching prior (2.11) is not a HPD matching prior. Also since

$$\frac{\partial}{\partial\theta_1}\{I_{11}^{-\frac{3}{2}}L_{111}\} \neq 0,$$

thus the second order matching prior (2.11) does not match the alternative coverage probabilities (Mukerjee and Reid, 1999), and now

$$\frac{\partial}{\partial\theta_1}\{(I^{11})^2 L_{111}\pi_m^{(2)}\} \neq 0,$$

so the second order matching prior (2.11) is not a CDF matching prior (Mukerjee and Ghosh, 1997).

2.2. The reference priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. We derive the reference priors for different groups of ordering of (θ_1, θ_2). Then due to the orthogonality of the parameters, following Datta and Ghosh (1995b), choosing rectangular compacts for each θ_1 and θ_2 when θ_1 is the parameter of interest, the reference priors are given as follows.

For the the stress-strength reliability model (2.6), if θ_1 is the parameter of interest, then the reference prior distributions for group of ordering of $\{(\theta_1, \theta_2)\}$ is

$$\pi_1(\theta_1, \theta_2) \;\propto\; \theta_1^{-1}(1-\theta_1)^{-1}\theta_2^{-1}.$$

For group of ordering of $\{\theta_1, \theta_2\}$, the reference prior is

$$\pi_2(\theta_1, \theta_2) \;\propto\; \theta_1^{-1}(1-\theta_1)^{-1}\theta_2^{-1}.$$

Conclusively, from the above reference priors, we know that the one-at-a-time reference prior π_2 and Jeffreys' prior π_1 are the second order matching prior, and all priors are the same.

3. Implementation of the Bayesian procedure

We investigate the propriety of posteriors for a general class of priors which include the reference and the matching priors. We consider the class of priors

$$\pi(\theta_1, \theta_2) \propto \theta_1^{-a}(1-\theta_1)^{-a}\theta_2^{-b}, \tag{3.1}$$

where $a > 0$ and $b > 0$. The following general theorem can be proved.

Theorem 3.1 The posterior distribution of (θ_1, θ_2) under the prior π, in (3.1), is proper if $bm - a + 1 > 0$ and $bn - a + 1 > 0$.

Proof: Note that the joint posterior for θ_1 and θ_2 given $\mathbf{x}$ and $\mathbf{y}$ is

$$\pi(\theta_1, \theta_2|\mathbf{x}, \mathbf{y}) \propto \theta_1^{-a}(1-\theta_1)^{-a}\theta_2^{-b+1}\left[\prod_{i=1}^{n}(1+x_i)^{\theta_1^{\frac{-m}{n+m}}(1-\theta_1)^{\frac{m}{n+m}}\theta_2^{\frac{1}{n+m}}}\right] \times \left[\prod_{i=1}^{m}(1+y_i)^{\theta_1^{\frac{n}{n+m}}(1-\theta_1)^{\frac{-n}{n+m}}\theta_2^{\frac{1}{n+m}}}\right]. \quad (3.2)$$

Let $\omega = \theta_2^{\frac{1}{n+m}}$. Then integrating with respect to ω in (3.2), we can get

$$\pi(\theta_1|\mathbf{x}, \mathbf{y}) \propto \theta_1^{mb-a}(1-\theta_1)^{-mb-a}\left[\sum_{i=1}^{n}\log(1+x_i) + \frac{\theta_1}{1-\theta_1}\sum_{i=1}^{m}\log(1+y_y)\right]^{-b(n+m)} \quad (3.3)$$

Let $\delta = \frac{\theta_1}{1-\theta_1}$. Then we obtain the posterior

$$\pi(\delta|\mathbf{x}, \mathbf{y}) \propto \delta^{mb-a}(1+\delta)^{2a-2}\left[1+\delta\frac{\sum_{i=1}^{m}\log(1+y_y)}{\sum_{i=1}^{n}\log(1+x_i)}\right]^{-b(n+m)}. \quad (3.4)$$

Thus the posterior (3.4) is proper if $bm - a + 1 > 0$ and $bn - a + 1 > 0$. This completes the proof. □

Theorem 3.2 Under the prior (3.1), the marginal posterior density of θ_1 is given by

$$\pi(\theta_1|\mathbf{x}, \mathbf{y}) \propto \frac{\theta_1^{mb-a}}{(1-\theta_1)^{mb+a}}\left[\sum_{i=1}^{n}\log(1+x_i) + \frac{\theta_1}{1-\theta_1}\sum_{i=1}^{m}\log(1+y_y)\right]^{-b(n+m)}. \quad (3.5)$$

Note that the marginal density of θ_1 requires an one dimensional integration. Therefore we have the marginal posterior density of θ_1, and so it is easy to compute the marginal moment of θ_1 which is a Bayes estimator of θ_1 under the squared error loss function.

4. Numerical studies

We evaluate the frequentist coverage probability by investigating the credible interval of the marginal posterior density of θ_1 under the reference prior given in Section 3 for several configurations (η, β) and (n, m). That is to say, the frequentist coverage of a $(1-\alpha)$th posterior quantile should be close to $1-\alpha$. This is done numerically. Table 4.1 gives numerical values of the frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for the proposed prior. The computation of these numerical values is based on the following algorithm for any fixed (η, β) and any prespecified probability value α. Here α is 0.05 (0.95). Let $\theta_1^{\pi}(\alpha|\mathbf{X}, \mathbf{Y})$ be the αth posterior quantile of θ_1 given $\mathbf{X}$ and $\mathbf{Y}$. That is,

$F(\theta_1^\pi(\alpha|\mathbf{X},\mathbf{Y})|\mathbf{X},\mathbf{Y}) = \alpha$, where $F(\cdot|\mathbf{X},\mathbf{Y})$ is the marginal posterior distribution of θ_1. Then the frequentist coverage probability of this one sided credible interval of θ_1 is

$$P_{(\theta_1,\theta_2)}(\alpha;\theta_1) = P_{(\theta_1,\theta_2)}(0 < \theta_1 \leq \theta_1^\pi(\alpha|\mathbf{X},\mathbf{Y})). \tag{4.1}$$

The estimated $P_{(\theta_1,\theta_2)}(\alpha;\theta_1)$ when $\alpha = 0.05(0.95)$ is shown in Table 4.1. In particular, for fixed n, m and (η, β), we take $10,000$ independent random samples of $\mathbf{X} = (X_1, \cdots, X_n)$ and $\mathbf{Y} = (Y_1, \cdots, Y_m)$ from the Pareto distributions, respectively.

In Table 4.1, we can observe that the proposed prior meets very well the target coverage probabilities even for the small sample sizes. Also note that the results of table are not much sensitive to the change of the values of (θ_1, η).

Example. This example taken from Rezaei, et al. (2010), and the data has been generated using $n = m = 20$, $\eta = 1.0$ and $\beta = 1.4$ with common scale parameter $\lambda = 1.0$. The X values are 0.0024, 1.0863, 19.3212, 6.5356, 6.1724, 4.9743, 0.0648, 1.7140, 0.4599, 1.8520, 0.1250, 0.0185, 10.4438, 0.3367, 0.0088, 0.9618, 1.8178, 0.7337, 0.0296, 0.1922 and the corresponding Y values are 0.0359, 0.7108, 2.0499, 4.7506, 2.4195, 5.0290, 1.4026, 0.2223, 4.9126, 0.0538, 0.5266, 0.7632, 0.7383, 1.3550, 0.0206, 3.3423, 0.5894, 1.1491, 0.6478, 0.0824.

Table 4.1 Frequentist coverage probability of 0.05 (0.95) posterior quantiles of θ_1

			η		
θ_1	n	m	1.0	10.0	100.0
0.1	3	3	0.050(0.955)	0.051(0.951)	0.051(0.953)
		5	0.053(0.948)	0.049(0.950)	0.051(0.950)
		10	0.052(0.950)	0.051(0.951)	0.053(0.949)
	5	3	0.049(0.949)	0.052(0.952)	0.051(0.948)
		5	0.048(0.953)	0.054(0.950)	0.053(0.950)
		10	0.050(0.952)	0.050(0.951)	0.049(0.948)
	10	3	0.048(0.952)	0.050(0.951)	0.052(0.953)
		5	0.049(0.952)	0.051(0.949)	0.047(0.950)
		10	0.049(0.946)	0.054(0.948)	0.051(0.949)
0.3	3	3	0.048(0.951)	0.049(0.951)	0.049(0.949)
		5	0.051(0.955)	0.053(0.954)	0.052(0.950)
		10	0.044(0.951)	0.048(0.949)	0.052(0.945)
	5	3	0.052(0.950)	0.048(0.949)	0.047(0.951)
		5	0.051(0.946)	0.049(0.950)	0.050(0.950)
		10	0.047(0.953)	0.050(0.952)	0.052(0.950)
	10	3	0.051(0.949)	0.052(0.948)	0.051(0.954)
		5	0.050(0.952)	0.050(0.950)	0.049(0.951)
		10	0.051(0.953)	0.050(0.952)	0.050(0.948)
0.5	3	3	0.045(0.950)	0.050(0.951)	0.052(0.950)
		5	0.050(0.951)	0.048(0.953)	0.050(0.953)
		10	0.045(0.951)	0.049(0.951)	0.054(0.949)
	5	3	0.050(0.949)	0.048(0.947)	0.051(0.951)
		5	0.046(0.952)	0.053(0.946)	0.053(0.951)
		10	0.053(0.948)	0.048(0.950)	0.049(0.950)
	10	3	0.051(0.947)	0.047(0.951)	0.050(0.948)
		5	0.052(0.949)	0.051(0.953)	0.052(0.951)
		10	0.053(0.951)	0.047(0.951)	0.047(0.950)
0.7	3	3	0.047(0.951)	0.050(0.952)	0.050(0.948)
		5	0.051(0.948)	0.047(0.949)	0.055(0.955)
		10	0.052(0.951)	0.046(0.951)	0.050(0.951)
	5	3	0.051(0.948)	0.048(0.949)	0.047(0.947)
		5	0.049(0.948)	0.050(0.951)	0.047(0.951)
		10	0.050(0.946)	0.048(0.954)	0.052(0.949)
	10	3	0.055(0.952)	0.053(0.951)	0.047(0.947)
		5	0.052(0.946)	0.050(0.948)	0.050(0.953)
		10	0.048(0.951)	0.052(0.949)	0.050(0.957)

Table 4.1 (Continue) Frequentist coverage probability of 0.05 (0.95) posterior quantiles of θ_1

			η		
θ_1	n	m	1.0	10.0	100.0
0.9	3	3	0.053(0.951)	0.055(0.951)	0.047(0.950)
		5	0.045(0.948)	0.049(0.947)	0.051(0.951)
		10	0.052(0.953)	0.047(0.949)	0.051(0.946)
	5	3	0.053(0.952)	0.048(0.952)	0.048(0.948)
		5	0.055(0.948)	0.049(0.947)	0.052(0.952)
		10	0.049(0.948)	0.047(0.954)	0.052(0.949)
	10	3	0.051(0.948)	0.050(0.950)	0.052(0.951)
		5	0.048(0.951)	0.048(0.950)	0.048(0.952)
		10	0.049(0.949)	0.051(0.949)	0.047(0.950)

From the results of Rezaei, et al. (2010), the maximum likelihood estimate (MLE) and estimate of the percentile bootstrap method of θ_1 are 0.5353 and 0.5361, respectively, and also the 95% confidence intervals of θ_1 based on the MLE, the percentile bootstrap method and the bootstrap-t method are (0.381, 0.684), (0.380, 0.691) and (0.359, 0.702), respectively.

Bayes estimate and the 95% credible interval based on the reference prior are 0.5344 and (0.381, 0.684), respectively. The Bayes estimate based on the reference prior, the MLE and estimate by the bootstrap method give almost same results. And the confidence intervals based on the MLE, the percentile bootstrap method and the reference prior give the similar results but the confidence interval based on the MLE and the Bayesian credible interval are shorter than the confidence intervals based on the bootstrap methods.

5. Concluding remarks

In the Pareto models, we have found the second order matching prior and the reference priors for the stress-strength reliability. We revealed that the second order matching prior is not a HPD matching prior and is not a CDF matching prior, and also does not match the alternative coverage probabilities up to the second order. It turns out that the reference prior and Jeffreys' prior are the second order matching prior. As illustrated in our numerical study, the reference prior meets very well the target coverage probabilities. Thus we recommend the use of the reference prior for Bayesian inference of the stress-strength reliability in two independent Pareto distributions.

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