압축센싱 기반의 무선통신 시스템

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I. Abstract

As a result of quickly growing data, a digital transmission system is required to deal with the challenge of acquiring signals at a very high sampling rate. Fortunately, the CS (Compressed Sensing or Compressive Sensing) theory, a new concept based on theoretical results of signal reconstruction, can be employed to exploit the sparsity of the received signals. Then, they can be adequately reconstructed from a set of their random projections, leading to dramatic reduction in the sampling rate and in the use of ADC (Analog-to-Digital Converter) resources. The goal of this article is provide an overview of the basic CS theory and to survey some important compressed sensing applications in wireless communications.

II. Introduction

A number of signal processing algorithms have recently been developed for capturing and extracting information from continuous signals. The Nyquist-Shannon sampling theorem provides a bridge between continuous signals and discrete signals. The theorem shows that a bandlimited analog signal that has been sampled can be perfectly reconstructed from an infinite sequence of samples if the sampling rate exceeds twice the maximum frequency present in the signal; this is called the Nyquist rate. Since signal processing moved from analog to digital domain, digitalization has made digital systems more robust, flexible, and cheaper than the analog systems.

Nowadays, as a result of the quickly growing data, the digital transmission system is required to deal with the challenge of acquiring signals at a very high sampling rate. This fact is usually ignored by signal processing algorithms which assume that it can be solved by quick development of hardware technologies. In reality, however, this requires extremely expensive and complex ADCs (Analog-to-Digital Converters). For instance, many coherent UWB (Ultra Wide Band) systems need to acquire a UWB pulse by using an ADC with several GHz sampling rate or even higher on the receiver side.

Fortunately, the CS (Compressed Sensing or Compressive Sensing) theory, a new concept based on the theoretical results of signal reconstruction, can be employed to exploit the "sparsity" of the received signals. They can then be adequately reconstructed from a set of their random projections. This leads to a dramatic reduction in the sampling rate and in the use of ADC resources [1]. The fundamental difference between the CS and the classical sampling is a manner in with the two frameworks deal with signal recovery (i.e., the problem of recovering the signal from the measurements). In the Shannon-Nyquist framework, signal recovery is achieved through interpolation, a linear process that requires little computation and has a simple interpretation. However, CS signal recovery is achieved by using nonlinear and relatively expensive optimization-based or iterative algorithms [2-4]. Most of the CS literature has focused on improving speed and accuracy of this process. CS is considered promising and is drawing a lot of attention from scientists and engineers in different disciplines.

The goal of this article is to provide an overview of the basic CS theory and to survey some important compressed sensing applications in wireless communications. The next section reviews the background of the CS theory. Section III introduces some CS applications that have the signals sparse in the time domain. In Section IV, CS applications with the signals sparse in the frequency domain are presented. The last section discusses the current research of CS applications and the future approaches.

Ⅲ. Compressed Sensing

Referring to [1,5,6], we briefly describe the CS theory in this section. Consider a real-valued, finite-length, one-dimensional, and discrete-time signal $\boldsymbol{x}$ that can be viewed as an $N\times 1$ column vector in R^N. Any signal in R^N can be represented in $N\times 1$ vectors $\{\psi_n\}_{n=1}^{N}$ called a set of basis vectors. So, a signal $\boldsymbol{x}$ can be formulated as

$$\boldsymbol{x}=\sum_{n=1}^{N}\theta_n\psi_n \quad \text{or} \quad \boldsymbol{x}=\boldsymbol{\Psi\theta}, \qquad (1)$$

where $\boldsymbol{\Psi}$ is the $N\times N$ basis matrix with the vectors $\{\psi_n\}_{n=1}^{N}$ as columns and $\boldsymbol{\theta}$ is the $N\times 1$ column vector of the weighting coefficients. Thus, $\boldsymbol{x}$ is the representation of the signal in the time domain, while $\boldsymbol{\theta}$ is the representation in the $\boldsymbol{\Psi}$ domain.

The signal $\boldsymbol{x}$ is said to be K-sparse if it is a linear combination of only K $(\leq N)$ basis vectors; this means that only K elements of the coefficients $\boldsymbol{\theta}$ are nonzero and the remaining $N-K$ elements are zero. If $\boldsymbol{x}$ has a few large K coefficients and many small coefficients, then it is called compressible.

The CS method measures the signal by computing M inner products between $\boldsymbol{x}$ and a collection of vectors $\{\phi_m\}_{m=1}^{M}$, as in $y_m=\langle \boldsymbol{x},\phi_m\rangle$. Arranging the measurements in an $M\times 1$ vector $\boldsymbol{y}$ and the measurement vectors ϕ_m^T as rows in an $M\times N$ matrix $\boldsymbol{\Phi}$, $\boldsymbol{y}$ can be expressed by projecting $\boldsymbol{x}$ on the matrix $\boldsymbol{\Phi}$ as

$$\boldsymbol{y}=\boldsymbol{\Phi x}=\boldsymbol{\Phi\Psi\theta}=\boldsymbol{\Theta\theta}, \qquad (2)$$

where $\boldsymbol{\Theta}=\boldsymbol{\Phi\Psi}$ is the $M\times N$ matrix. The rate M/N is then called the *compressive rate*, which expresses how much the sampling rate can be reduced.

When the sparsity basis $\boldsymbol{\Psi}$ and the matrix $\boldsymbol{\Phi}$ are incoherent, the CS theory indicates that the original sparse coefficients can be well-recovered from the compressive measurement $\boldsymbol{y}$ via minimum l_1-norm reconstruction. This is known as solving a convex optimization based on the ℓ_1-norm

$$\hat{\boldsymbol{\theta}}=\arg\min\|\boldsymbol{\theta}'\|_1 \ \text{ such that } \ \boldsymbol{\Theta\theta}'=\boldsymbol{y}, \qquad (3)$$

where $\boldsymbol{\theta}'$ is the $N\times 1$ vector that satisfies $\boldsymbol{\Theta\theta}'=\boldsymbol{y}$ and $\hat{\boldsymbol{\theta}}$ is the recovered K-sparse coefficients.

As presented above, sparse signals can be recovered from a few measurements; however, the CS has to deal with both nearly sparse signals and noise. Because signals are usually not sparse but approximately sparse, any real data will be affected by a small amount of noise. Cand s *et al.* introduced the concept of RIP (Restricted Isometry Property) [7] and used it to prove many theorems in the field of CS. The RIP makes the CS robust to nearly sparse signals. The RIP definition states that for each integer K, the isometry constant δ_k of a matrix $\boldsymbol{\Theta}$ is defined as the smallest number by which

$$(1-\delta_k)\|\boldsymbol{\theta}\|_2^2 \leq \|\boldsymbol{\Theta\theta}\|_2^2 \leq (1+\delta_k)\|\boldsymbol{\theta}\|_2^2 \qquad (4)$$

holds for all K-sparse vectors $\boldsymbol{\theta}$. Here, $\|.\|_2$ is the ℓ_2-norm of a vector. It is useful that K-sparse vectors $\boldsymbol{\theta}$ cannot be in the null space of $\boldsymbol{\Theta}$; otherwise, it cannot be reconstructed. Additionally, the problems of noise effect are considered by the recovery algorithms, such as a combinatorial optimization algorithm proposed by Haupt and Nowak [8].

Ⅳ. Sparse Signals in the Time Domain

This section focuses on the CS-based applications for signals that are sparse in the time domain. That is, a signal is a linear combination of the time-shifted basis vectors.

1. CS-based AIC

While Paredes *et al.* have proposed a compressed sensing based AIC (Analog-to-Informaion Converter) for UWB IR (Impulse Radio) [9], Meng *et al.* proposed an AIC for 60 GHz UWB [10]. Both approaches rely on the fact that transmitting an ultra-short pulse through a multipath channel leads to a received signal that can be approximated by a linear combination of a few atoms from a pre-defined dictionary. In other words, the received signal is sparse.

For a general UWB multipath channel model, we consider the typical tap-delay-line model described in [11] and represent the CIR (Channel Impulse Response) $h(t)$ as

$$h(t) = \sum_{\ell=1}^{L} \alpha_\ell \delta(t - \tau_\ell), \tag{5}$$

where L represents the number of resolvable paths, α_ℓ denotes the gain of the ℓ-path, and τ_ℓ is the time delay of the ℓ-path.

Without loss of generality, we consider a simple communications model of transmitting a pulse $w(t)$ through the multipath channel $h(t)$. The received UWB signal $r(t)$ without noise or composite pulse-multipath channel $h_c(t)$ can be modeled as

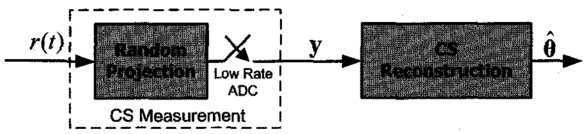
$$r(t) = h_c(t) = w(t) * h(t) = \sum_{\ell=1}^{L} \alpha_\ell w(t - \tau_\ell). \tag{6}$$

Obviously, the received UWB signal $r(t)$ is the form of scaled and delayed versions of the pulse $w(t)$ as shown in ⟨Fig. 1⟩. Then, the sparsity basis $\boldsymbol{\Psi}$ can be generated based on the following set.

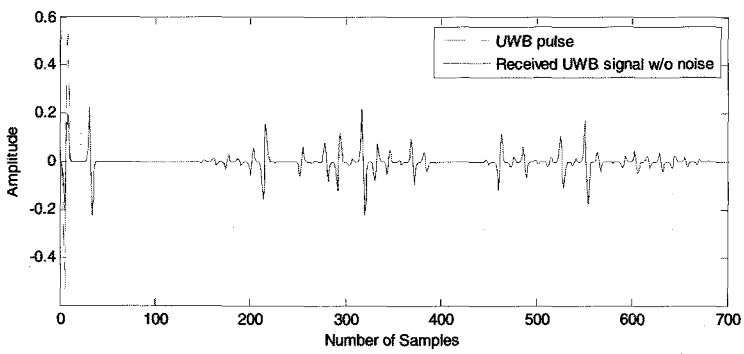
$$\boldsymbol{D} = \{d_n(t)\}_{n=1}^{N} = \{w(t-(n-1) \cdot \Delta)\}_{n=1}^{N}, \tag{7}$$

where an atom $d_n(t)$ is a delayed version of a UWB received pulse $w(t)$ with time resolution Δ.

In this approach, CS-based AIC architecture can be depicted as ⟨Fig. 2⟩. There are two main blocks: CS measurement (including random projection and low-rate ADC) and CS reconstruction. Since the random projection can be performed in the analog



⟨Fig. 2⟩ CS-based AIC for the signals sparse in time domain



⟨Fig. 1⟩ A received UWB signal without noise in form of scaled and delayed versions of a UWB pulse domain by using random filters [12] or by a PN (Pseudo Noise) sequence generator plus integrator [10], the received UWB signal feeding the random projection block can be an analog signal. Next, the low-rate ADC samples the randomly-projected signal at a sub-Nyquist rate. In the CS reconstruction block, the samples of the randomly-projected signal are assumed to be processed by an efficient CS recovery algorithm. The recovery algorithm outputs a sparse coefficient vector $\hat{\boldsymbol{\theta}}$ for regenerating the transmitted signal by using (1).

As a result of applying the CS theory, the sampling rate is reduced significantly. In [9], since the authors show that CS reconstructing operates with implicit de-noising, the compressive rate M/N is reduced to 0.24 without degrading the BER (Bit Error Rate) performance. Meanwhile, the compressive rate can be reduced to 0.04 in 60 GHz UWB systems, as seen in [10].

2. Channel Estimation

In the previous sub-section, CS research only focuses on reducing the sampling rate. However, CS can be extended to a much broader range of statistical inference tasks, and is particularly well-suited for applications in wireless communications such as channel estimation [13,14].

Let us consider the composite pulse-multipath channel $h_c(t)$, given by (6). The number of multipath components in the UWB channel may be large, but it can be limited to estimate only L_c most significant paths that compose the UWB CIR. Here, assume that the composite-multipath channel is L_c-sparse and consider the UWB signal measurement and reconstruction, as described in ⟨Fig. 2⟩. The output of the CS reconstruction block is the sparse coefficient vector $\hat{\boldsymbol{\theta}}=[\hat{\theta}_1,\cdots,\hat{\theta}_N]^T$ which contains the multipath gains. From this vector, we can determine the estimated gain $\hat{\alpha}_\ell$ and the estimated delay $\hat{\tau}_\ell$ of the ℓ -path of the multipath CIR by locating the values and indices of the non-zero elements of the $\hat{\boldsymbol{\theta}}$.

Since path gains and delays of the channel $h(t)$ are estimated at the receiver, a CS rake receiver was proposed for a noisy communication environment [13]. Furthermore, based on the sparse coefficient vector $\hat{\boldsymbol{\theta}}$, we have extended to propose a ToA (Time of Arrival) estimation scheme which finds the threshold between noise and signal for detecting the first path arrival of a received signal [15]. It exploits the sampling reduction of CS benefit and provides high accuracy performance.

3. Compressed Detection

Wang *et al.* recently proposed a GLRT (Generalized Likelihood Radio Test) detector for IR UWB based *only* on CS measurement [16,17]. The proposed receiver does not require a high-rate ADC and complex implementation for CS recovery algorithms.

Let us consider the IR UWB system where binary information symbols are conveyed by a stream of ultra-short pulses $w(t)$. In particular, N_f antipodal pulses $w(t)$ are repeated to transmit one binary symbol. The transmitted burst includes N_p pilot symbols that are not modulated and N_s antipodal modulated symbols. Assume that the multipath CIR $h(t)$ is time-invariant during the duration of each stream of bits. Then, the CS measurement for the pilot symbols is expressed as

$$\boldsymbol{y}_p[n]=\boldsymbol{\Phi x}_0+\boldsymbol{\Phi z}_p[n],\ \ n=0,\cdots,N_pN_f-1, \quad (8)$$

where $\boldsymbol{x}_0$ is the digitalized representation of the noise-free received signal and $\boldsymbol{z}_p[n]$ is the digitalized representation of the noise within a projection interval.

Similarly, the j-th data modulated symbol is measured as

$$\boldsymbol{y}_{i|j}[n]=b_j\boldsymbol{\Phi x}_0+\boldsymbol{\Phi z}_s[n],\ \ n=0,\cdots,N_f-1,\ \ (j=0,\cdots,N_s-1) \quad (9)$$

where $b_j \in \{-1,+1\}$ is the binary data bit, and $\boldsymbol{z}_s[n]$ is noise vector within a data-symbol interval.

Assuming that the data bits are sent with equal *a priori* probability, there are two hypotheses for the compressed detection of the j-th symbol b_j:

$$H_0: \; \boldsymbol{y}_s[n] = -\boldsymbol{\Phi x}_0 + \boldsymbol{\Phi z}_s[n], \; (b_j = -1)$$
$$H_1: \; \boldsymbol{y}_s[n] = +\boldsymbol{\Phi x}_0 + \boldsymbol{\Phi z}_s[n], \; (b_j = +1) \quad (10)$$

To distinguish between the two hypotheses, the likelihood ratio is given as

$$likelihood(\boldsymbol{y}_s) = \frac{pdf(\boldsymbol{y}_s;\widehat{\boldsymbol{\Phi x}}_0, H_1)}{pdf(\boldsymbol{y}_s;\widehat{\boldsymbol{\Phi x}}_0, H_0)} \underset{H_1}{\overset{H_0}{\lessgtr}} 1, \quad (11)$$

where $\widehat{\boldsymbol{\Phi x}_0}$ is the estimation of $\boldsymbol{\Phi x}_0$, which is obtained by the ML (Maximum Likelihood) estimation of $\boldsymbol{\Phi x}_0$ as

$$\widehat{\boldsymbol{\Phi x}}_0 = \frac{1}{N_p N_f} \sum_{n=0}^{N_p N_f - 1} \boldsymbol{y}_p[n] = \boldsymbol{\Phi x}_0 + \boldsymbol{\Phi} \hat{\boldsymbol{z}}_p, \quad (12)$$

where $\hat{\boldsymbol{z}}_p = \frac{1}{N_p N_f} \sum_{n=0}^{N_p N_f - 1} \boldsymbol{z}_p[n]$. After some manipulations, a test statistic is obtained as follows.

$$test(\boldsymbol{y}_s) = (\boldsymbol{\Phi x} + \boldsymbol{\Phi} \bar{\boldsymbol{z}}_p)^T (N_0 \boldsymbol{B\Phi\Phi}^T)^{-1} (b_j \boldsymbol{\Phi x} + \boldsymbol{\Phi} \bar{\boldsymbol{z}}_s)^T \quad (13)$$

where $\bar{\boldsymbol{z}}_s = \frac{1}{N_f} \sum_{n=0}^{N_f - 1} \boldsymbol{z}_s[n]$.

Although the compressed detection can provide superior BER performance, it requires a large number of measurements [16,17]. Hence, Wang *et al.* proposed an improved compressive detection method called *subspace compressive detection*, which uses the *subspace measurement matrix* tailored to the signal structure to achieve fewer measurements as well as better detection performance [17,18]. In this approach, the subspace measurement matrix $\overline{\boldsymbol{\Phi}}$ is proposed as

$$\overline{\boldsymbol{\Phi}} = \boldsymbol{G}_{iid} (\boldsymbol{\Psi}^T \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi}^T, \quad (14)$$

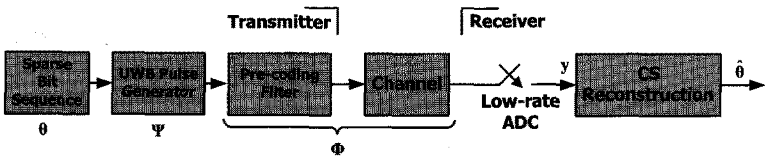
where $\boldsymbol{G}_{iid}$ is an $M \times N$ identically distributed random matrix and the basis $\boldsymbol{\Psi}$ is assumed to be known. This improvement leads to a simplified receiver design and provides a competitive performance.

4. Pre-Filtering Applications Based on CS

In the previous sub-sections, CS measurement and CS reconstruction are both implemented in the receivers. This leads to high circuitry complexity in the receivers. To reduce the complexity on the receiver side, P. Zhang *et al.* [19] have proposed the pre-filtering CS-based UWB systems that can move the complexity of CS measurement to the transmitters. The authors proposed the following system architecture for a pre-filtering CS-based UWB system in ⟨Fig. 3⟩. A UWB signal is transmitted by feeding a sparse bit sequence through a UWB pulse generator and a pre-coding filter. After going through the channel, the received signal is directly sampled using a low-rate ADC and then processed by a recovery algorithm. The measurement matrix $\boldsymbol{\Phi}$ is now a projecting matrix consisting of the pre-coding FIR (Finite Impulse Response) filter and the CIR. That is, the channel itself is a part of the measurement matrix in the CS measurement phase.

Let us consider K-PPM (Pulse Position modulation) scheme which is used to modulate the sparse bit sequence. Each PPM symbol is said to be K-sparse signal, because there are N positions and only K pulses in each symbol. The output of the UWB pulse generator can be written as

$$x(t) = \sum_{n=0}^{N-1} \theta_n d_n(t) \text{ or } \boldsymbol{x} = \boldsymbol{\Psi\theta}, \quad (15)$$



⟨Fig. 3⟩ A pre-filtering CS-based system where $d_n(t) = w(t - nT_p)$ with $w(t)$ as the UWB pulse and T_p as the UWB pulse duration, and $\boldsymbol{D} = [d_0(t), \cdots, d_{N-1}(t)]$. Note that this application focuses on exploiting the sparsity of the transmitted signal and not the sparsity of the multipath CIR. We can model the pre-coding filter and channel as a FIR filter $F(t)$ by combining the FIR filter $f(t)$ of the pre-coding filter and the multipath CIR $h(t)$ as

$$F(t) = f(t) * h(t). \quad (16)$$

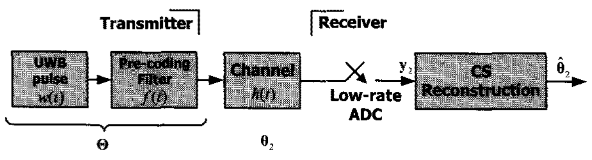
Then the received signal $y(t) = F(t) * x(t)$ is down sampled by a low rate ADC and formulated as

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\theta}, \quad (17)$$

where the measurement matrix $\boldsymbol{\Phi}$ is the algebraic representation of the FIR filter $F(t)$. Now, the communication problem becomes a CS problem as in (2). Then, the measurements $\boldsymbol{y}$ is processed as usual to get the estimated coefficient vector $\hat{\boldsymbol{\theta}}$.

However, the multipath CIR $h(t)$ must be known *a priori* at the receiver in this system. Thus, it requires a scheme that allows this system to estimate $h(t)$ without adding any circuitry complexity. The authors and H. Yu *et al.* both proposed similar approaches for estimating the channel in the pre-filtering system as in ⟨Fig. 4⟩ [14,19]. Similarly to the channel estimation scheme in subsection III.2, by transmitting the probing UWB pulse and basing on the sparsity of the multipath CIR $h(t)$, we can estimate the path gains α_ℓ and the path delays τ_ℓ, which are two important parameters of the CIR $h(t)$. That is, it takes the CS measurement on $h(t)$ by using the probing pulse and the precoding filter $w(t) * f(t)$. Then the CS problem is formulated as

$$\boldsymbol{y}_2 = w(t) * f(t) * h(t) = \boldsymbol{\Theta}_2 \boldsymbol{\theta}_2 \quad (18)$$



⟨Fig. 4⟩ A block diagram of channel estimation

where $\boldsymbol{\Theta}_2$ is the matrix derived from $w(t) * f(t)$ and $\boldsymbol{\theta}_2$ is the vector whose elements are the path gains α_ℓ of the multipath CIR $h(t)$. The channel estimation problem then becomes the CS reconstruction problem which can be solved by a CS recovery algorithm.

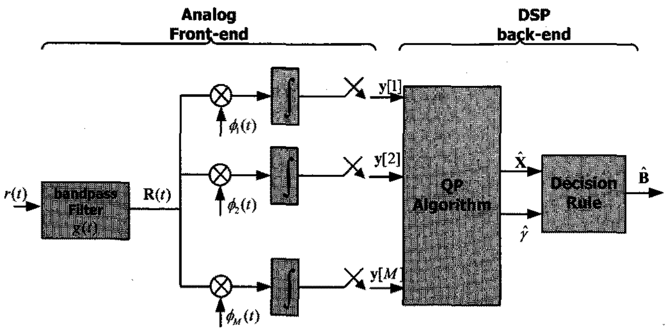
5. CS-based bursty Communications

Oka *et al.* proposed a compressed sensing receiver for IR UWB communication in WSNs (Wireless Sensor Networks) that are characterized by bursty transmission and power constraints [20]. This flexible and robust receiver performs a joint decoding of timing and amplitude information. The proposed receiver bypasses the requirement of high-rate ADC. Instead, it uses an analog front-end consisting of a bank of correlators with test functions, a low-rate ADC, and a DSP back-end based on a computationally efficient QP (Quadratic Program). The architecture of the CS-based bursty receiver is depicted in ⟨Fig. 5⟩.

After going through the bandpass-filter $g(t)$ with center frequency f_c and bandwidth Ω, the received signal $R(t)$ is formulated as

$$R(t) = \sum_{k=0}^{K-1} B^k h_{total}(t - kT_{baund} - v) + W(t) \quad (19)$$

where the bit B^k is an element of the burst with N bits $\boldsymbol{B}$, $h_{total}(t)$ is the total impulse response which is the convolution of the transmitted pulse $w(t)$, the channel $h(t)$, and the low-pass filter response $g(t)$. T_{baund} is the interval between two consecutive bits, v is the ToA of the received signal with the maximum value assumed to be $v_{\max}$, and $W(t)$ is the band-limited zero-mean additive Gaussian noise. $R(t)$ is then fed into a bank of M parallel analog correlators, followed by M integrators. The ensemble of test functions is denoted by $\{\phi_m(t)\}_{m=1}^{M}$. The ensemble of test functions can be simply the set of square waves with frequencies selected deterministically and uniformly from the signal band $[f_c - \Omega/2, f_c + \Omega/2]$, which is seen to perform as well as the PN ensemble. Then we can write the



⟨Fig. 5⟩ CS-based receiver for bursty communications

measurement in matrix form as follows.

$$y = \Phi R = \Phi H X + \Phi W, \quad (20)$$

where Φ is the $M \times N$ matrix represented for the ensemble of test functions $\{\phi_m(t)\}_{m=1}^{M}$ and the $N \times \Lambda_x$ matrix H is represented for the total CIR $h_{total}(t)$. In addition, the $N \times 1$ vector R, the $\Lambda_x \times 1$ vector X, and the $N \times 1$ vector W are the algebraic representations of the received signal $R(t)$, the burst of bits B, and the bandlimited noise, respectively.

The ML demodulation of B based on the measurement of (20), can be described for solving ℓ_1 –minimization problem as

$$\hat{X} = \arg\min_{X} (Y - \Phi HX)^T (\Phi G G \Phi)^{-1} (Y - \Phi HX), \quad (21)$$

where G is the matrix representation of the filter impulse response and X is a potential solution. Instead of solving the complex ML problem of (21), the authors considered the relaxation of the ML demodulation problem (21) as

$$\begin{aligned} &\widetilde{X}[n] = \widetilde{Z}[n] - \widetilde{Z}[n+N] \\ &\widetilde{Z} = \min f^T z + \frac{1}{2} z^T Q z \\ &\text{such that } z \geq 0, \ [\xi(a,\ell_1,\ell_2), \xi(a,\ell_1,\ell_2)] z = K \end{aligned} \quad (22)$$

where

$Q = \begin{bmatrix} H^T \Phi^T (\Phi G G^T \Phi^T)^{-1} \Phi H & -H^T \Phi^T (\Phi G G^T \Phi^T)^{-1} \Phi H \\ -H^T \Phi^T (\Phi G G^T \Phi^T)^{-1} \Phi H & H^T \Phi^T (\Phi G G^T \Phi^T)^{-1} \Phi H \end{bmatrix}$, $f = [-y^T (\Phi G G^T \Phi^T)^{-1} \Phi H, y^T (\Phi G G^T \Phi^T)^{-1} \Phi H]$, $\xi(a,\ell_1,\ell_2)[n] = \begin{cases} 1.0, & n \in [a+\ell_1, a+\ell_2] \\ \mho, & otherwise \end{cases}$ with $\mho$ is a large number, and $\widetilde{Z} = [\max(\widetilde{X},0), abs(\min(\widetilde{X},0))]$. There are several efficient large-scale methods to solve the QP problem [21] so that it reduces the complexity of solving the original problem (21).

The QP demodulation is then performed in two stages. In the first stage, the QP problem (22) is solved by inputting $\xi(a=0,\ell_1=0,\ell_2=\Gamma)$ corresponding to the full ToA uncertainty $\Gamma = v_{\max} f_s$ with f_s as the virtual Nyquist sampling frequency. The result of this stage $\widetilde{X}_1$ is used to estimate the ToA $\tilde{v}$. Denote $\tilde{\gamma}$ is the quantized version of $\tilde{v}$, that is, $\tilde{\gamma} = round(\tilde{v} f_s)$. Thus, $\tilde{\gamma}$ maximizes the correlation of the delay version of $\widetilde{X}_1$ and the template $\xi(0,0,0)$

$$\tilde{\gamma} = \arg\max_{n_0 = 0,\cdots,\Gamma} \sum_n |\widetilde{X}_1[n-n_0]| \xi(0,0,0)[n]. \quad (23)$$

The second stage solves the QP problem again by using $\xi(a=\tilde{\gamma},\ell_1=0,\ell_2=0)$ to obtain the solution $\widetilde{X}_2$. A simple rule was proposed to demodulate the information bits B.

$$\widehat{B}^k = sign(\widetilde{X}_2[\tilde{\gamma} + kN_{baund}]), \ k = 0,\cdots,K-1. \quad (24)$$

The authors presented an innovative compressed sensing receiver with the ability to detect a burst of bits, while other compressed detection schemes only detect each bit or symbol for each process of reconstructing signals by CS recovery algorithms. Moreover, it can estimate the ToA of a received signal with high accuracy and reduce the complexity of CS reconstruction implementation in receivers due to relaxing the CS problem to the QP problem which can be solved by some efficient algorithms. However, the authors assumed that the CIR $h(t)$ is known in the receiver; although it can be estimated by some channel estimation schemes, the inaccuracy of the estimated channel $h(t)$ may degrade the BER performance.

Ⅴ. Sparse Signals in the Frequency Domain

1. CS-based AIC

In many applications, signals are sparse in the frequency domain. That is, the number of significant frequency components is often much smaller than the bandlimit allows. So, this sparsity can be exploited to design a CS-based AIC for reducing high sampling rate [22-24].

Kirolos *et al.* proposed a type of sampling system called a random demodulator [22] that can be used to acquire signals. The block diagram for the random demodulator is illustrated in ⟨Fig. 6⟩.

As seen in ⟨Fig. 6⟩, there are three main blocks: demodulation, filtering and uniform sampling. The signal is first modulated by a high-rate PN sequence of ± 1, called the chipping sequence $p_c(t)$. The purpose of the demodulation is to spread the frequency content of the signal into the entire spectrum so that it leaves a signature that can be detected by a low-rate ADC. To understand the details of how the random demodulator works, we can study its properties by using tools from matrix analysis and functional analysis [23].

Assume that the signal $x(t)$ is bandlimted, periodic and K-sparse in frequency domain. Consider the following mathematical model for sparse signal in the frequency domain.

$$f(t) = \sum_{\omega \in \Omega} a_\omega e^{-2\pi i \omega t} \quad \text{for } t \in [0,1), \tag{25}$$

where Ω is a set of K integer-valued frequencies with $\Omega \subset \{0, \pm 1, \pm 2, \cdots, \pm(\omega_{\max}/2-1), \omega_{\max}/2\}$. Moreover, $\omega_{\max}/2$ is a positive integer that exceeds the highest frequency of the continuous-time signal x, and a_ω is a complex-valued amplitude. Note that we are only interested in the case of $K \ll \omega_{\max}/2$. Let us define the discrete representation of input signal $\boldsymbol{s}$ in the frequency domain as follows.

$$\boldsymbol{f} = \boldsymbol{F}\boldsymbol{s}, \tag{26}$$

where $\boldsymbol{F}$ is $w_{\max} \times w_{\max}$ permuted DFT (Discrete Fourier Transform) matrix.

Let $\epsilon_0, \cdots, \epsilon_{w_{\max}-1}$ be the PN chipping sequence. The modulation of signal by PN chipping sequence is now the map

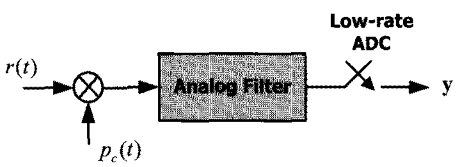
$$\boldsymbol{f} \rightarrow \boldsymbol{D}_c \boldsymbol{f}, \tag{27}$$

where $\boldsymbol{D}_c = \begin{bmatrix} \epsilon_0 & & 0 \\ & \epsilon_1 & \\ & & \ddots & \\ 0 & & & \epsilon_{w_{\max}-1} \end{bmatrix}$. Next, considering the action of the low-pass filter and the sampler, assume that the sampling rate is f_s and each sample is the sum of $\omega_{\max}/f_s$ consecutive entries of the demodulated signal. Thus, we consider a matrix $\boldsymbol{H}$ in which each row has $f_s \times w_{\max}$ consecutive unit elements as the following example:

$$\boldsymbol{H} = \begin{bmatrix} 1\,1 \cdots 1 & & & 0 \\ & 1\,1 \cdots 1 & & \\ & & \ddots & \\ 0 & & & 1\,1 \cdots 1 \end{bmatrix}. \tag{28}$$

To the end, the action of random demodulator is formulated as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{D}_c\boldsymbol{F}\boldsymbol{s} = \boldsymbol{\Phi}\boldsymbol{F}\boldsymbol{s}, \tag{29}$$



⟨Fig. 6⟩ A block diagram of the random demodulator where Φ is considered as the measurement matrix in the CS theory. The measurement matrix has the RIP [23] so that the samples contain enough information to approximate general signals extremely well by CS recovery algorithms.

2. Interference Cancellation

There are two types of classical interference cancellations: analog cancellation and digital cancellation. Analog cancellation assumes that the interfering signal has enough known parameters that it can be identified and removed in the analog domain before signal acquisition. It is implied that the interference content is not interesting to the acquisition system or its user and can be safely discarded. Digital cancellation assumes that the interference is bandlimited and follows a similar approach after sampling the signal at the Nyquist rate dictated by the combined bandwidth of the signal of interest and the interference. The compressive measurement acquired by CS-based systems usually includes undesired interference. Since CS-based signal acquisition and processing has been shown to be more susceptible to noise and interference than classical methods [25], it seems prudent to eliminate as much noise and interference as possible prior to processing.

Davenport *et al.* have developed an efficient compressive domain filtering algorithm [26] that eliminates signal interference, while preserving the RIP for the set of signals of interest. Thus, the projected measurements retain sufficient information to enable the direct recovery of this signal of interest, or alternatively to enable the use of efficient compressive domain algorithms for further processing.

Suppose that signal $x \in R^N$ consists of two components:

$$x = x_s + x_I, \tag{30}$$

where x_s represents the signal of interest and x_I represents interference. The measurements of both components are acquired simultaneously as

$$y = \Phi(x_s + x_I). \tag{31}$$

The goal is to remove the contribution of interference x_I to the measurement y, while preserving the information on x_s. Assume that the sparsity basis Ψ is the same both for x_s and x_I. Without loss of generality, assume that this sparsity basis is the canonical (identity) basis and x_I belongs to a K-dimensional subspace of R^N having basis $\{e_j\}_{j \in J}$, where e_j denotes the vector of all zeros with a 1 in the j-th position and J denotes a set of indices. Note that if Φ_J denotes the matrix consisting of the columns of Φ indexed by set J, then Φx_I lies in $R(\Phi_J)$. To cancel the interference signals, we need to construct a linear operator P that operates on the measurements y and maps $R(\Phi_J)$ to zero; i.e., the null space of P should be equal to $R(\Phi_J)$. There are a variety of methods for constructing P. However, to ease the computational cost of applying P, the authors have constructed P as follows.

$$P = I - \Phi_J \Phi_J^{\dagger}, \tag{32}$$

where $\Phi_J^{\dagger} \equiv \left(\Phi_J^* \Phi_J\right)^{-1} \Phi_J^*$ is the pseudoinverse. Note that since $\Phi x_I \in R(\Phi_J)$, there exists a vector $\alpha \in R^{K_1}$ such that

$$\begin{aligned} P\Phi x_I &= P\Phi_J \alpha \\ &= (I - \Phi_J(\Phi_J^* \Phi_J)^{-1} \Phi_J^*)\Phi_J \alpha \\ &= \Phi_J \alpha - \Phi_J \alpha = 0 \end{aligned} \tag{33}$$

Thus P eliminates the interference x_I from the samples y.

From (30) and (33), $Py = P\Phi(x_s + x_I) = P\Phi x_s$. The authors have already proved that the matrix $P\Phi$ satisfies a relaxed version of RIP. This means that $P\Phi x_s$ contains sufficient information about x_s and can be reconstructed efficiently by CS recovery algorithms. Davenport *et al.* also proposed three methods of interference cancelations, namely cancel-then-recover, modified recovery and recovery-then-cancel. Their simulation results showed that the cancel-then-recover method performed significantly better than both of the other methods.

Davenport *et al.* proposed a simple and efficient approach, however they require the assumption of known basis $\{e_j\}_{j\in J}$ of the interference signal. At this point, it may not be practical due to the diversity of interference sources. While Wang *et al.* were interested in NBI (Narrowband Interference) mitigation [27], they proposed a method that estimates NBI subspace and cancels the most significant NBI components.

Ⅵ. Discussion and Conclusion

In this article, we presented a survey on CS-based applications in wireless communications. The CS-based applications can be broken down into two types: CS-applications for signals sparse in the time domain and CS-applications for signals sparse in the frequency domain. The survey is based on some recently typical research of applying CS into wireless communication: AIC implementation, channel estimation, compressed detection, pre-filtering filter, bursty communications, and interference mitigation.

Even though CS-based applications already showed promising results, researchers are still dealing with the following challenges: How to model signals well-represented by K vectors of the basis Ψ, how to minimize the number of CS measurements M, and how to efficiently reconstruct signals. For signals sparse in the time domain, they can be modeled as the scaled and delayed versions of the signal waveform but it is still challenging for signals sparse in the frequency domain. This means it requires the prior-known basis Ψ or some other technique to estimate the basis Ψ. There is a trade-off between the number of CS measurements and the accuracy of reconstructing sparse signals. If the number of CS measurements increases, the accuracy of reconstructed signal will be improved and vice versa. Additionally, increasing the number of CS measurements leads to high complexity of implementing the CS measurements and CS recovery algorithms. Therefore, it should be improved by using some technologies such as subspace measurement matrix in [17,18,27].

The CS theory is promising in wireless communications and still challenging to researchers. Thus, a lot of theory and applications of CS will be proposed for CS-based applications in wireless communications.

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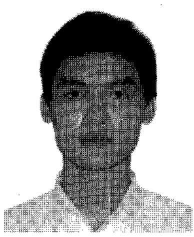

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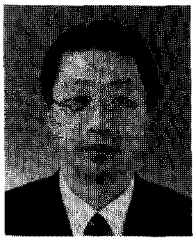

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