International Journal of Reliability and Applications
Vol. 12, No. 2, pp. 117-122, 2011

Optimal replacement strategy under repair warranty with age-dependent minimal repair cost

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Abstract. In this paper, we suggest the optimal replacement policy following the expiration of repair warranty when the cost of minimal repair depends on the age of system. To do so, we first explain the replacement model under repair warranty. And then the optimal replacement policy following the expiration of repair warranty is discussed from the user's point of view. The criterion used to determine the optimality of the replacement model is the expected cost rate per unit time, which is obtained from the expected cycle length and the expected total cost for our replacement model. The numerical examples are given for illustrative purpose.

Key Words: *Expected cost rate per unit time, repair warranty, replacement model*

1. INTRODUCTION

A certain type of warranty by the manufacturer is usually provided to the user at the sale of its system. Usually, two types of warranty policies are widely offered: replacement warranty and repair warranty. Under the replacement warranty, the system which fails during its warranty period is replaced by a new one at a full cost (or at a prorated cost) to the manufacturer. However, under the repair warranty, the system is minimally repaired at each failure during the warranty period. As a result, a number of maintenance models following the expiration of warranty have been proposed and discussed in the literature. Sahin and Polatoglu (1996), Jung and Park (2003), Chen and Chien (2007), Jung, Han and Park (2008) and Chien (2008a) consider the maintenance policies following the expiration of replacement warranty. On the other hand, Yeh, Chen and Lin (2007) deal with the replacement policies under repair warranty.

Although many maintenance policies after warranty are proposed in the literature and its optimality is considered, most of them assume that the cost of minimal repair is constant, not depending on the age of system. However, it is more practical to assume that the minimal repair cost varies with time. Thus we consider the replacement strategy

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following the expiration of warranty when the cost of minimal repair depends on the age of system.

In this paper, we suggest the optimal replacement policy following the expiration of repair warranty when the cost of minimal repair depends on the age of system. If the system fails during the original warranty period, it is minimally repaired by the manufacturer at a partially cost to the user. Furthermore, the manufacturer guarantees a satisfactory service only during the original warranty period. On the other hand, if the system failure occurs after the original warranty period, the user is fully responsible for each failure of the system. And we assume that the minimal repair cost varies with time. At the end of maintenance period, the system is replaced by a new one. As the criterion to determine the optimal maintenance strategy, we adopt the expected cost rate per unit time from the user's perspective. All maintenance costs of the system incurred after the warranty is expired are paid by the user. Given the cost structures during the life cycle of the system, we determine the optimal maintenance period following the expiration of repair warranty.

In Section 2, we describe the replacement model following the expiration of repair warranty when the cost of minimal repair depends on the age of system. Section 3 derives the explicit expressions for the expected total cost and the expected cost rate per unit time for our replacement mode and then the optimal replacement policy is obtained. In Section 4, the numerical examples are presented for illustrative purpose.

Nomenclature

T	time to failure of a system
$F(t)$, $f(t)$	life distribution and probability density function of T
w	non-renewing warranty period
$h(t)$	failure rate function
τ	length of maintenance period after the warranty is expired
$c_m(t)$	cost of minimal repair
c_r	cost of replacement at the end of the maintenance period
$ECL(\tau)$	expected cycle length
$ETC(\tau)$	expected total cost
$C(\tau)$	expected cost rate per unit time

2. REPLACEMENT MODEL UNDER WARRANTY

2.1 Model

In this section, we consider the replacement model following the expiration of repair warranty when the cost of minimal repair depends on the age of system. Under repair warranty, the manufacturer guarantees a satisfactory service only during the original warranty period and the failed system is minimally repaired by the manufacturer at partially cost to the user during the original warranty period w. At the end of maintenance period of a fixed length, the system is replaced by a new one, regardless of its current age. Once the original non-renewing warranty is expired, the user is fully responsible for each failure of the system during the entire maintenance period. The replacement model after warranty, which is being considered in this paper, is depicted in the diagram given in Figure 2.1.

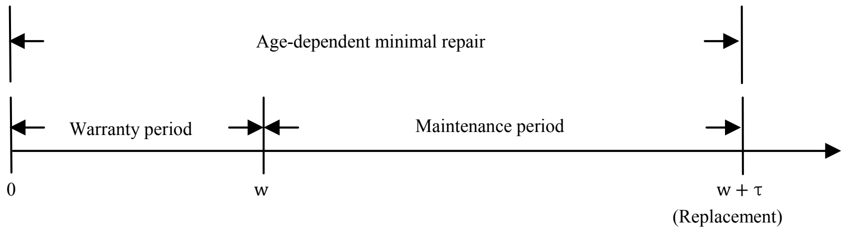

Figure 2.1. Replacement model under repair warranty

2.2 Expected cost rate

To obtain the expected cost rate per unit time for our model, we need the expected cycle length and the expected total cost. Firstly, we consider the expected cycle length for the replacement model. Under the repair warranty policy, the life cycle starts with the original purchase of the system and ends when the system is replaced by a new one at the end of the maintenance period, regardless of the number of replacements during the warranty period and the maintenance period. Thus, the expected cycle length is always equal to $w + \tau$. That is,

$$\mathrm{ECL}(\tau) = w + \tau. \tag{2.1}$$

In the next, we evaluate the expected total cost, which is charged to the user during the life cycle of the system, for our replacement model. Let C_W, C_M, and C_R be the cost-related random variables denoting cost of warranty period, cost of maintenance period and cost of replacement at the end of the life cycle. Then, the expected total cost can be expressed as

$$\mathrm{ETC}(\tau) = E(C_W) + E(C_M) + E(C_R). \tag{2.2}$$

Each of the expected costs, given in Eq. (2.2), can be obtained as follows:

$$E(C_W) = \frac{1}{\gamma}\int_0^w C_m(t)h(t)\,dt.$$

$$E(C_M) = \int_w^{w+\tau} C_m(t)h(t)\,dt,$$

and $E(C_R) = c_r$.

Here $C_m(t)$ is the cost of minimal repair for $t \geq 0$, where t denotes the time to failure of the system and $C_m(t)$ is assumed to be a non-decreasing continuous function of t. Hence as the system ages, it becomes more expensive to perform minimal repair. Also, the value of γ is the greater than zero. Thus, the expected total cost during the life cycle of the system, given in (2.2), can be obtained as follows.

$$\mathrm{ETC}(\tau) = \frac{1}{\gamma}\int_0^w C_m(t)h(t)\,dt + \int_w^{w+\tau} C_m(t)h(t)\,dt + c_r, \tag{2.3}$$

Using the expected cycle length and the expected total cost, given in (2.1) and (2.3) respectively, the expected cost rate per unit time for our replacement model can be expressed as

$$C(\tau) = \frac{\mathrm{ETC}(\tau)}{\mathrm{ECL}(\tau)} = \frac{1}{w+\tau}\left\{\frac{1}{\gamma}\int_0^w C_m(t)h(t)\,dt + \int_w^{w+\tau} C_m(t)h(t)\,dt + c_r\right\}. \tag{2.4}$$

If $\gamma = \infty$, then Eq. (2.4) is reduced to the following expected cost rate per unit time, which is the expected cost rate for the replacement policy under free repair warranty.

$$C(\tau) = \frac{1}{w+\tau}\left\{\int_{w}^{w+\tau} C_m(t)h(t)\,dt + c_r\right\}.$$

2.3 Optimization

In this section, we derive the optimal replacement policy following the expiration of repair warranty when the cost of minimal repair depends on the age of system. As for the criterion of optimality, we seek the value of τ, the length of replacement period, which makes the expected cost rate per unit time to be minimized. To determine the value of τ which minimizes $C(\tau)$, given in Eq. (2.4), we differentiate $C(\tau)$ with respect to τ and set it equal to 0. Consequently, we obtain

$$\left((w+\tau)\frac{1}{\gamma}Q(w+\tau) - \frac{1}{\gamma}\int_0^{w} Q(t)\,dt - \int_{w}^{w+\tau} Q(t)\,dt\right) = c_r, \tag{2.5}$$

where $Q(t) = C_m(t)h(t)$.

If the expected cost rate function, given in Eq. (2.4), is known to be pseudo-convex, it has exactly one local minimum and thus a unique global minimum exists. Sahin and Polatoglu (1996) utilize the concept of pseudo-convexity of the cost function to determine the optimal maintenance policy.

Lemma 2.1. *If F is an IFR distribution with strictly increasing failure rate function and* $C_m(t)$ *is non-decreasing function in* t, *then* $C(\tau)$, *given in Eq. (2.4), is pseudo-convex in* $\tau \geq 0$.

Theorem 2.2. *If F is an IFR distribution with strictly increasing failure rate function and* $C_m(t)$ *is non-decreasing function in* t *, then* $\tau^* = 0$ *if and only if* $\left(\frac{1}{\gamma}wQ(w) - \frac{1}{\gamma}\int_0^{w} Q(t)\,dt\right) \geq c_r$ *and* $0 < \tau^* < \infty$ *is the unique solution of Eq. (2.5) if and only if* $\left(\frac{1}{\gamma}wQ(w) - \frac{1}{\gamma}\int_0^{w} Q(t)\,dt\right)t < c_r$.

Using the property of pseudo-convexity of $C(\tau)$ and Eq. (2.5), it is straightforward to prove Theorem 2.2. It follows from Theorem 2.2 that the value of τ satisfying Eq. (2.5) is the optimal replacement period following expiration of repair warranty when the cost of minimal repair depends on the age of system.

3. NUMERICAL EXAMPLES

This section considers numerical examples to illustrate the optimal maintenance policy derived in Section 2. The failure times of a system are assumed to follow a Weibull distribution with the failure rate of $h(t) = \beta\lambda^{\beta}t^{\beta-1}$ for $t \geq 0$. As a special case, we take $\lambda = 1$, $w = 0.5$, $\gamma = 0.5$ and $C_m(t) = ct$, where $c > 0$ is a constant and consider various choices of β, c_r, and y. Using such parameter values, we determine the optimal length of maintenance period, denoted by τ^*, and investigate the pattern changes of τ^* and $C(\tau^*)$.

The optimal value of τ, denoted by τ^*, is determined to minimize the expected cost rate $C(\tau)$ of Eq. (2.4) and Table 3.1 presents the values of $\tau^*$and its resulting expected cost rate for various choice of β. For instance, in Table 3.1, when $\beta = 3, c = 1, \gamma = 10$ and $c_r = 20$, the optimal replacement period τ^* is equal to 0.73510(unit time) and its resulting expected cost rate is 16.068014(unit cost). From Table 3.1, it is observed that the optimal maintenance period becomes shorter as the value of β increases for a fixed c_r. In addition, we note that the length of the optimal maintenance period becomes longer as the value of c_r increases for fixed β.

Table 3.1. Optimal maintenance policies with age-dependent minimal repair cost

β	c	Optimal policy	c_r		
			10	20	30
3	0.5	τ^*	0.73510	0.89338	1.00021
		$C(\tau^*)$	9.53401	17.11341	24.01562
	1	τ^*	0.60343	0.73510	0.82417
		$C(\tau^*)$	10.54706	19.06801	26.87234
	2	τ^*	0.49429	0.60343	0.67747
		$C(\tau^*)$	11.59259	21.09412	29.85158
4	0.5	τ^*	0.62848	0.73252	0.80074
		$C(\tau^*)$	9.984730	18.42741	26.31290
	1	τ^*	0.53857	0.62848	0.68749
		$C(\tau^*)$	10.76960	19.96946	28.59529
	2	τ^*	0.46105	0.53857	0.58957
		$C(\tau^*)$	11.56716	21.53920	30.92697
5	0.5	τ^*	0.57667	0.65401	0.70375
		$C(\tau^*)$	10.20480	19.14479	27.61871
	1	τ^*	0.50815	0.57667	0.62077
		$C(\tau^*)$	10.84288	20.40960	29.50205
	2	τ^*	0.44749	0.50815	0.54724
		$C(\tau^*)$	11.48566	21.68576	31.40718

4. CONCLUSION

Although many maintenance policies after warranty are proposed in the literature and its optimality is considered, most of them assume that the cost of minimal repair is constant, not depending on the age of system. Therefore, in this paper, we consider the replacement model following the expiration of repair warranty when the cost of minimal repair depends on the age of system. As a criterion of the optimality, we utilize the expected cost rate per unit time during the life cycle from the user's perspective and suggest the optimal maintenance period following the expiration of repair warranty. In addition, we investigate the pattern changes of those objective parameters of our interests, such as length of maintenance period following the expiration of warranty, and expected cost rate per unit time.

ACKNOWLEDGEMENTS

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2010-0009320).

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