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Fuzzy programming for improving redundancy-reliability allocation problems in series-parallel systems

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Abstract. Redundancy-reliability allocation problems in multi-stage series-parallel systems are addressed in this study. Fuzzy programming techniques are proposed for finding satisfactory solutions. First, a multi-objective programming model is formulated for simultaneously maximizing system reliability and minimizing system total cost. Due to the nature of uncertainty in the problem, the fuzzy set theory and technique are used to convert the deterministic multi-objective programming model into a fuzzy nonlinear programming problem. A heuristic method is developed to get satisfactory solutions for the fuzzy nonlinear programming problem. A Pareto optimal solution is found with maximal degree of satisfaction from the interception area of fuzzy sets. A case study that is related to the electronic control unit installed on aircraft engine over-speed protection system is used to implement the developed approach. Results suggest that the developed fuzzy multi-objective programming model can effectively resolve the fuzzy and uncertain problem when design goals and constraints are not clearly confirmed at the initial conceptual design phase.

Key Words: *Fuzzy nonlinear programming, engine protection system, reliability allocation*

1. INTRODUCTION

For a system design with low reliability requirement, the designer can adopt series-parallel-systems techniques to improve system reliability and redundancy allocation. However, without further consideration, series-parallel-systems design techniques will increase system complexity, cost, weight, volume, and power dissipation. These constrained elements shall be considered when series-parallel-systems are applied.

In practice, solution methods for series-parallel-systems with redundancy-reliability allocation problems can be categorized into two methods: active redundant model and standby redundant model. An active redundant model adopts several parallel components,

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in which each component shall be actively operated. The entire system will operate well if a specific number of components within this system operate normally. A standby redundant model adopts several parallel components as well. In order to make sure that the whole system can be operated functionally, certain numbers of components are required to be operated normally. Besides, when some of parallel components fail, components in standby will be operated by using switch devices. The failure rates of non-operated components and switching devices are both zero. If the failure rate of switching devices is excluded, the system reliability of standby redundant model is higher than that of active redundant model. Although the system reliability of both models is high, these two models could cause higher cost, higher weight, and higher volume. Especially, when some switch devices are adopted by the standby redundant model, an additional cost will increase in accordance with the number of switch devices used. On the other hand, if the failure rate of switching devices is included, we need to further study the relationship among the whole system reliability, the contact reliability of switching devices, and the conditional dynamic/static system reliability.

The objective of reliability maximization is always pursued while the system cost became higher, or the objective of system cost minimization is obtained while the system reliability was sacrificed for traditional single objective optimization method. If multi-objective optimization method is adopted to solve reliability allocation trade-off problems of series-parallel systems, one could consider the optimization of system reliability and total system cost at the same time. Also one could consider the constrained factors of weight and volume. The system reliability requirement with the above design disciplines could be achieved. When the product reliability demonstration is carried out, one didn't need to spend too much cost and time. Several objectives could be conflicted each other. The decision procedures were also very complicated. It also involved different levels of uncertainties, such as characteristics of expert information, qualitative statements and fuzzy, etc. In order to solve these uncertain factors, this study proposed fuzzy multi-objective optimization decision method, which combined improved search heuristic algorithm. We hoped that this algorithm could solve the optimization decision problems of two or two above objectives. This algorithm could also find out an optimal solution, which could give the highest degree of satisfaction with respect to all the fuzzy sets (including fuzzy objectives and fuzzy constraints) in the fuzzy decision space. The purpose of this paper is to find out several effective solutions and a Pareto optimal solution with fuzzy multi-objective programming method to solve the optimal design of reliability allocation and total system cost for series-parallel systems.

2. LITERTURE REVIEW

During many practical design situations, reliability allocation problems turn into more complicated when several goals conflict each other. Sakawa (1988) considered surrogate worth tradeoff method for multi-objective models of reliability allocation problems. Inagaki et al. (1988) used interactive optimal design with a minimal cost and weight, and maximal system reliability.

In order to solve reliability redundant optimization problems, Misra and Sharma (1991) launched the research of using bound search techniques, which were integer programming methods. Li (1996) proposed a bound dynamic programming (BDP) method that could solve reliability redundant optimization problems. Through the usage of dynamic programming method and bound areas of the problems, the BDP solution procedures for reliability redundant optimization problems are more economic and efficient than Misra's integer programming algorithm. Any heuristic techniques could be used to acquire a new and a better boundary point from any given boundary points. Li and Jia (1997) further used a partial bound enumeration (PBE) technique that could solve reliability redundant optimization problems.

Baxter and Harche (1992) used exact algorithm to solve the optimal reliability allocation of series-parallel systems. Chern et al. (1991) also used exact algorithms and parametric non-linear integer programming methods to solve the application of the reliability optimization problems with multiple constraints for the series-parallel systems. Gopal et al. (1980) utilized improved heuristic method to solve optimal allocation of redundancy for series-parallel systems. Kuo et al. (1987) utilized heuristic method to solve optimal system reliability by redundancy allocation, utilized Lagrangian multiplier and branch-and-bound techniques to solve reliability optimization problems, respectively. Nakagawa and Miyazaki (1981) utilized heuristic method, surrogate constraints algorithm and multi-objective reliability optimization method, respectively, to solve reliability optimization problems and its experimental comparison. Petrovic (1991) utilized heuristic method to improve decision support of system reliability by redundancy allocation. Sharma and Misra (1990) utilized heuristic method to optimize system reliability and developed an effective algorithm to solve integer programming problems of reliability optimization, respectively. Xu, et al. (1990) utilized heuristic method by the allocation of reliability redundancy to solve optimal constraints of improving system reliability.

Ida et al. (1994) utilized meta-heuristic method by GA to solve system reliability optimization with several failure modes. Painton and Campbell (1995) utilized meta-heuristic method by GA to solve system reliability optimization problems. Yokota et al. (1996) utilized meta-heuristic method to solve system reliability optimization with several failure modes, mixed integer non-linear programming problems and its application, respectively.

Gen et al. (1989) utilized multi-objective programming method to solve optimal reliability for large-scale series-parallel systems. Misra and Sharma (1991) utilized meta-heuristic method and an effective reliability design tool to solve integer programming problems, utilized multi-objective programming method to solve multi-objective redundancy optimization problems, respectively. Sakawa (1981) utilized multi-objective reliability optimization method to solve the optimal reliability design of large-scale series-parallel systems. El-Neweihi et al. (1986) utilized optimal allocation of inter-exchangeable component method to solve the optimal component allocation of series-parallel and parallel-series systems. Prasad and Raghavachari (1998) utilized heuristic method to solve optimal component allocation of series-parallel networks and utilized optimal allocation of inter-exchangeable component method to solve the optimal component allocation of series-parallel and parallel-series systems, respectively.

Kuo and Prasad (2000) consider that exact algorithm of reliability optimization problems was not often easy to solve. Even though there was a solution, the effectiveness of exact algorithm was also restricted. In order to solve the optimal redundant allocation problems, they therefore recommended us to focus on the following four approaches: heuristic method, meta-heuristic method, multi-objective optimization method, and optimal allocation of inter-exchangeable component method. Park (1987) utilized fuzzy set theory to analyze two components series system of reliability allocation problems, which are subjected to single constraint. Zedeh and Bellman (1990) designed a fuzzy-decision environment to provide different kinds of total solutions for design problems. Huang (1997) proposed fuzzy multi-objective optimization decision method, which could provide two or two above goals of reliability optimization decisions.

3. RELIABILITY MODEL FOR SERIES-PARALLEL SYSTEMS

Figure 1 displays a diagram for an N stages series-parallel system with redundancy-reliability allocation problems. In this system, several parallel and identical components are arrayed in each stage. While the series-parallel system with redundancy can be applied to increase the system reliability, this technique may inevitably add more complexity, weight, volume, or cost to the design system. It would be better that a multi-objective programming model is developed to tackle this problem.

When modeling the target problem, the objectives are two-fold. One is to determine optimal design reliability for each component and the other to select an optimal number of components within each stage. That is, the overall system reliability is maximized and the overall system cost is minimized. In addition, several constrained design criteria, such as minimum requirements for system reliability, system cost, system volume, and system weight, are considered in this model. In order to develop a mathematical design model for the engine protection systems, we define the following decision variables and parameters.

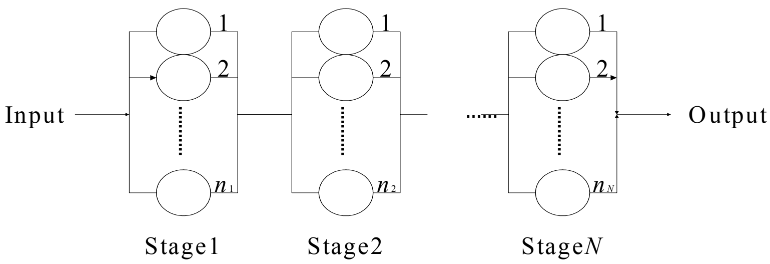


Figure 3.1. Diagram for N-stages series-parallel systems

Decision Variable**:**

R_i = represent the component reliability within the *i*-th stage；

n_i = represent the number of components within the *i*-th stage;

f_1 = represent the overall system reliability;

f_2 = represent the overall system cost.

Parameter**:**

$C_i(R_i)$ = represent the component cost within the i-th stage;
w_i = represent the component weight within the i-th stage;
v_i = represent the component volume within the i-th stage;
R = represent the lower limit for the overall system reliability;
C = represent the upper limit for the overall system cost;
W = represent the maximum limit for the system weight;
V = represent the maximum limit for the system volume;
N = represent the number of stages in the design system;
N_{high} = represent the maximum number of components within each stage;
N_{low} = represent the minimum number of components within each stage;
R_{high} = represent the maximum limit of reliability within each stage;
R_{low} = represent the minimum requirement of reliability within each stage.

Then a multi-objective mathematical reliability design model may be given as follows.

$$\text{Maximize} \quad f_1 = \prod_{i=1}^{N} \left[1 - (1 - R_i)^{n_i} \right] \tag{3.1}$$

$$\text{Minimize} \quad f_2 = \sum_{i=1}^{N} C_i(R_i) \tag{3.2}$$

subject to

$$\prod_{i=1}^{N} \left[1 - (1 - R_i)^{n_i} \right] \geq R \tag{3.3}$$

$$\sum_{i=1}^{N} w_i \, n_i \exp(n_i / N) \leq W \tag{3.4}$$

$$\sum_{i=1}^{N} v_i n_i^2 \leq V \tag{3.5}$$

$$\sum_{i=1}^{N} C_i(R_i)\left[n_i + \exp(n_i / N) \right] \leq C \tag{3.6}$$

$$N_{low} \le n_i \le N_{high}, \; i = 1,\dots,N \tag{3.7}$$

$$R_{low} \le R_i \le R_{high}, \; i = 1,\dots,N \,. \tag{3.8}$$

The objective function (3.1) is used to maximize the overall system reliability for the engine protection systems, while the objective function (3.2) is used to minimize the overall system cost. Constraint (3.3) is used to set the minimum requirement for the system reliability. Constraint (3.4) is used to set the maximum limit for the system weight. Constraint (3.5) is used to set the maximum limit for the system volume. Constraint (3.6) is used to set the maximum limit for the system cost, where $C_i(R_i)$ is the unit cost for the component with R_i reliability in the *i-th* stage. Constraint (3.7) denotes the allowable range of component number for each stage and constraint (3.8) is used to specify the range of reliability for each component in each stage.

In constraint (3.6), $C_i(R_i)$ can be formulated as: $C_i(R_i) = \alpha_i / \lambda_i^{\beta_i}$, where α_i and β_i are constant and characteristics factors for each component in the *i-th* stage and λ_i is the failure rate of component in the *i-th* stage. Furthermore, by the relationship of $R_i = \exp(-\lambda_i t)$, we can obtain $C_i(R_i) = \alpha_i \cdot [-t/\ln(R_i)]^{\beta_i}$, where t is the active operation time. Hence, constraint (3.6) can be reformulated as:

$$\sum_{i=1}^{N} \alpha_i \times \left[{-t}/{\ln(R_i)} \right]^{\beta_i} \times [n_i + \exp(n_i / N)] \le C \tag{3.9}$$

The developed reliability design model is one type of multi-objective mixed integer nonlinear programming problem. This is a NP-hard problem. In addition, information about the reliability, cost, weight, and volume parameters in the model can be uncertain or incomplete in terms of data collection in the early design stage of system life cycle. It would be better to apply fuzzy set techniques to solve this problem.

4. FUZZY PROGRAMMING MODEL FOR SERIES-PARALLEL SYSTEMS

The fuzzy set theory (Zadeh, 1985) is applied to construct a fuzzy nonlinear programming model for solving the series-parallel systems. First, those objective functions and constraints in the multi-objective programming model are treated as fuzzy objective functions and fuzzy constraints using membership functions to quantify uncertain parameters. The following notations for developing a fuzzy nonlinear model are provided.

Notation:

$\overline{}$ = fuzzy information;

$\cap$ = fuzzy intersection;

$\overline{F}_i$ = the *i-th* fuzzy objective function (*i =1,…, k*);

$\overline{G}_j$ = the *j-th* fuzzy constraint (*j =1,…, m*) ;

$\overline{D}$ = the fuzzy decision set;

μ_i = the *i-th* fuzzy membership function;

α_i = the degree of satisfaction for the *i-th* objective function;

α_j = the degree of satisfaction for the *j-th* fuzzy constraint;

Φ = the fuzzy set for the decision space;

$A(\Phi)$ = the overall satisfaction;

R_S = the reliability goal which is set up by designer;

∇R = difference between reliability goal and minimal reliability limit;

C_S = the cost goal which is set up by designer;

∇C = difference between cost goal and maximal cost limit;

w_s = the system weight;

v_s = the system volume.

The membership function for the fuzzy reliability objective function and the degree of satisfaction of reliability function may be given as follows. The operational range varies from minimal reliability limit to reliability goal.

$$\alpha_{i=1} = \mu_{\overline{F}_1}(R_S) = \begin{cases} 0 & R_S \le R \\ \dfrac{R_S - R}{\nabla R} & R < R_S < R + \nabla R \\ 1 & R_S \ge R + \nabla R \end{cases} \tag{4.1}$$

The membership function for fuzzy cost objective function and the degree of satisfaction of cost function may be expressed as follows. The operational range varies from cost goal to maximal cost limit.

$$\alpha_{i=2} = \mu_{\overline{F}_2}(C_S) = \begin{cases} 1 & C_S \le C \\ \dfrac{C + \nabla C - C_S}{\nabla C} & C < C_S < C + \nabla C \\ 0 & C_S \ge C + \nabla C \end{cases} \tag{4.2}$$

The membership function for the degree of satisfaction of weight constraint may be expressed as follows. The operational range varies from 0 to maximal weight limit.

$$\alpha_{j=1} = \mu_{\overline{G}_1}(w_S) = \begin{cases} 0 & w_S > W \\ 1 & w_S \le W \end{cases} \tag{4.3}$$

The membership function for the degree of satisfaction of volume constraint may be expressed as follows. The operational range varies from 0 to maximal volume limit.

$$\alpha_{j=2} = \mu_{\overline{G}_2}(v_S) = \begin{cases} 0 & v_S > V \\ 1 & v_S \le V \end{cases} \tag{4.4}$$

Assume Φ is a fuzzy set of decision space and α(Φ) denotes the degree of overall satisfaction for the developed engine protection systems. The degree of overall satisfaction, α(Φ), within this decision space may be expressed as follows.

$$\begin{aligned} \alpha(\Phi) = \mu_{\overline{D}}(R^*) &= \left[\mu_{\overline{F}_1}(R_s) \cap \mu_{\overline{F}_2}(C_s)\right] \cap \left[\mu_{\overline{G}_1}(w_s) \cap \mu_{\overline{G}_2}(v_s)\right] \\ &= \left[\mu_{\overline{F}_1}(R_S) \cap \mu_{\overline{F}_2}(C_S) \cap \mu_{\overline{G}_1}(w_S) \cap \mu_{\overline{G}_2}(v_S)\right] \end{aligned} \tag{4.5}$$

If the fuzzy objective functions (4.1), (4.2) and the fuzzy constraints (4.3), (4.4) are known with certainty, the degree of satisfaction for those fuzzy functions are available. Then the overall satisfaction can be obtained via seeking the intersection area of those fuzzy functions. Mathematically, it is equivalent to find feasible solutions, optimal solution (R^*) and the maximal α(Φ) in the following fuzzy nonlinear programming problem.

$$\text{Maximize} \quad \alpha(\Phi) \tag{4.6}$$

subject to

$$\alpha(\Phi_{\overline{F}}) \;\square\; \mu_{\overline{F}_1}(R_S) \cap \mu_{\overline{F}_2}(C_S) \tag{4.7}$$

$$\alpha(\Phi_{\overline{G}}) \;\square\; \mu_{\overline{G}_1}(w_S) \cap \mu_{\overline{G}_2}(v_S) \tag{4.8}$$

$$\alpha_{i=1} = \mu_{\bar{F}_1}(R_S) = \begin{cases} 0 & R_S \leq R \\ \dfrac{R_S - R}{\nabla R} & R < R_S < R + \nabla R \\ 1 & R_S \geq R + \nabla R \end{cases} \tag{4.9}$$

$$\alpha_{i=2} = \mu_{\bar{F}_2}(C_S) = \begin{cases} 1 & C_S \leq C \\ \dfrac{C + \nabla C - C_S}{\nabla C} & C < C_S < C + \nabla C \\ 0 & C_S \geq C + \nabla C \end{cases} \tag{4.10}$$

$$\alpha_{j=1} = \mu_{\bar{G}_1}(w_S) = \begin{cases} 0 & w_S > W \\ 1 & w_S \leq W \end{cases} \tag{4.11}$$

$$\alpha_{j=2} = \mu_{\bar{G}_2}(v_S) = \begin{cases} 0 & v_S > V \\ 1 & v_S \leq V \end{cases} \tag{4.12}$$

$$0 \leq \alpha(\Phi_{\bar{F}}) \leq 1 \tag{4.13}$$

$$0 \leq \alpha(\Phi_{\bar{G}}) \leq 1 \tag{4.14}$$

The objective function (4.6) is used to maximize the degree of overall satisfaction for the design system. Constraint (4.7) is used to set the minimum satisfaction requirement for the reliability and cost objectives. Constraint (4.8) is used to set the minimum satisfaction requirement for the weight and volume functions. Constrain (4.9) is used to specify the degree of satisfaction of reliability function. Constraint (4.10) is used to specify the degree of satisfaction of cost function. Constraint (4.11) is used to specify the degree of satisfaction of weight function. Constraint (4.12) is used to specify the degree of satisfaction of volume function. Constraint (4.13) provides the range between 0 and 1 for the degree of overall satisfaction about the objective functions. Constraint (4.14) provides the range between 0 and 1 for the degree of overall satisfaction. The developed model can allow one to achieve a maximum overall satisfaction value while satisfying multi-objective fuzzy objective functions and fuzzy constraints within a fuzzy decision space.

The developed fuzzy programming model is one type of nonlinear problem, in which the fuzzy multi-objective programming problem is converted into a deterministic single objective problem. For the developed fuzzy programming model, an α–search heuristic method is devised to generate a group of satisfactory solutions. The procedure of the α-search heuristic method is given as follows:

Step 0. Initialization:

Set K = the maximal number of iterations, N = the number of stages, R_{low} = lower bounds for reliability, R_{high} = upper bounds for reliability, $\triangle R_i$ = interval of increment for reliability, N_{low} = minimal components within each stage, N_{high} = maximal components within each stage, $\Phi\{\ \}$ = solution set, Q = maximal number of elements in solution set.

Step 1. Initialization:

Provide an initial solution: $k = 1$, $R_i^k = R_{low}$, $n_i^k = N_{low}$, $i = 1,\ldots,N$. Place them into the solution set $\Phi(\,)$.

Step 2. Validation:

Compute and compare the overall satisfaction $\alpha^k(\Phi)$ for the incumbent solution. If

$\alpha^k(\Phi) > \alpha^{k-1}(\Phi)$, replace the incumbent solution with previous solution in the solution set. If $n_i^k = N_{high}$ and $R_i^k = R_{high}$, $i = 1,...,N,$ go to Step 4. Otherwise, go to Step 3.

Step 3. Improvement:

If $R_i^k = R_{high}$, then $n_i^k = n_i^k+1$ and $R_i^k = R_{low}$. Otherwise, $R_i^k = R_i^{k+1} + \triangle R_i$, $k = k+1$.

Compute the values for:

1. Reliability fuzzy set and its degree of satisfaction;
2. Cost fuzzy set and its degree of satisfaction;
3. Weight fuzzy set and its degree of satisfaction;
4. Volume fuzzy set and its degree of satisfaction.

Return to Step 2

Step 4. Closing:

Generate a group of satisfactory solutions.

A computer programming language, Delphi 7.0, is used to code and compile the above procedure in the developed α–search heuristic. A graphic user-interface is also provided for simulating alternative solutions

5. CASE STUDY

In this study, the developed approach is implemented to a design problem of over-speed protection system in turbo engines. The mission of this problem is to design a protection system during the over-speed operation of turbo engines. Figure 2 displays a functional block diagram for an over-speed protection system that is installed in turbo engines. This protection system consists of one electronic control valve and three mechanical valves, which provide over-speed protection for the turbo engine in a continuous way. Due to the incomplete or uncertain information about the design parameters during the early design phase, fuzzy set techniques combined with reliability design model is utilized to provide solution methods.

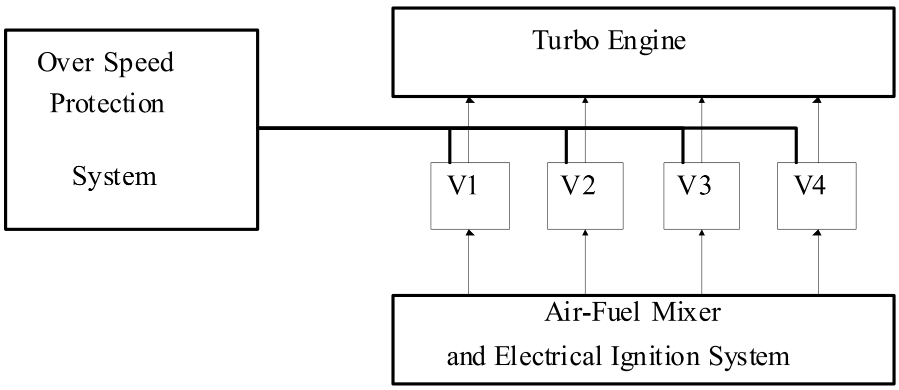


Figure 5.1. Diagram for over-speed protection system of turbo engine

Table 5.1 provides the design data for this case study, which includes number of stages, reliability and cost goal, limits for reliability, cost, weight, and volume, and operational time. Table 5.2 provides the physical characteristics of redundant components for different stages. Figures 5.2, 5.3, 5.4, and 5.5 provide the membership function for reliability, cost, weight, and volume, respectively. The intersection area of these four membership functions and its individual fuzzy set can lead us to find out feasible solutions as long as we maximize the degree of overall satisfaction, α(Φ), which is shown in Figure 5.6.

Table 5.1. Design data of case study.

Number of stages	N = 4
Reliability goal which is set up by designer	$R_s = 0.99$
Cost goal which is set up by designer	$C_s = 300$
Lower limit of reliability	R = 0.90
Upper limit of cost	C = 400
Upper limit of weight	W = 500
Upper limit of volume	V = 250
Operation time	T = 1000 Hours

Table 5.2. Physical characteristics of redundant components for each stage

Stage	α_i	β_i	v_i	w_i
1st	1.0×10^{-5}	1.5	1	6
2nd	2.3×10^{-5}	1.5	2	6
3rd	0.3×10^{-5}	1.5	3	8
4th	2.3×10^{-5}	1.5	2	7

$$\alpha_1 = \mu_{\overline{F}_1}(R_S) = \begin{cases} 0 & R_S \le 0.9 \\ \dfrac{R_S - 0.9}{0.09} & 0.9 < R_S < 0.99 \\ 1 & R_S = 0.99 \end{cases}$$

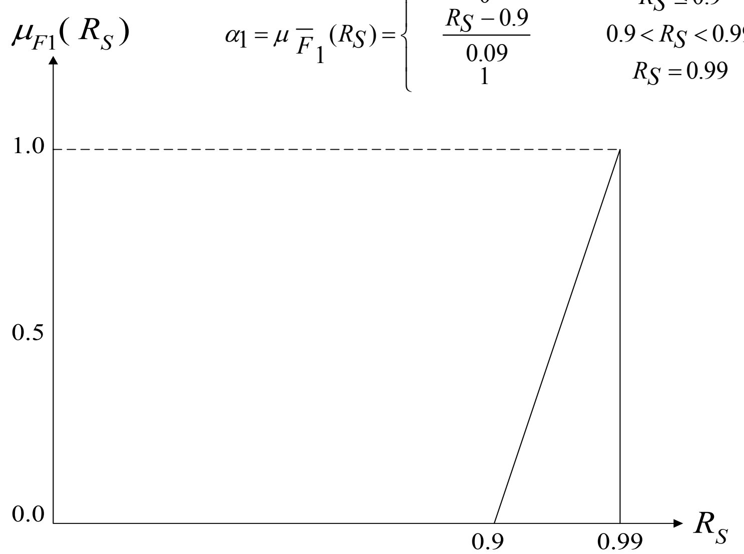


Figure 5.2. Membership function for reliability

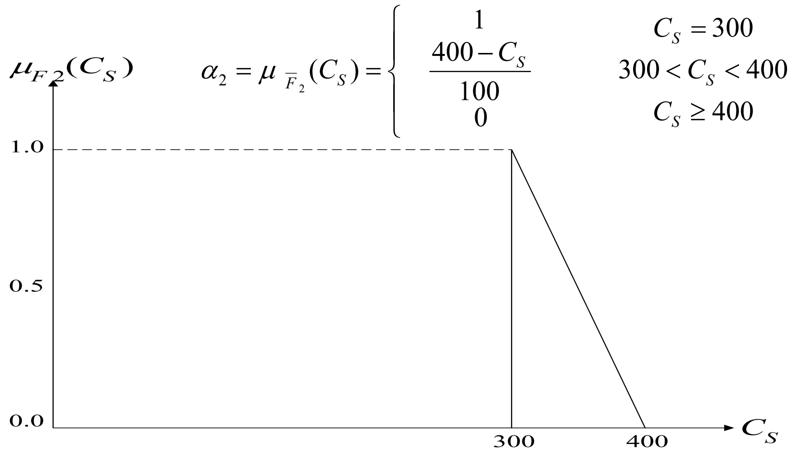


Figure 5.3. Membership function for cost

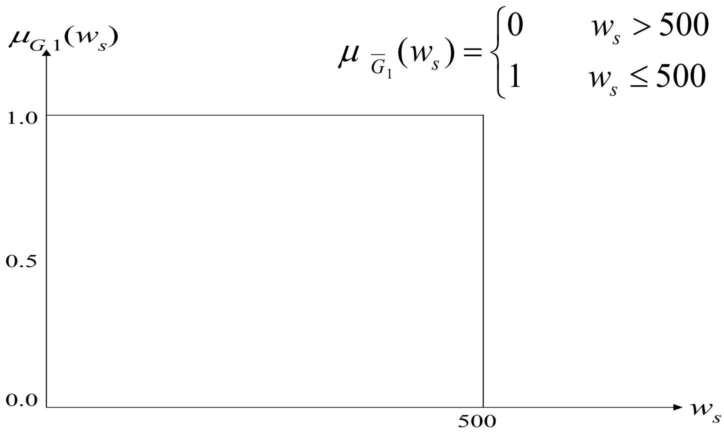


Figure 5.4. Membership function for weight

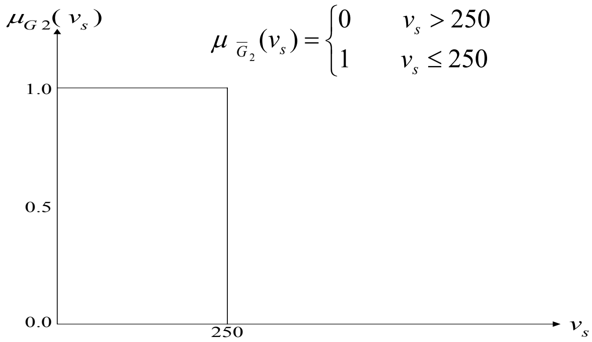


Figure 5.5. Membership function for volume

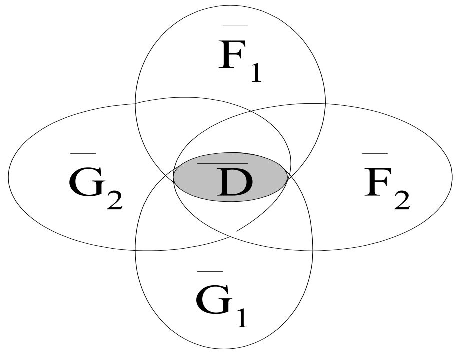


Figure 5.6. Decision space $\overline{D}$

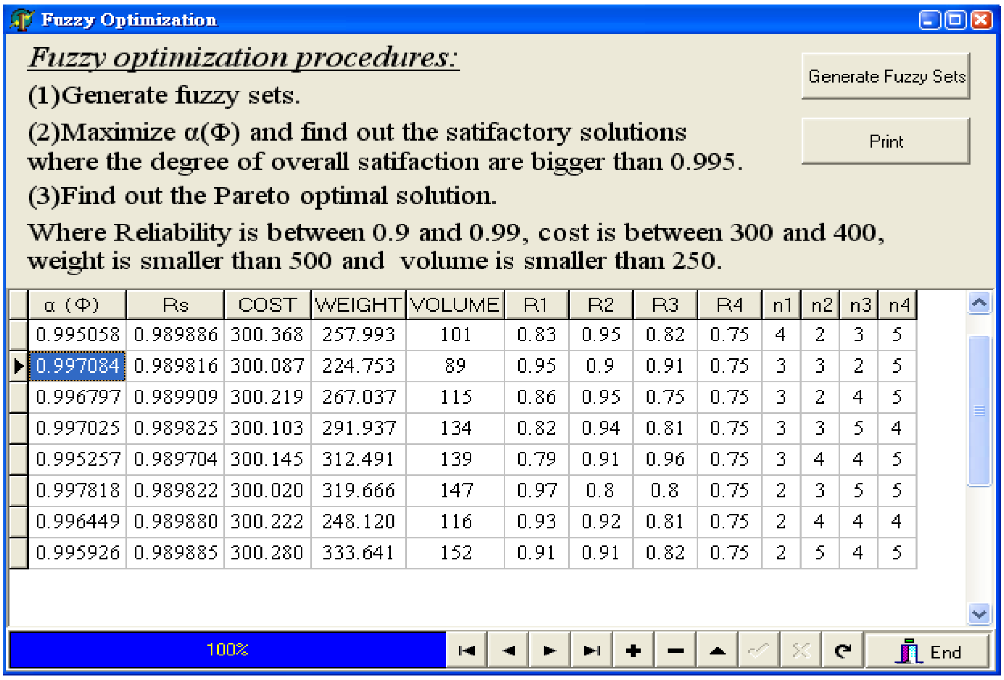
Table 5.3 then provides some results obtained from the application to the fuzzy nonlinear programming problem. If we consider all the enumerations of α(Φ) which is just bigger than 0.995, then we can find out eight combinations of system reliability, system cost, system weight, and system volume. Each combination is associated with one value for α(Φ). We can see that the highest value of α(Φ) didn't necessarily imply the best combination of reliability, cost, weight, and volume for the design system. The values for reliability and cost didn't vary much with the change of α(Φ), While the values of weight and volume change significantly. Hence, system weight and system volume play the key role in determining the most satisfactory solution. From Table 3, the most satisfactory solution appears when α(Φ) equals 0.997084 as R_s=0.989816, C=300.087、, W=224.753, and V=89.0. This is also a Pareto optimal solution from the interception area of fuzzy sets by applying fuzzy goal programming method.

Table 5.3. Satisfactory solutions obtained by the heuristic

Satisfactory solution with α(Φ) > 0.995	Characteristics for each satisfactory solution			
	Reliability	Cost	Weight	Volume
0.995058	0.989886	300.368	257.993	101.0
0.995257	0.989704	300.145	312.491	139.0
0.995926	0.989885	300.280	333.641	152.0
0.996449	0.989880	300.222	248.120	116.0
0.996797	0.989909	300.219	267.037	115.0
0.997025	0.989825	300.103	291.937	134.0
0.997084	0.989816	300.087	224.753	89.0
0.997818	0.989822	300.020	319.666	147.0

The obtained results suggest that the developed fuzzy nonlinear programming technique can provide higher quality solutions regardless of size and complexity of problems. When the information about the design parameters is uncertain or incomplete for the series-parallel systems with redundancy, the developed fuzzy goal programming technique can be apply to provide satisfactory solutions for decision maker. Results also suggest that fuzzy multi-objective programming can effectively resolve the fuzzy and uncertain problem when design goals and constraints are not still clearly confirmed at the initial conceptual design phase.

A computer programming language, Delphi 7.0, is used to code and compile the algorithm. A graphic user-interface is illustrated in Figure 8. Finally we can draw a reliability block diagram for this case based on the outcomes of the following Delphi programs, which is shown in Figure 5.8.



α (Φ)	Rs	COST	WEIGHT	VOLUME	R1	R2	R3	R4	n1	n2	n3	n4
0.995058	0.989886	300.368	257.993	101	0.83	0.95	0.82	0.75	4	2	3	5
0.997084	0.989816	300.087	224.753	89	0.95	0.9	0.91	0.75	3	3	2	5
0.996797	0.989909	300.219	267.037	115	0.86	0.95	0.75	0.75	3	2	4	5
0.997025	0.989825	300.103	291.937	134	0.82	0.94	0.81	0.75	3	3	5	4
0.995257	0.989704	300.145	312.491	139	0.79	0.91	0.96	0.75	3	4	4	5
0.997818	0.989822	300.020	319.666	147	0.97	0.8	0.8	0.75	2	3	5	5
0.996449	0.989880	300.222	248.120	116	0.93	0.92	0.81	0.75	2	4	4	4
0.995926	0.989885	300.280	333.641	152	0.91	0.91	0.82	0.75	2	5	4	5

Figure 5.7. A graphic user-interface for the case study

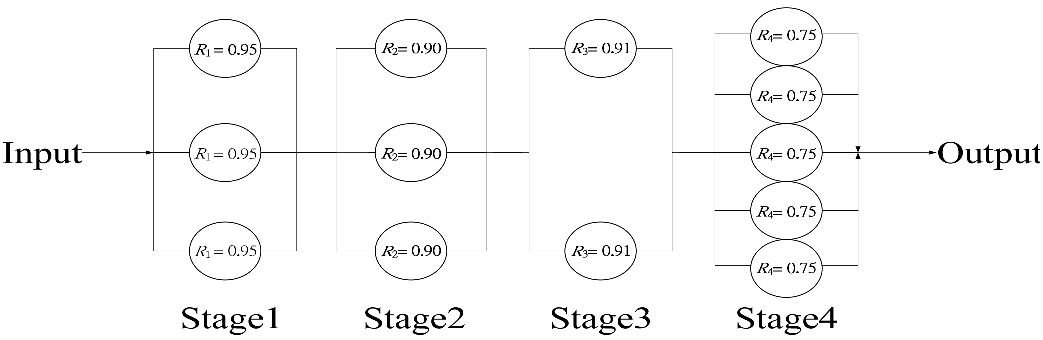


Figure 5.8. A reliability block diagram for the case study

6. CONCLUDING REMARKS

When design-in system reliability is low, series-parallel systems are adopted as a design guideline for improving system reliability. However, this design guideline will increase the total system cost and weight. Hence this design guideline seldom meets the practical requirement. In this study, fuzzy goal programming techniques are applied to deal with multi-stage series-parallel systems with redundancy problem. The developed fuzzy goal programming model can provide the most satisfactory solutions for determination of system/component reliability and number of components at each stage. A heuristic search method and the associated graphical user-interface are devised.

A Pareto optimal solution is found with several degree of satisfaction from the interception area of fuzzy sets. The obtained Pareto optimal solution of fuzzy multi-objective programming method is better than that of goal programming method. A case study that relates to the electronic control unit installed on aircraft's engine over-speed protection system is used to implement the developed approach. Results suggest that fuzzy multi-objective programming can effectively resolve the fuzzy and uncertain problem when design goals and constraints are not still clearly confirmed at the initial conceptual design phase. These models can also be applied efficiently and effectively for proper decision-making procedures when ill-structured situations occur.

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