International Journal of Reliability and Applications
Vol. 12, No. 1, pp. 15-39, 2011

Performance evaluation and reliability analysis of a complex system with three possibilities in repair with the application of copula

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Abstract. This paper deals with the reliability analysis of a complex system with three possibilities at the time of repair. The considered system consists of two subsystems A and Bin series configuration (1-out-of-2: F). Subsystem A has *n* units which are connected in series whereas subsystem B consists of *n* units in parallel configuration. The configuration of subsystem A is of 1-out-of-*n*: F whereas subsystem B is of *k*-out-of-*n*: D and *k*+1-out-of-*n*: F nature. System has three states: Good, degraded and failed. Supplementary variable technique has been used for mathematical formulation of the model. Laplace transform is being utilized to solve the mathematical equation. Reliability, Availability, M.T.T.F., Busy Period and Cost effectiveness of the system have been computed. The repairs from state S_7 to S_0, S_8 to S_0, S_9 to S_0 and S_{11} to S_0 have two types namely exponential and general. Joint probability distribution of repair rate from S_7 to S_0, S_8 to S_0, S_9 to S_0 and S_{11} to S_0 is computed by Gumbel-Hougaard family of copula. Some particular cases of the system have also been derived to see the practical importance of the model.

Key Words: *Reliability, availability, M.T.T.F., busy period, sensitivity analysis, cost analysis, supplementary variable technique, Gumbel-Hougaard copula*

1. INTRODUCTION

The study of repairable systems is an important topic in reliability. Repairman is one of the essential parts of repairable systems, and can affect the economic benefit of the systems, directly or indirectly. The work forms of repairman may affect the performance

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of the system. Therefore, he plays an important role on improving the reliability of the repairable systems. Khaled M. El-Said (2010) analyzed the cost–benefit analysis of a two-unit cold standby system with two-stage repair of failed unit. The repair process is divided into two stages. In the first stage, the repairing process of the unit is started but it does not get completed; instead the process is completed in the second stage. A. M. Rashad, M. Salah EL-Sherbeny and Z. M. Hussien (2009) considered two-unit cold standby system with two types of repair-minor (regular) and major (expert). The first repairman (minor) will remain with the repair facility assuming that he might not be able to do some complex repair within some tolerable time. The second repairman (major) is called to the system if and only if the (minor) repairman is unable to do the job within time or when the two units are failed, whichever occurs first. DequanYue, Wuyi Yue and Hongjuan Qi (2008) studied a machine repairable system with spares and two repairmen. The first repairman never takes vacations and always available for serving the failed units. The second repairman leaves for a vacation under some conditions. Linmin Hu and Jiandong (2009) analyzed reliability characteristics of a three-dissimilar unit repairable system with one repairman, with assumption that the repairman leaves for a vacation if there are no failed units waiting for repair in the system. When he comes from the vacation and find the failed units, he starts to repair the failed units and if there are no failed units then he goes for another vacation. Mangey Ram and S. B. Singh (2008) studied the complex system considered consists of two independent repairable subsystems A and B in series. The system is analyzed under preemptive-resume repair policy. Subsystem A is assumed to be priority unit while subsystem is non-priority unit. Here, whenever subsystem B is under repair and at the same time subsystem A fails, the repairing of subsystem B is stopped and subsystem A is taken for the repair. The repair of subsystem B is started from the point where it was left earlier as soon as repair of A is completed. Rakesh Gupta, C. K. Goel and ArchanaTomer (2010) considered two non-identical unit standby system model with single repair facility which appears and disappears from the system randomly assuming that when the repairman starts the repair of a failed unit, he does not leave the system till all the units are repaired that has failed during his service. S. K. Singh (1989) considered two-unit cold standby system with the assumption that the service facility appears and disappears from the system randomly. S. M. Rizwan, H. Chauhan and G. Taneja (2005) examined one unit and two unit systems with the concept of accident during inspection and possibility of multiple post repair inspection with the assumption that accident takes place during inspection of repaired unit and the harms the repairman in such a way that he is not capable to do the job. Then another repairman is called to repair the failed unit. There may be many situations in practical life such as: (i) The repair cost of failed unit is much greater than the cost of new unit. In such condition the failed unit should be replace by new unit (ii) The repairman disappears at the time of repair of failed unit. In this case the system stops working till the repairman comes and repairs the failed unit (iii) Sometimes when a repairman is busy in repair of failed unit, an accident takes place during repair. The repairman is injured in accident and cannot do his job further.

Keeping the above facts in mind, the aim of this paper is to analyze the reliability of a complex system including all three possibilities at the time of repair of failed unit which have not been considered simultaneously by earlier researchers. The considered system consists of two subsystems A and B in series configuration (1-out-of-2: F). Subsystem A has n units, connected in series whereas subsystem B consists of n units in parallel. The configuration of subsystem A is of 1-out-of-n: F whereas subsystem B is of k-out-of-n: D and k+1-out-of-n: F nature. System has three states: Good, degraded and failed. Here, following three possibilities have been considered for the failure of subsystem: (i) If the repairman is available when a subsystem fails then he repairs the failed subsystem and if the repairman disappears from the system at the time of repair of failed subsystem. Then system starts working when a repairman comes and repairs the failed subsystem. (ii) When the repairman is busy in the repair of failed subsystem, an accident takes place during the repair. The repairman is injured and does not capable to continue the job, in this situation the new repairman is called to do the job. The new repairman repairs the failed subsystem afresh with more care and attention. (iii) If the subsystem is taken maximum time for repair then it should be replaced by new unit. The repair times follow exponential and general time distributions respectively. At states S_7, S_8, S_9 and S_{11} both the subsystems are completely failed and being repaired with two different repair rates. The repairs from state S_7 to S_0, S_8 to S_0, S_9 to S_0 and S_{11} to S_0 have two types namely exponential and general. Joint probability distribution of repair rate from S_7 to S_0, S_8 to S_0, S_9 to S_0 and S_{11} to S_0 is computed by Gumbel-Hougaard family of copula (2006). The system is studied by using the supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula to obtain reliability, availability, M.T.T.F., busy period, sensitivity analysis and cost effectiveness of the system. At last some numerical examples have been taken to discuss the particular cases.

2. ASSUMPTIONS

1) Initially the system is in perfectly good state i.e. all the units are functioning perfectly.
2) At $t = 0$ all the components are perfect and $t > 0$ they start operating.
3) The system consists of two subsystem A and B are connected in series (1-out-of-2: F). Subsystem A has n units are connected in series whereas subsystem B consists of n units in parallel configuration.
4) The configuration of subsystem A is of 1-out-of-n: F whereas subsystem B is of k-out-of-n: D and k+1-out-of-n: F nature.
5) System has three states: Good, degraded and failed.
6) When a subsystem fails, we consider three possibilities.
   (a) If the repairman is available with the system when the subsystem has failed. Then he repairs the subsystem and if he disappears from the system at the time of repair of failed subsystem. Then the system will start when he comes after a particular amount of time and repairs the failed subsystem.
   (b) Repairman may meet with an accident when he is busy in repair of the failed subsystem in a way that he is not capable to repair the failed subsystem. Then new repairman is called to do the job. We assumed that the new repairman repaired the failed subsystem with more carefully so that no further accident occurs.
   (c) If the repairman does not repair the failed subsystem within maximum repair time then he replaced failed subsystem by new one.

7) The repaired subsystem is as good as new and is immediately reconnected to the system.
8) Transition from the completely failed state S_7 to the initial state S_0, completely failed state S_8 to the initial state S_0, completely failed state S_9 to initial state S_0, completely failed states S_{11} to initial state S_0 follows two different distributions.
9) Joint probability distribution of repair rate from S_7 to S_0, S_8 to S_0, S_9 to S_0 and S_{11} to S_0 is computed by Gumbel-Hougaard family of copula.

3. STATE SPECIFICATION

G: Good state, D: Degraded state, F_r: Failed unit under repair, F: Failed unit

Table 3.1. State specification

States	Subsystems		System state
	A	B	
S_0	G	G	G
S_1	F_r	G	F_r
S_2	F	G	F
S_3	F_r	G	F_r
S_4	F_r	G	F_r
S_5	F_r	G	F_r
S_6	G	D	D
S_7	F_r	F_r	F_r
S_8	F_r	F_r	F_r
S_9	F_r	F_r	F_r
S_{10}	F	F	F
S_{11}	F_r	F_r	F_r
S_{12}	G	F_r	F_r
S_{13}	G	F_r	F_r
S_{14}	G	F_r	F_r
S_{15}	G	F	F
S_{16}	G	F_r	F_r

4. BLOCK AND STATE TRANSITION DIAGRAM

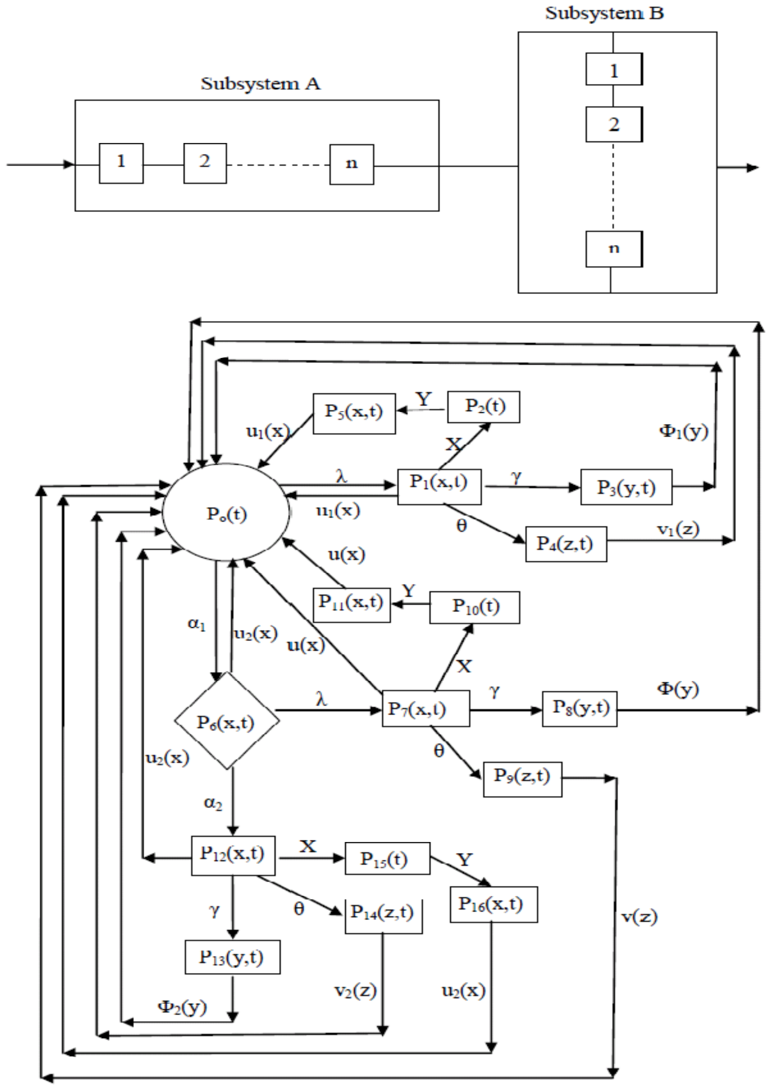

Figure 4.1. Block and state transition diagram

5. NOTATIONS

λ : Constant failure rate of subsystem A

$\alpha_{1,}\,\alpha_2$: Constant failure rates of subsystem B. Where $\alpha_1 = \sum_{i=1}^{K} \alpha_i$ and $\alpha_2 = \sum_{j=K+1}^{n} \alpha_j$

$P_i(t)$: Probability that the system is in S_i state at instant t for I = 1 to 16.

$\overline{P_i}(s)$: Laplace transform of $P_i(t)$
γ : Accident rate
θ : Maximum repair time
$u_1(x)$: Repair rate of subsystem A
$u_2(x)$: Repair rate of subsystem B
$\phi_1(y)$: Repair rate of subsystem A after an accident takes place
$\phi_2(y)$: Repair rate of subsystem B after an accident takes place
$v_1(z)$: Replacement rate of subsystem A
$v_2(z)$: Replacement rate of subsystem B
$E_p(t)$: Expected profit during the interval (0, *t*].
$K_1\ K_2$: Revenue per unit time and service cost per unit time respectively
$\phi(x)$: Coupled repair rate
$F_x(x)/F$: Marginal distribution of random variables, where $F_x(x) = e^x$ and $F = u_2(x), \phi_2(x), v_2(x)$

Letting $F_x(x) = e^x$ and $F = u_2(x), \phi_2(x), v_2(x)$, the expression for joint probability (failed state S_7 to good state S_0, failed state S_8 to good state S_0, failed state S_9 to good state S_0 and failed state S_{11} to initial state S_0) according to Gumbel-Hougaard family of copula is given by

$$u(x) = \exp\left[x^\theta + \log(u_2(x))^\theta\right]^{\frac{1}{\theta}},\ \phi(y) = \exp\left[y^\theta + \log(\phi_2(y))^\theta\right]^{\frac{1}{\theta}}$$
$$v(z) = \exp\left[z^\theta + \log(v_2(z))^\theta\right]^{\frac{1}{\theta}}$$

6. FORMULATION OF MATHEMATICAL MODEL

By probability consideration and continuity arguments the following difference-differential equations governing the behavior of the system seems to be good.

$$\left(\frac{d}{dt} + \lambda + \alpha_1\right)P_0(t) = \int_0^\infty u_1(x)\,P_5(x,t)\,dx + \int_0^\infty u_1(x)\,P_1(x,t)\,dx + \int_0^\infty v_1(z)\,P_4(z,t)\,dx + \int_0^\infty \phi_1(y)$$
$$P_3(y,t)\,dy + \int_0^\infty \phi_2(y)\,P_{13}(y,t)\,dy + \int_0^\infty v_2(z)\,P_{14}(z,t)\,dz + \int_0^\infty v(z)\,P_9(z,t)\,dz$$
$$\int_0^\infty \phi(y)\,P_8(y,t)\,dy + \int_0^\infty u(x)\,P_{11}(x,t)\,dx + \int_0^\infty u_2(x)\,P_{16}(x,t)\,dx + \int_0^\infty u_2(x)$$
$$P_6(x,t)dx + \int_0^\infty u_2(x)\,P_{12}(x,t)\,dx + \int_0^\infty u(x)P_7(x,t)dx \tag{6.1}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+X+\gamma+\theta+u_1(x)\right)P_1(x,t)=0 \tag{6.2}$$

$$\left(\frac{d}{dt}+Y\right)P_2(t)=0 \tag{6.3}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_1(y)\right)P_3(y,t)=0 \tag{6.4}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+v_1(z)\right)P_4(z,t)=0 \tag{6.5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_1(x)\right)P_5(x,t)=0 \tag{6.6}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)+\lambda+\alpha_2\right)P_6(x,t)=0 \tag{6.7}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u(x)+\gamma+X+\theta\right)P_7(x,t)=0 \tag{6.8}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi(y)\right)P_8(y,t)=0 \tag{6.9}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+v(z)\right)P_9(z,t)=0 \tag{6.10}$$

$$\left(\frac{d}{dt}+Y\right)P_{10}(t)=XP_7(t) \tag{6.11}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u(x)\right)P_{11}(x,t)=0 \tag{6.12}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+X+\theta+\gamma+u_2(x)\right)P_{12}(x,t)=0 \tag{6.13}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_2(y)\right)P_{13}(y,t)=0 \tag{6.14}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+v_2(z)\right)P_{14}(z,t)=0 \tag{6.15}$$

$$\left(\frac{d}{dt}+Y\right)P_{15}(t)=XP_{12}(t) \tag{6.16}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)\right)P_{16}(x,t)=0 \tag{6.17}$$

Boundary conditions:

$$P_1(0,t) = \gamma P_0(t) \tag{6.18}$$

$$P_3(0,t) = \gamma P_1(t) \tag{6.19}$$

$$P_4(0,t) = \theta P_1(t) \tag{6.20}$$

$$P_5(0,t) = Y P_2(t) \tag{6.21}$$

$$P_6(0,t) = \alpha_1 P_0(t) \tag{6.22}$$

$$P_7(0,t) = \lambda P_6(t) \tag{6.23}$$

$$P_8(0,t) = \gamma P_7(t) \tag{6.24}$$

$$P_9(0,t) = \theta P_7(t) \tag{6.25}$$

$$P_{11}(0,t) = Y P_{10}(t) \tag{6.26}$$

$$P_{12}(0,t) = \alpha_2 P_6(t) \tag{6.27}$$

$$P_{13}(0,t) = \gamma P_{12}(t) \tag{6.28}$$

$$P_{14}(0,t) = \theta P_{12}(t) \tag{6.29}$$

$$P_{16}(0,t) = Y P_{15}(t) \tag{6.30}$$

Initial condition:

$P_0(0) = 1$, and other state probabilities are zero at t=0.

7. SOLUTION OF THE MODEL

Taking Laplace transformation of (6.1) to (6.30) and on further simplification, one may obtain:

$$\overline{P}_0(s) = \frac{1}{A(s)} \tag{7.1}$$

$$\overline{P}_1(s) = \lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\frac{1}{A(s)} \tag{7.2}$$

$$\overline{P}_2(s) = \frac{X}{(s+Y)}\lambda\left[\frac{1-\overline{S}u_1(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\frac{1}{A(s)} \tag{7.3}$$

$$\overline{P}_3(s)=\gamma\,\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\left[\frac{1-\overline{S}\phi_1(s)}{s}\right]\frac{1}{A(s)} \tag{7.4}$$

$$'\overline{P}_4(s)=\theta\,\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\left[\frac{1-\overline{S}v_1(s)}{s}\right]\frac{1}{A(s)} \tag{7.5}$$

$$\overline{P}_5(s)=\frac{XY}{(s+Y)}\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]\frac{1}{A(s)} \tag{7.6}$$

$$\overline{P}_6(s)=\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\frac{1}{A(s)} \tag{7.7}$$

$$\overline{P}_7(s)=\lambda\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\frac{1}{A(s)} \tag{7.8}$$

$$\overline{P}_8(s)=\gamma\,\lambda\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_{2)}}\right]\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\left[\frac{1-\overline{S}_\phi(s)}{s}\right]\frac{1}{A(s)} \tag{7.9}$$

$$\overline{P}_9(s)=\theta\,\lambda\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\left[\frac{1-\overline{S}_v(s)}{s}\right]\frac{1}{A(s)} \tag{7.10}$$

$$\overline{P}_{10}(s)=\frac{X}{(s+Y)}\lambda\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\frac{1}{A(s)} \tag{7.11}$$

$$\overline{P}_{11}(s)=\frac{XY}{(s+Y)}\lambda\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\left[\frac{1-\overline{S}_u(s)}{s}\right]\frac{1}{A(s)} \tag{7.12}$$

$$\overline{P}_{12}(s)=\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\frac{1}{A(s)} \tag{7.13}$$

$$\overline{P}_{13}(s)=\gamma\,\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\left[\frac{1-\overline{S}_{\phi_2}(s)}{(s)}\right]\frac{1}{A(s)} \tag{7.14}$$

$$\overline{P}_{14}(s)=\theta\,\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\left[\frac{1-\overline{S}_{v_2}(s)}{s}\right]\frac{1}{A(s)} \tag{7.15}$$

$$\overline{P}_{15}(s)=\frac{X}{(s+Y)}\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\frac{1}{A(s)} \tag{7.16}$$

$$\overline{P}_{16}(s)=\frac{XY}{(s+Y)}\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right]\frac{1}{A(s)} \tag{7.17}$$

where

$$\begin{aligned}
A(s)={}&(s+\lambda+\alpha_1)-\frac{XY}{(s+Y)}\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\overline{S}_{u_1}(s)-\lambda\overline{S}_{u_1}(s+X+\theta+\gamma)-\\
&\theta\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\overline{S}_{v_1}(s)-\gamma\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\overline{S}_{\phi_1}(s)-\gamma\alpha_2\\
&\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\overline{S}_{\phi_2}(s)-\theta\alpha_2\,\alpha_1\\
&\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\overline{S}_{v_2}(s)-\theta\lambda\alpha_1\\
&\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u}(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\overline{S}_{v}(s)-\gamma\lambda\alpha_1\\
&\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u}(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\overline{S}_{\phi}(s)-\frac{XY}{(s+Y)}\lambda\alpha_1\\
&\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u}(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\overline{S}_{u}(s)-\frac{XY}{(s+Y)}\alpha_2\,\alpha_1\\
&\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\overline{S}_{u_2}(s)-\alpha_1\overline{S}_{u_2}(s+\lambda+\alpha_2)\\
&-\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\overline{S}_{u_2}(s+X+\theta+\gamma)-\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\\
&\overline{S}_{u}(s+\gamma+X+\theta)
\end{aligned} \tag{7.18}$$

It is worth nothing that

$$\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_3(s)+\overline{P}_4(s)+\overline{P}_5(s)+\overline{P}_6(s)+\overline{P}_7(s)+\overline{P}_8(s)+\overline{P}_9(s)+\overline{P}_{10}(s)+$$
$$\overline{P}_{11}(s)+\overline{P}_{12}(s)+\overline{P}_{13}(s)+\overline{P}_{14}(s)+\overline{P}_{15}(s)+\overline{P}_{16}(s)=\frac{1}{s} \tag{7.19}$$

8. EVALUATION OF LAPLACE TRANSFORMATION OF UP AND DOWN STATE PROBABILITIES

The Laplace Transformation of the probabilities that the system is in operable and down state at time '*t*' can be evaluated as follows.

$$\overline{P}_{up}(s)=\overline{P}_0(s)+\overline{P}_6(s)$$
$$=\frac{1}{A(s)}+\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\frac{1}{A(s)} \tag{8.1}$$

$$\overline{P}_{down}(s)=\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_3(s)+\overline{P}_4(s)+\overline{P}_5(s)+\overline{P}_7(s)+\overline{P}_8(s)+\overline{P}_9(s)+\overline{P}_{10}(s)+$$
$$\overline{P}_{11}(s)+\overline{P}_{12}(s)+\overline{P}_{13}(s)+\overline{P}_{14}(s)+\overline{P}_{15}(s)+\overline{P}_{16}(s)$$
$$=\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\frac{1}{A(s)}+\frac{X}{(s+Y)}\lambda\left[\frac{1-\overline{S}u_1(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\frac{1}{A(s)}$$
$$\gamma\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\left[\frac{1-\overline{S}\phi_1(s)}{s}\right]\frac{1}{A(s)}+\theta\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]$$
$$\left[\frac{1-\overline{S}v_1(s)}{s}\right]\frac{1}{A(s)}+\theta\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\left[\frac{1-\overline{S}v_1(s)}{s}\right]\frac{1}{A(s)}+\frac{XY}{(s+Y)}$$
$$\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\gamma+\theta)}{(s+X+\gamma+\theta)}\right]\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]\frac{1}{A(s)}+\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]$$
$$\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\frac{1}{A(s)}+\gamma\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_{2)}}\right]\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]$$
$$\left[\frac{1-\overline{S}_\phi(s)}{s}\right]\frac{1}{A(s)}+\theta\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\left[\frac{1-\overline{S}_v(s)}{s}\right]$$

$$\frac{1}{A(s)}+\frac{X}{(s+Y)}\lambda\alpha_1\left[\frac{1-\bar{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\bar{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\frac{1}{A(s)}+\frac{XY}{(s+Y)}$$
$$\lambda\alpha_1\left[\frac{1-\bar{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\bar{S}_u(s+\gamma+X+\theta)}{(s+\gamma+X+\theta)}\right]\left[\frac{1-\bar{S}_u(s)}{s}\right]\frac{1}{A(s)}+\alpha_2\,\alpha_1$$
$$\left[\frac{1-\bar{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\bar{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\frac{1}{A(s)}+\gamma\alpha_2\,\alpha_1\left[\frac{1-\bar{S}_{\phi_2}(s)}{(s)}\right]$$
$$\left[\frac{1-\bar{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\bar{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\frac{1}{A(s)}+\theta\alpha_2\,\alpha_1\left[\frac{1-\bar{S}_{v_2}(s)}{s}\right]$$
$$\left[\frac{1-\bar{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\bar{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\frac{1}{A(s)}+\frac{X}{(s+Y)}\alpha_2\,\alpha_1$$
$$\left[\frac{1-\bar{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\bar{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\frac{1}{A(s)}+\frac{XY}{(s+Y)}\alpha_2\,\alpha_1$$
$$\left[\frac{1-\bar{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\bar{S}_{u_2}(s+X+\theta+\gamma)}{(s+X+\theta+\gamma)}\right]\left[\frac{1-\bar{S}_{u_2}(s)}{s}\right] \quad . \tag{8.2}$$

9. ASYMPTOTIC BEHAVIOR OF THE SYSTEM

Using Abel's lemma in Laplace transformation viz.

$$\lim_{s\to 0}\left\{s\bar{F}(s)\right\}=\lim_{t\to\infty}F(t)=F(say)$$

provided that the limit on right hand exists in (8.1) and (8.2), the time independent up and down state probabilities are obtained as follows:

$$P_{up}=\frac{1}{A(0)}+\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\frac{1}{A(0)} \tag{9.1}$$

$$P_{down}=\lambda\left[\frac{1}{(X+\gamma+\theta+u_1)}\right]\frac{1}{A(0)}+\frac{X}{(s+Y)}\lambda\left[\frac{1}{(X+\gamma+\theta+u_1)}\right]\frac{1}{A(0)}+\gamma\lambda$$
$$\left[\frac{1}{(X+\gamma+\theta+u_1)}\right]\left[\frac{1}{s+\phi_1}\right]\frac{1}{A(0)}+\theta\lambda\left[\frac{1}{(X+\gamma+\theta+u_1)}\right]\left[\frac{1}{s+v_1}\right]$$
$$\frac{1}{A(0)}+\theta\lambda\left[\frac{1}{(X+\gamma+\theta+u_1)}\right]\left[\frac{1}{s+v_1}\right]\frac{1}{A(0)}+\frac{XY}{(s+Y)}\lambda\left[\frac{1)}{(X+\gamma+\theta+u_1)}\right]$$
$$\left[\frac{1}{(s+u_1}\right]\frac{1}{A(0)}+\lambda\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\left[\frac{1}{(\gamma+X+\theta+u)}\right]\frac{1}{A(0)}+\gamma\lambda\alpha_1$$

$$\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\left[\frac{1}{(\gamma+X+\theta+u)}\right]\left[\frac{1}{s+\phi}\right]\frac{1}{A(0)}+\theta\lambda\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]$$
$$\left[\frac{1}{(\gamma+X+\theta+u)}\right]\left[\frac{1}{s+v}\right]\frac{1}{A(0)}+\frac{X}{(s+Y)}\lambda\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]$$
$$\left[\frac{1}{(\gamma+X+\theta+u)}\right]\frac{1}{A(0)}+\frac{XY}{(s+Y)}\lambda\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\left[\frac{1}{(\gamma+X+\theta+u)}\right]$$
$$\left[\frac{1}{s+u}\right]\frac{1}{A(0)}+\alpha_2\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\left[\frac{1}{(X+\theta+\gamma+u_2)}\right]\frac{1}{A(0)}+\gamma\alpha_2\alpha_1$$
$$\left[\frac{1}{(s+\phi_2)}\right]\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\left[\frac{1}{(X+\theta+\gamma+u_2)}\right]\frac{1}{A(0)}+\theta\alpha_2\alpha_1\left[\frac{1}{s+v_2}\right]$$
$$\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\left[\frac{1}{(X+\theta+\gamma+u_2)}\right]\frac{1}{A(0)}+\frac{X}{(s+Y)}\alpha_2\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]$$
$$\left[\frac{1}{(X+\theta+\gamma+u_2)}\right]\frac{1}{A(0)}+\frac{XY}{(s+Y)}\alpha_2\alpha_1\left[\frac{1}{(\lambda+\alpha_2+u_2)}\right]\left[\frac{1}{(X+\theta+\gamma+u_2)}\right]$$
$$\left[\frac{1}{s+u_2}\right] \tag{9.2}$$

10. PARTICULAR CASE

(1) When repair follows exponential distribution

In this case the result can be derived by putting

$$\bar{S}_{u_1}(s+X+\gamma+\theta)=\frac{u_1(x)}{s+X+\gamma+\theta+u_1(x)},\bar{S}_{u_2}(s+\lambda+\alpha_2)=\frac{u_2(x)}{s+\lambda+\alpha_2+u_2(x)}$$
$$\bar{S}_{u_2}(s+X+\theta+\gamma)=\frac{u_2(x)}{s+X+\theta+\gamma+u_2(x)},\bar{S}_{u}(s+\gamma+X+\theta)=\frac{u(x)}{s+\gamma+X+\theta+u(x)}$$
$$\bar{S}_{\phi_1}(s)=\frac{\phi_1(x)}{s+\phi_1(x)},\bar{S}_{u}(s)=\frac{u(x)}{s+u(x)},\bar{S}_{v}(s)=\frac{v(x)}{s+v(x)},\bar{S}_{\phi_2}(s)=\frac{\phi_2(x)}{s+\phi_2(x)}$$
$$\bar{S}_{u_1}(s)=\frac{u_1(x)}{s+u_1(x)}\bar{S}_{u_2}(s)=\frac{u_2(x)}{s+u_2(x)},\bar{S}_{v_1}(s)=\frac{v_1(x)}{s+v_1(x)},\bar{S}_{\phi}(s)=\frac{\phi(x)}{s+\phi(x)} \tag{10.1}$$

in equations (7.1)-(7.18) , we get

$$\bar{P}_0(s)=\frac{1}{A(s)} \tag{10.2}$$

$$\overline{P}_1(s)=\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\overline{P}_0(s) \tag{10.3}$$

$$\overline{P}_2(s)=\frac{X}{(s+Y)}\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\overline{P}_0(s) \tag{10.4}$$

$$\overline{P}_3(s)=\gamma\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\left[\frac{1}{s+\phi_1(x)}\right]\overline{P}_0(s) \tag{10.5}$$

$$'\overline{P}_4(s)=\theta\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\left[\frac{1}{s+v_1(x)}\right]\overline{P}_0(s) \tag{10.6}$$

$$\overline{P}_5(s)=\frac{XY}{(s+Y)}\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\left[\frac{1}{s+u_1(x)}\right]\overline{P}_0(s) \tag{10.7}$$

$$\overline{P}_6(s)=\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\overline{P}_0(s) \tag{10.8}$$

$$\overline{P}_7(s)=\lambda\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\overline{P}_0(s) \tag{10.9}$$

$$\overline{P}_8(s)=\gamma\lambda\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\left[\frac{1}{s+\phi(x)}\right]\overline{P}_0(s) \tag{10.10}$$

$$\overline{P}_9(s)=\theta\lambda\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\left[\frac{1}{s+v(x)}\right]\overline{P}_0(s) \tag{10.11}$$

$$\overline{P}_{10}(s)=\frac{X}{(s+Y)}\lambda\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\overline{P}_0(s) \tag{10.12}$$

$$\overline{P}_{11}(s)=\frac{XY}{(s+Y)}\lambda\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\left[\frac{1}{s+u(x)}\right]\overline{P}_0(s) \tag{10.13}$$

$$\overline{P}_{12}(s)=\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\overline{P}_0(s) \tag{10.14}$$

$$\overline{P}_{13}(s)=\gamma\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\left[\frac{1}{s+\phi_2(x)}\right]\overline{P}_0(s) \tag{10.15}$$

$$\overline{P}_{14}(s)=\theta\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\left[\frac{1}{s+v_2(x)}\right]\overline{P}_0(s) \tag{10.16}$$

$$\overline{P}_{15}(s)=\frac{X}{(s+Y)}\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\overline{P}_0(s) \tag{10.17}$$

$$\overline{P}_{16}(s)=\frac{XY}{(s+Y)}\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\left[\frac{1}{s+u_2(x)}\right]\overline{P}_0(s) \tag{10.18}$$

where

$$\begin{aligned}
A(s)=(s+\lambda+\alpha_1)-&\frac{XY}{(s+Y)}\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\frac{u_1(x)}{s+u_1(x)}-\lambda\frac{u_1(x)}{s+X+\theta+\gamma+u_1(x)}-\\
&\theta\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\frac{v_1(x)}{s+v_1(x)}-\gamma\lambda\left[\frac{1}{(s+X+\gamma+\theta+u_1(x))}\right]\frac{\phi_1(x)}{s+\phi_1(x)}-\\
&-\gamma\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\frac{\phi_2(x)}{s+\phi_2(x)}-\theta\alpha_2\alpha_1\\
&\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\frac{v_2(x)}{s+v_2(x)}-\theta\lambda\alpha_1\\
&\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\frac{v(x)}{s+v(x)}-\gamma\lambda\alpha_1\\
&\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\frac{\phi(x)}{s+\phi(x)}-\frac{XY}{(s+Y)}\\
&\lambda\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+\gamma+X+\theta+u(x))}\right]\frac{u(x)}{s+u(x)}\\
&-\frac{XY}{(s+Y)}\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\left[\frac{1}{(s+X+\theta+\gamma+u_2(x))}\right]\\
&\frac{u_2(x)}{s+u_2(x)}-\alpha_1\frac{u_2(x)}{s+\lambda+\alpha_2+u_2(x)}-\alpha_2\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\\
&\frac{u_2(x)}{s+X+\theta+\gamma+u_2(x)}-\lambda\alpha_1\left[\frac{1}{(s+\lambda+\alpha_2+u_2(x))}\right]\\
&\frac{u(x)}{s+\gamma+X+\theta+u(x)}
\end{aligned}$$

(2) When accident do not take place when the failed unit is repaired by the repairman.

In this case the result can be derived by putting γ=0 in equations (7.1-18), Laplace transformation of various state probabilities are as follows:

$$\overline{P}_0(s)=\frac{1}{A(s)} \tag{10.19}$$

$$\overline{P}_1(s)=\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\theta)}{(s+X+\theta)}\right]\frac{1}{A(s)} \tag{10.20}$$

$$\overline{P}_2(s)=\frac{X}{(s+Y)}\lambda\left[\frac{1-\overline{S}u_1(s+X+\theta)}{(s+X+\theta)}\right]\frac{1}{A(s)} \tag{10.21}$$

$$\overline{P}_3(s)=0 \tag{10.22}$$

$$`\overline{P}_4(s)=\theta\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\theta)}{(s+X+\theta)}\right]\left[\frac{1-\overline{S}v_1(s)}{s}\right]\frac{1}{A(s)} \tag{10.23}$$

$$\overline{P}_5(s)=\frac{XY}{(s+Y)}\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\theta)}{(s+X+\theta)}\right]\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]\frac{1}{A(s)} \tag{10.24}$$

$$\overline{P}_6(s)=\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\frac{1}{A(s)} \tag{10.25}$$

$$\overline{P}_7(s)=\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+X+\theta)}{(s+X+\theta)}\right]\frac{1}{A(s)} \tag{10.26}$$

$$\overline{P}_8(s)=0 \tag{10.27}$$

$$\overline{P}_9(s)=\theta\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+X+\theta)}{(s+X+\theta)}\right]\left[\frac{1-\overline{S}_v(s)}{s}\right]\frac{1}{A(s)} \tag{10.28}$$

$$\overline{P}_{10}(s)=\frac{X}{(s+Y)}\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+X+\theta)}{(s+X+\theta)}\right]\frac{1}{A(s)} \tag{10.29}$$

$$\overline{P}_{11}(s)=\frac{XY}{(s+Y)}\lambda\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_u(s+X+\theta)}{(s+X+\theta)}\right]\left[\frac{1-\overline{S}_u(s)}{s}\right]\frac{1}{A(s)}\tag{10.30}$$

$$\overline{P}_{12}(s)=\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta)}{(s+X+\theta)}\right]\frac{1}{A(s)}\tag{10.31}$$

$$\overline{P}_{13}(s)=0\tag{10.32}$$

$$\overline{P}_{14}(s)=\theta\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta)}{(s+X+\theta)}\right]\left[\frac{1-\overline{S}_{v_2}(s)}{s}\right]\frac{1}{A(s)}\tag{10.33}$$

$$\overline{P}_{15}(s)=\frac{X}{(s+Y)}\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta)}{(s+X+\theta)}\right]\frac{1}{A(s)}\tag{10.34}$$

$$\overline{P}_{16}(s)=\frac{XY}{(s+Y)}\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\left[\frac{1-\overline{S}_{u_2}(s+X+\theta)}{(s+X+\theta)}\right]\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right]\frac{1}{A(s)}\tag{10.35}$$

where

$$\begin{aligned}A(s)={}&(s+\lambda+\alpha_1)-\frac{XY}{(s+Y)}\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\theta)}{(s+X+\theta)}\right]\overline{S}_{u_1}(s)-\lambda\overline{S}_{u_1}(s+X+\theta)-\\&\theta\lambda\left[\frac{1-\overline{S}_{u_1}(s+X+\theta)}{(s+X+\theta)}\right]\overline{S}_{v_1}(s)-\theta\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\\&\left[\frac{1-\overline{S}_{u_2}(s+X+\theta)}{(s+X+\theta)}\right]\overline{S}_{v_2}(s)-\theta\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\\&\left[\frac{1-\overline{S}_u(s+X+\theta)}{(s+X+\theta)}\right]\overline{S}_v(s)-\frac{XY}{(s+Y)}\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\\&\left[\frac{1-\overline{S}_u(s+X+\theta)}{(s+X+\theta)}\right]\overline{S}_u(s)-\frac{XY}{(s+Y)}\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\\&\left[\frac{1-\overline{S}_{u_2}(s+X+\theta)}{(s+X+\theta)}\right]\overline{S}_{u_2}(s)-\alpha_1\,\overline{S}_{u_2}(s+\lambda+\alpha_2)-\alpha_2\,\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\\&\overline{S}_{u_2}(s+X+\theta)-\lambda\alpha_1\left[\frac{1-\overline{S}_{u_2}(s+\lambda+\alpha_2)}{(s+\lambda+\alpha_2)}\right]\overline{S}_u(s+X+\theta)\end{aligned}\tag{10.36}$$

11. NUMERICAL COMPUTATION

(1) Reliability analysis

Assuming that the equation (10.1) holds then the reliability of the system is given by

$$\overline{R}(s) = \frac{1}{s+\lambda+\alpha_1} + \frac{\alpha_1}{(s+\lambda+\alpha_2)(s+\lambda+\alpha_1)} \tag{11.1}$$

Taking inverse Laplace Transformation, we have

$$R(t) = \frac{-\alpha_2 e^{(-(\lambda+\alpha_1)t)} + \alpha_1 e^{(-(\lambda+\alpha_2)t)}}{-\alpha_2+\alpha_1} \tag{11.2}$$

For numerical illustration, let us consider the values λ = 0.1, α_1 = 0.2, α_2 = 0.3 and t = 0, 1, 2, 3, …. Using these values in equation (11.2), we compute the Table 11.1 and corresponding graph has been shown in Figure 11.1.

(2) Availability analysis

Assuming the parameters λ = 0.1, α_1 = 0.2, α_2 = 0.3, γ = 0.4, θ = 0.5, X = 0.6, Y = 0.08, repair rates $\Phi = \Phi_1 = \Phi_2 = u = u_1 = u_2 = v = v_1 = v_2 = 1$, and x = y = z = 1, and letting that equation (10.1) holds. Putting all these values in equation (8.1) and taking inverse Laplace transformation, we get

$$P_{up} = 0.002330353689\exp(-2.505671778\,t) - 0.0655477237\exp(-1.526615558\,t) + 0.1436820545\exp(-1.133297414\,t) + 0.2743740253\exp(-0.1144152498\,t) + 0.6451612903 \tag{11.3}$$

Now in equation (11.3) setting t = 0,1,2,3,4,5,6,7,8,9,10, one can obtain Table 11.2. The same is shown in Figure 11.2 which shows the variation of availability with respect to time.

(3) M.T.T.F. analysis

Letting the equation (10.1) holds then the mean time to failure (M.T.T.F.) of the system is given by

$$M.T.T.F. = \lim_{s\to 0}\overline{R}(s) = \frac{\lambda+\alpha_2+\alpha_1}{(\lambda+\alpha_2)(\lambda+\alpha_1)} \tag{11.4}$$

1) Taking the values α_1 = 0.2, α_2 = 0.3, and λ = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, one can obtain Table11.3 which shows how M.T.T.F. changes with respect to λ.

2) Varying α_1 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, and keeping other parameter fixed at λ = 0.1, α_2 = 0.3, one can get the change of M.T.T.F. with respect to α_1 as given in Table11.4.

3) Assuming λ = 0.1, α_1 = 0.2, and α_2 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10. The variation of M.T.T.F. with respect to α_2 is obtained which is given in Table11.5.

(4) Busy period analysis

Let the equation (10.1) holds then Mean time to repair (M.T.T.R.) of the system is given by

$$M.T.T.R. = \lim_{s\to 0} \overline{P}_{down}(s)$$

Letting $\Phi = \Phi_1 = \Phi_2 = u = u_1 = u_2 = v = v_1 = v_2 = 1$, $x = y = z = 1$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\gamma = 0.4$, $\theta = 0.5$, $X = 0.6$, $Y = 0.08$ and varying λ as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, one can obtain Table 11.6. This is shown in Figure 11.6 which demonstrates how busy period changes with respect to λ.

(5) Sensitivity analysis

Assuming that the equation (10.1) holds. We first perform a sensitivity analysis for changes in R(t) resulting from changes in system parameters λ, α_1 and α_2. Differentiating equation (11.1) with respect to λ, we get

$$\frac{\partial R(s)}{\partial \lambda} = -\frac{1}{(s+\lambda+\alpha_1)^2} - \frac{\alpha_1}{(s+\lambda+\alpha_2)^2 (s+\lambda+\alpha_1)} - \frac{\alpha_1}{(s+\lambda+\alpha_2)(s+\lambda+\alpha_1)^2}$$

Taking inverse Laplace transform, we obtain

$$\frac{\partial R(t)}{\partial \lambda} = \frac{t\, e^{(-(\lambda+\alpha_1)t}\alpha_2 - t\, e^{(-(\lambda+\alpha_2)t}\alpha_1}{-\alpha_2+\alpha_1}$$

Using the same procedure described above, we can get $\frac{\partial R(t)}{\partial \alpha_1}$ *and* $\frac{\partial R(t)}{\partial \alpha_2}$.

Then we perform a sensitivity analysis of changes in M.T.T.F. with respect to λ, α_1, and α_2. Differentiating equation (11.4) with respect to λ, we obtain

$$\frac{\partial MTTF}{\partial \lambda} = \frac{1}{(\lambda+\alpha_2)(\lambda+\alpha_1)} - \frac{(\lambda+\alpha_2+\alpha_1)}{(\lambda+\alpha_2)^2(\lambda+\alpha_1)} - \frac{(\lambda+\alpha_2+\alpha_1)}{(\lambda+\alpha_2)(\lambda+\alpha_1)^2}$$

Using the same procedure, $\frac{\partial MTTF}{\partial \alpha_1}$ and $\frac{\partial MTTF}{\partial \alpha_2}$ can be obtained. Numerical results of the sensitivity analysis for the system reliability and the MTTF are presented in Figures11.7, 8, 9, 10, 11 and 12.

(6) Cost analysis

Assuming $\Phi = \Phi_1 = \Phi_2 = u = u_1 = u_2 = v = v_1 = v_2 = 1$, $x = y = z = 1$, $\lambda = 0.1$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\gamma = 0.4$, $\theta = 0.5$, $X = 0.6$, $Y = 0.08$. Furthermore if the repair follows exponential distribution (i.e. equation (10.1) holds), putting all these values in equation (8.1) and taking inverse laplace transform, one can obtain equation (11.3).

If the service facility is always available, then expected profit during the interval (0, *t*] is given by

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

where K_1 and $K_2$are the revenue per unit time and service cost per unit time respectively, then

$$E_p(t) = K_1(-0.0009300315027\exp(-2.505671778t) + 0.04293662760\exp(-1.5266$$
$$15558t) - 0.1267823016\exp(-1.133297414t) - 2.398054680\exp(-0.11441$$
$$52498t) + 0.6451612903t) + 2.482830386) - tK_2$$

Keeping K_1=1 and changing K_2 at 0.1, 0.2, 0.3, 0.4, 0.5, one can obtain Table 11.10 which is depicted by Figure 11.13.

Time	Reliability
0	1
1	0.8818145700
2	0.7477769798
3	0.6173205552
4	0.4997895997
5	0.3987199139
6	0.3144607580
7	0.2457491596
8	0.1906294519
9	0.1469690933
10	0.1127299273

Table 11.1. Time vs. Reliability

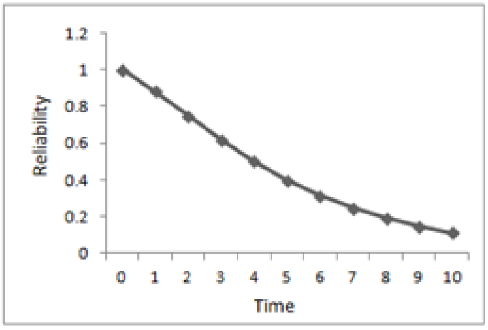

Figure 11.1. Time vs. Reliability

Time	Availability
0	1
1	0.9220820146
2	0.8752317809
3	0.8439443755
4	0.8201728250
5	0.8004703654
6	0.7834175908
7	0.7683837875
8	0.7550335375
9	0.7431457397
10	0.7325494011

Table 11.2. Time vs. Availability

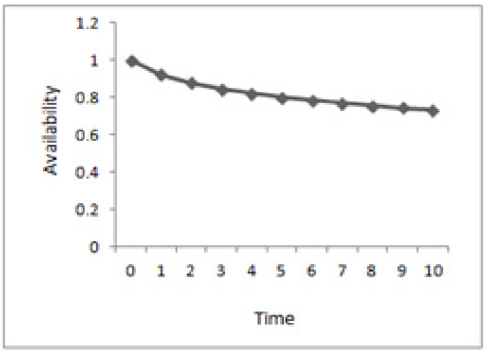

Figure 11.2. Time vs. Availability

λ	M.T.T.F.
0.01	7.834101381
0.02	7.386363636
0.03	6.982872200
0.04	6.617647058
0.05	6.285714284
0.06	5.982905985
0.07	5.705705707
0.08	5.451127818
0.09	5.216622459
0.10	5.000000000

Table 11.3. λ vs. MTTF

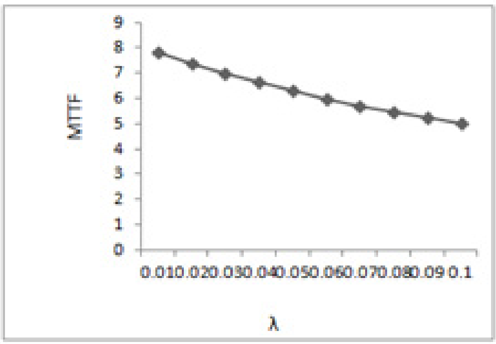

Figure 11.3. λ vs. MTTF

α_1	M.T.T.F.
0.01	9.318181818
0.02	8.750000000
0.03	8.269230769
0.04	7.857142857
0.05	7.500000000
0.06	7.187500000
0.07	6.911764706
0.08	6.666666667
0.09	6.447368421
0.10	6.250000000

Table 11.4. α_1 vs. MTTF

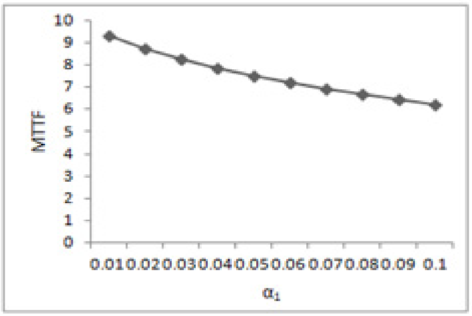

Figure 11.4. α_1 vs. MTTF

α_2	M.T.T.F.
0.01	9.393939393
0.02	8.888888890
0.03	8.461538460
0.04	8.095238097
0.05	7.777777777
0.06	7.500000000
0.07	7.254901960
0.08	7.037037037
0.09	6.842105263
0.10	6.666666667

Table 11.5. α_2 vs. MTTF

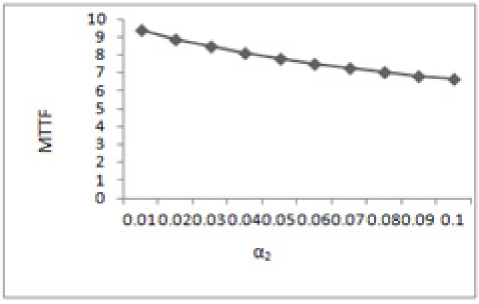

Figure 11.5. α_2 vs. MTTF

λ	Busy Period
0.01	1.091966558
0.02	1.245179063
0.03	1.384766263
0.04	1.512437811
0.05	1.629629630
0.06	1.737556562
0.07	1.837253311
0.08	1.929606626
0.09	2.015380799
0.10	2.095238095

Table 11.6. λ vs. Busy Period

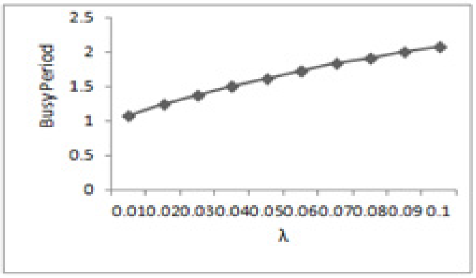

Figure 11.6. λ vs. Busy Period

λ	$\frac{\partial MTTF}{\partial \lambda}$
0.01	-47.21555635
0.02	-42.45222108
0.03	-38.34530213
0.04	-34.78229527
0.05	-31.67346938
0.06	-28.94659946
0.07	-26.54305958
0.08	-24.41489061
0.09	-22.52257535
0.10	-20.83333334

Table 11.7. Sensitivity analysis for MTTF with respect to λ

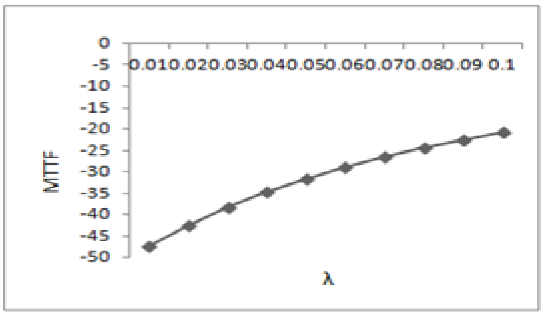

Figure 11.7. Sensitivity analysis for MTTF with respect to λ

α_1	$\frac{\partial MTTF}{\partial \alpha_1}$
0.01	-61.98347107
0.02	-52.08333334
0.03	-44.37869823
0.04	-38.26530612
0.05	-33.33333333
0.06	-29.29687500
0.07	-25.95155710
0.08	-23.14814815
0.09	-20.77562327
0.10	-18.75000000

Table 11.8. Sensitivity analysis for MTTF with respect to α_1

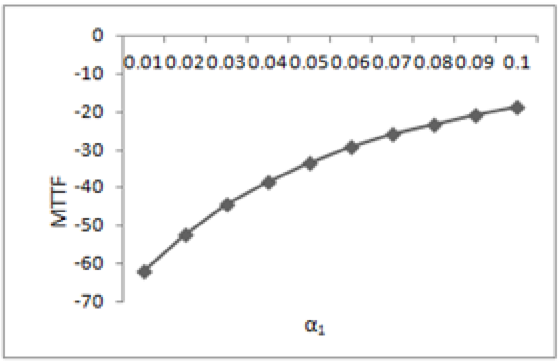

Figure 11.8. Sensitivity analysis for MTTF With respect to α_1

α_2	$\frac{\partial MTTF}{\partial \alpha_2}$
0.01	-55.09641873
0.02	-46.29629629
0.03	-39.44773176
0.04	-34.01360546
0.05	-29.62962965
0.06	-26.04166667
0.07	-23.06805076
0.08	-20.57613168
0.09	-18.46722068
0.10	-16.66666666

Table 11.9. Sensitivity analysis for MTTF with respect to α_2

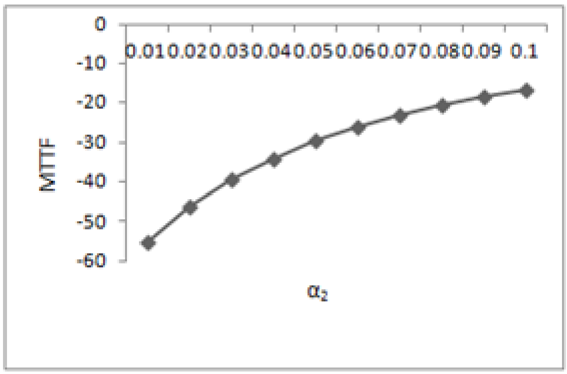

Figure 11.9. Sensitivity analysis for MTTF with respect to α_2

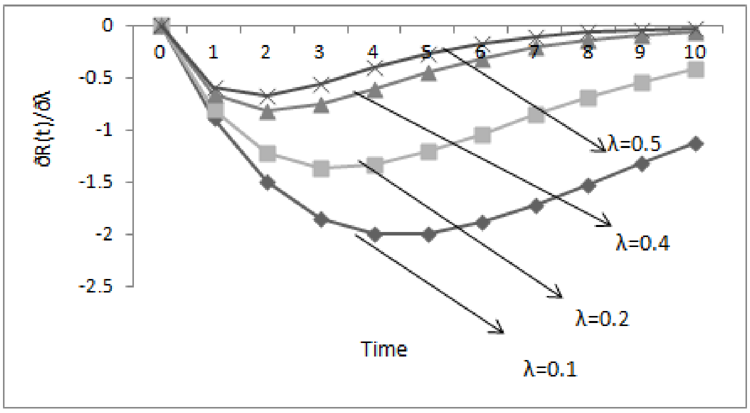

Figure 11.10. Sensitivity analysis for system reliability with various values of λ

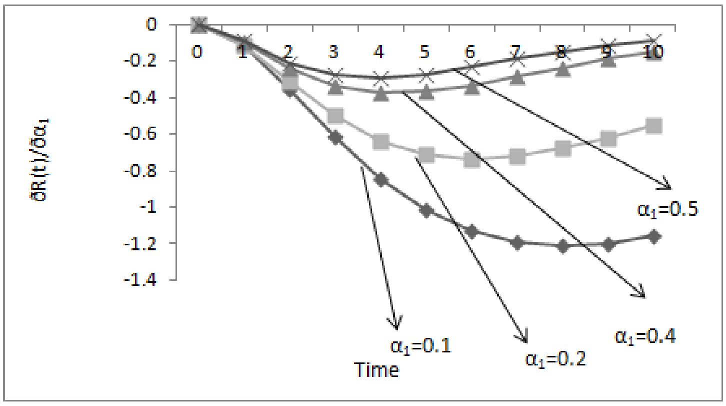

Figure 11.11. Sensitivity analysis for system reliability with various values of α_1

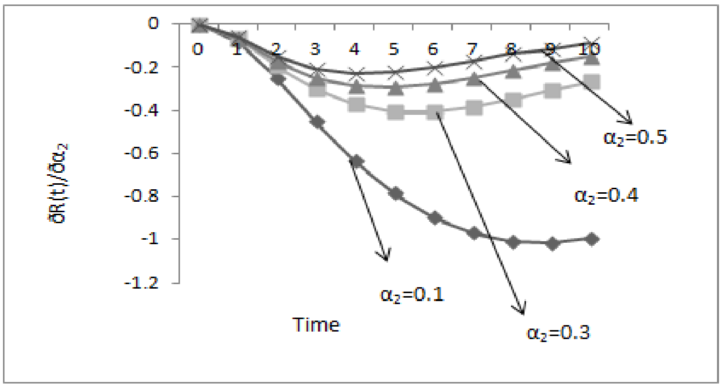

Figure 11.12. Sensitivity analysis for system reliability with various values of α_2

Table 11.10. Time vs. Expected Profit

Time	$E_p(t)$				
	K_2=0.1	K_2=0.2	K_2=0.3	K_2=0.4	K_2=0.5
0	0	0	0	0	0
1	0.857629472	0.757629472	0.657629472	0.557629472	0.457629472
2	1.654466194	1.454466194	1.254466194	1.054466194	0.854466194
3	2.413189453	2.113189453	1.813189453	1.513189453	1.213189453
4	3.144810992	2.744810992	2.344810992	1.944810992	1.544810992
5	3.854870839	3.354870839	2.854870839	2.354870839	1.854870839
6	4.546627106	3.946627106	3.346627106	2.746627106	2.146627106
7	5.222376092	4.522376092	3.822376092	3.122376092	2.422376092
8	5.883954727	5.083954727	4.283954727	3.483954727	2.683954727
9	6.532930141	5.632930141	4.732930141	3.832930141	2.932930141
10	7.170676408	6.170676408	5.170676408	4.170676408	3.170676408

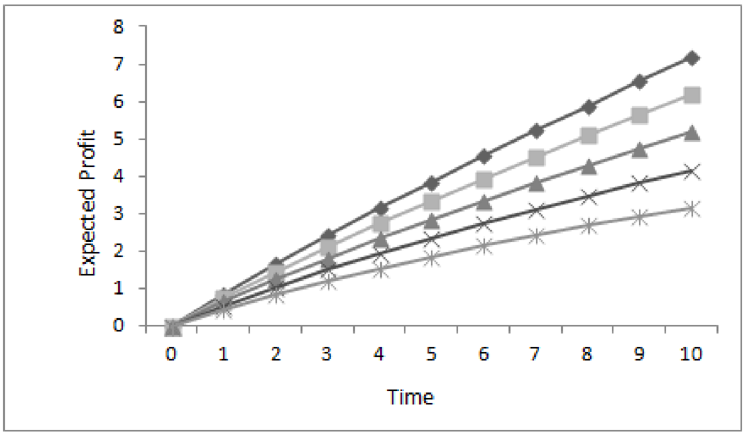

Figure 11.13. Time vs. Expected Profit

12. CONCLUSIONS

1) Figure 11.1 shows the graph of "Reliability vs. Time" and its values have given in Table 11.1. Analysis of Figure 11.1 yields that reliability of system decreases with the increment in time.
2) Figure 11.2 shows the graph of "Availability vs. Time" and its values have given in Table 11.2. Critical examination of Figure 11.2 concludes that availability decreases in the beginning but thereafter it decreases approximately in constant manner.
3) Figures 11.3, 4 and 5 are the graphs of "M.T.T.F. vs. λ", "M.T.T.F. vs. α_1" and "M.T.T.F. vs. α_2". The values have given in Tables 11.3, 4 and 5. Examination of Figures 11.3, 4 and 5, reveals that M.T.T.F. of considered system decreases as we increase in the values of failure rates λ, α_1 and α_2.
4) Figure 11.6 is the graph of "λ vs. Busy Period" and its values have given in Table 11.6. Observation of Figure 11.6 reveals that Busy Period increases as the value of λ increases.
5) The sensitivities of the system reliability with respect to system parameters λ, α_1 and α_2 are shown in Figures 11.10, 11 and 12. In Figure 11.10, along the time coordinate, we show the sensitivity of system reliability with respect to λ by varying λ from 0.1, 0.2, 0.4 and 0.5 when the other two parameter are fixed at $\alpha_1 = 0.2$ and $\alpha_2 = 0.3$. The sensitivities of various values of α_1 and α_2 on the system reliability are shown in Figure 11.11and Figure 11.12 respectively. The influence of λ, α_1 and α_2 on reliability increases as λ, α_1 and α_2 decreases. Moreover, Figures 11.7, 8 and 9 show that the gross sensitivity of various values of λ, α_1 and α_2 on the MTTF decreases from -47.21555635 to -20.83333334, -61.98347107 to -18.75000000 and – 55.09641873 to -16.66666666 as λ, α_1 and α_2 increases from 0.01 to 0.10.
6) Keeping revenue cost per unit time K_1 at value 1 and varying service cost K_2 as 0.1, 0.2, 0.3, 0.4 and 0.5, Figure 11.13 is obtained. This graph reveals an important conclusion that increasing service cost leads decrement in expected profit. The highest

and lowest values of expected profit are obtained to be 7.1706 and 0.4576 respectively for the assumed values. Expected profit decreases with the increment in K_2.

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