On the non-linearities of ship's restoring and the Froude-Krylov wave load part

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ABSTRACT: *When formulating a general, non-linear mathematical model of ship dynamics in waves the hydrostatic forces and moments along with the Froude-Krylov part of wave load are usually concerned. Normally radiation and the diffraction forces are regarded as linear ones. The paper discusses briefly few approaches, which can be used in this respect. The concerned models attempt to model the non-linearities of the surface waves; both regular and the irregular ones, and the non-linearities of the restoring forces and moments. The approach selected in the Laidyn method, which is meant for the evaluation of large amplitude motions in the 6 degrees-of-freedom, is presented in a bigger detail. The workability of the method is illustrated with the simulation of ship motions in irregular stern quartering waves.*

KEY WORDS: Froude-Krylov pressures; Nonlinear restoring; Stern quartering seas.

INTRODUCTION

The method called Laidyn (Matusiak, 2000b, 2001) is meant for the evaluation of ship motions in waves. Ship is regarded as a rigid intact body. The mathematical model behind the method comprises the elements of maneuvering and makes allowance for the non-linear large amplitudes motions in waves. The original version, meant for the regular waves only, was based on the so-called two-stage approach. At the first stage of this approach, linear approximation to the rigid body motion in waves is evaluated. A number of non-linearities involved in ship dynamics in waves are taken into account and the total response of ship in the six degrees-of-freedom is solved at the second stage. In particular the non-linearities of the rigid body dynamics, non-linear terms of the restoring and the Froude-Krylov forces, ship resistance, the forces developed by a propulsor and by a rudder are taken into account. Details of the method are given for instance in Matusiak (2007).

Extension of the method, aimed at dealing with the long-crested irregular waves, led to giving up the concept of the two-stage evaluation of the responses. Instead, a direct solution of ship response is evaluated in the time-domain. All other features of the method are preserved. Similarly as in the original method, the linear models represent the radiation forces and moments and also the diffraction part of the wave excitation acting on ship. It is worth noting that these forces and moments are oriented with the axes of the body-fixed co-ordinate system.

A linear surface wave theory of Airy is used to model surface waves. However, in order to take non-linearities of the Froude-Krylov loads into account, both the wetted surface of ship's hull and pressures are evaluated up to the actual position of free surface. This is done using a kinematical model involving a simple summation of the undisturbed component waves and knowing the position of a hull in space. Extrapolation of pressures beyond the linear model of Airy can in principle be done in two different ways. These are presented and discussed further in the paper

FORMULATION

For the sake of this paper completeness, a short description of the Laidyn method is presented in the following. A more detailed description can be found in the abovementioned

Equations of motion

Equations of ship rigid motions are given by a set of six expressions (1) given below (Matusiak, 2007) with u, v and w being the projections of the velocities of ship's centre of gravity in the Earth-fixed inertial co-ordinate system on the axes of the moving body-fixed system. The angular position of the ship is given by so-called modified Euler's angles denoted as ψ, θ and ϕ. Refer to Fig. 1 for the definitions of the inertia and body-fixed co-ordinate systems. In equations 1, X_g, Y_g, Z_g, K_g, M_g and N_g depict the components of global reaction force and moment vectors acting on the ship. These are given in the-body fixed co-ordinate system *xyz*. m and I_{ij}

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mean ship's mass and the components of the mass moment of inertia.

$$X_g - mg\sin\theta = m(\dot{u} + Qw - Rv)$$

$$Y_g + mg\cos\theta\sin\phi = m(\dot{u} + Rv - Pw)$$

$$Z_g - mg\cos\theta\cos\phi = m(\dot{u} + Pv - Qu)$$

$$K_g = I_x\dot{P} - I_{xy}\dot{Q} - I_{xy}\dot{R} + (I_xR - I_{zx}P - I_{zy}Q)Q$$
$$\quad - (I_yQ - I_{yz}R - I_{yx}P)R \qquad (1)$$

$$M_g = -I_{yx}\dot{P} - I_y\dot{Q} - I_{yz}\dot{R} + (I_xP - I_{xy}Q - I_{xy}R)R$$
$$\quad - (I_zR - I_{zx}P - I_{zy}Q)P$$

$$N_g = -I_{zx}\dot{P} - I_{zy}\dot{Q} - I_z\dot{R} + (I_yQ - I_{yz}R - I_{yx}P)P$$
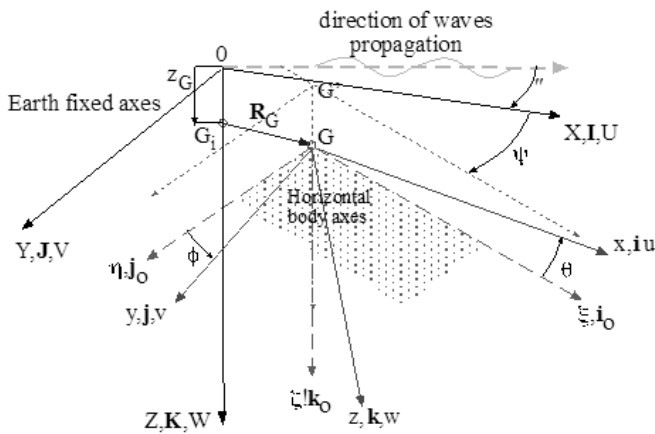$$\quad - (I_xP - I_{xy}Q - I_{xz}R)Q$$

Fig. 1 Co-ordinate systems used to describe ship motion (Matusiak, 2007)

The relation between the velocities of the ship's centre of gravity in the inertial co-ordinate system and their projections *u*, *v* and *w* on the axes of the moving body-fixed system is (Fossen, 1994; Clayton and Bishop, 1982)

$$\begin{Bmatrix} \dot{X}_G \\ \dot{Y}_G \\ \dot{Z}_G \end{Bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \begin{array}{c}\cos\psi\cos\phi\\-\sin\psi\cos\phi\end{array} & \begin{array}{c}\cos\psi\sin\theta\cos\phi\\+\sin\psi\sin\phi\end{array} \\ \sin\psi\cos\theta & \begin{array}{c}\sin\psi\sin\theta\sin\phi\\-\cos\psi\cos\phi\end{array} & \begin{array}{c}\sin\psi\sin\theta\cos\phi\\-\cos\psi\sin\phi\end{array} \\ -\sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

$$(2)$$

Moreover a relation between the angular velocity vector $\Omega = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ and the Euler's angles ψ, θ and ϕ is needed (Fossen, 1994; Clayton and Bishop, 1982)

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} \qquad (3)$$

Numerical Solution

Equations of motion 1 are solved numerically using the 4th order Runge-Kutta integration scheme yielding velocities *u, v, w, P, Q* and *R* in the co-ordinate system fixed with the moving ship. Equations 2 and 3 are used to integrate theses velocities into the ship's position in the inertial (Earth-fixed) co-ordinate system.

At each time step the components of global reaction force and moment vectors acting on the ship X_g, Y_g, Z_g, K_g, M_g and N_g have to be given. These include restoring, radiation and wave forces, ship resistance, the forces developed by a propulsor and a rudder. These are described in bigger detail in Matusiak (2001, 2002). An allowance for wind loading is included as well.

NON-LINEAR MODELS OF FROUDE-KRYLOV AND RESTORING FORCES AND MOMENTS

General on the non-linear models

There are a number of different approaches used in taking into account nonlinearities associated with restoring and Froude-Krylov forces and moments in waves. Some of these are direct extensions to the static buoyancy models. In this approach a simple or more sophisticated model of static lever (GZ-curve) in waves is considered (Hong et al., 2009; Vidic-Perunovic, 2009; Bulian and Francescutto 2008). A parametric variation of restoring term results in a Mathieu-type equation for roll motion, which in some cases gives a prediction of the so-called parametric roll resonance.

A multivariable Taylor expansion up to the third order can be used to model describe strongly coupled restoring terms of heave, roll and pitch. Also this approach is successfully used in a prediction of parametric roll resonance (Rodríguez et al., 2007).

Boundary element method, that is a panel method, either of a Rankine-type or utilizing a special Green function approach for unsteady free surface flows can be used in a linear or a non-linear form as well.

RANSE methods, that is the tools solving both the unsteady free surface flow problem using Reynolds-Averaged-Navier-Stokes equations and the body dynamics problem are already in use. As they are nearly free of any assumptions they may be regarded as the most sophisticated and reliable methods. However, their usage at present is mainly of a demonstrative nature only, as they require a lot of computer resources and take a lot of time to execute.

A more profound and detailed description of the methods used in predicting large amplitude motions in waves can be found in the reports of the ITTC Committees on Seakeeping and Stability.

In the next paragraph I will concentrate on a very restricted problem of modeling the non-linearities of hydrostatic and Froude-Krylov pressures in the panel-type seakeeping method.

Non-linear hydrostatic and Froude-Krylov forces and moments in the context of a panel representation of ship hull

When considering ship motions in waves, it is commonly believed that the most important contributors to the non-linearities in the external forces acting on a ship hull are the restoring and Froude-Krylov forces and moments. Evaluation of these is done using a wetted surface of ship hull represented by a discrete panel model. This takes into account both an instantaneous position of ship in space and pressure due to waves extending up to the actual water surface. This is done as follows.
Position of each control point c, that is a centre point of a panel, of Fig. 2 below is transformed from the body-fixed co-ordinate system (x_c, y_c, z_c) to the inertial Earth-fixed system (X_c, Y_c, Z_c) using transformation given by the following formula which is similar to the transformation 2.

$$\begin{Bmatrix} X_c \\ Y_c \\ Z_c \end{Bmatrix} = \begin{Bmatrix} X_G \\ Y_G \\ Z_G \end{Bmatrix}$$

$$+ \begin{bmatrix} \cos\psi\cos\theta & \begin{matrix}\cos\psi\cos\phi \\ -\sin\psi\cos\phi\end{matrix} & \begin{matrix}\cos\psi\sin\theta\cos\phi \\ +\sin\psi\sin\phi\end{matrix} \\ \sin\psi\cos\theta & \begin{matrix}\sin\psi\sin\theta\sin\phi \\ -\cos\psi\cos\phi\end{matrix} & \begin{matrix}\sin\psi\sin\theta\cos\phi \\ -\cos\psi\sin\phi\end{matrix} \\ -\sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{Bmatrix} x_c \\ y_c \\ z_c \end{Bmatrix}$$
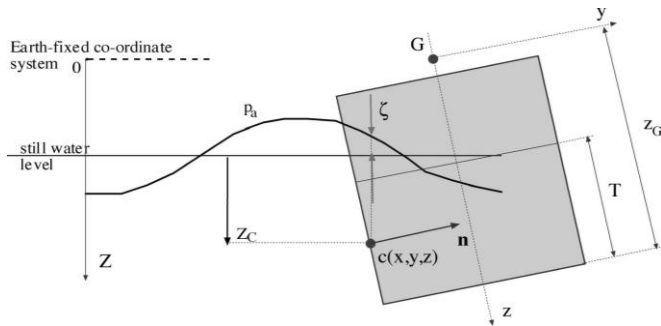
$$(4)$$

Fig. 2 Evaluation of hydrostatic and Froude-Krylov pressure.

Wave elevation above the control point c is given by a sum over the wave components N

$$\zeta(t) = \sum_{i=1}^{N} A_i \cos\left[k_i (X_c \cos\mu - Y_c \sin\mu) + \omega_i t + \delta_i \right] \quad (5)$$

where A_i and $k_i = \omega_i^2/g$ are wave amplitude and wave number corresponding to the i-th wave component. Phase angle δ_i of each wave component is a random number. Details on generating wave trains from a given wave spectra are given for instance in Naito (1995) or Matusiak (2000a).

There are three models for evaluating the pressure at point c. The first one is a linear Froude-Krylov pressure

model with the wetted surface extending up to the still water level. It is worth noting that restoring forces and moments are taking non-linearity into account. In this model pressure is given by the expression

$$p_c(t) = \rho g \left(\begin{matrix} Z_c + \sum_{i=1}^{N} A_i \exp\left(-k_i Z_c\right) \\ \times \cos\left[k_i \left(X_c \cos\mu - Y_c \sin\mu\right) - \omega_i t + \delta_i \right] \end{matrix} \right) \quad (6)$$

Two other models take into account the actual wetted surface. The pressures are evaluated for the immersed panels, that is for $Z_c + \zeta(t) > 0$.

The first of these models, which is presented in Faltinsen (1990), is similar to the one given by Formula 6 but with a linear extrapolation of pressure between the still water and actual water levels. Thus it can be understood as an extension of the linear model. The third model, called stretched pressure model, is given by the formula with the free surface raised by the amount of wave elevation (5) in the argument of the exponent function.

$$p_c(t) = \rho g \left(\begin{matrix} Z_c + \sum_{i=1}^{N} A_i \exp\left(-k_i Z_c + \zeta(t)\right) \\ \times \cos\left[k_i \left(X_c \cos\mu - Y_c \sin\mu\right) - \omega_i t + \delta_i \right] \end{matrix} \right) \quad (7)$$

The forces **F** and moments **M** are obtained by integrating the pressure (6 or 7) in the body fixed co-ordinate system. This integration is performed numerically by summing up the contribution from each wetted panel using

$$F_{F,K}^{total} = \sum_{i}^{M} F_{F,K,i}^{total} = \sum_{i}^{M} \rho_i \Delta S_i n_i$$

$$F_{F,K}^{total} = \sum_{i}^{M} r_i \times F_{F,K,i}^{total} \quad (8)$$

where the total number of the panels is denoted by M, ΔS_i is panel area, n_i unit vector normal to panel and r_i the position vector of the control point in the body-fixed co-ordinate system xyz.

SHIP BEHAVIOR IN IRREGULAR WAVES USING THREE DIFFERENT MODELS OF FROUDE-KRYLOV PRESSURE

Ship motions in irregular waves were evaluated using the above-described three models of Froude-Krylov pressures. The investigated vessel is the one used in the benchmark study initiated by the International Towing Tank Conference and presented in (Spanos and Papanikolaou, 2009). This is a containership of waterline length of $L_{PP} = 150$ m. Metacentric height was set to $GM_0 = 1.2$ [m] yielding the natural roll period of $T_\phi = 21$ [s]. A model of this ship was also investigated earlier in a similar study (ITTC, 2002).

In all three cases the same operational condition was set. The desired significant wave height was set to H_S=5 [*m*] and period T_1=7.3 [*s*]. In order to save the computing time, wave spectrum was represented by *N*=19 wave components only. Ship was set to the stern quartering long-crested waves at heading μ = 30 [*deg*]. In simulations ship is propelled with a propeller and steered with a rudder under a *PD*-control. Ship operated for 40 *minutes* in each numerical test. Summary of the results is presented in Table 1 below.

Each of the irregular realization of waves was different due to a randomness built into the waves' generation algorithm. Heave, roll and pitch motion components, as computed in time-domain, are presented in terms of their standard deviations, maxima and minima.

Table 1 Summary of the results

FK-linear	Wave [*m*]	Heave [*m*]	Heave (linear)	Roll [*deg*]	Roll (linear)	Pitch [*m*]	Pitch (linear)
stdev	1.15	0.20	0.19	3.99	1.09	0.58	0.64
Max	2.98	0.46	0.46	11.65	3.40	1.44	1.36
Min	-3.62	-0.52	-0.46	-9.78	-3.10	-0.98	-1.43
Faltinsen	Wave [*m*]	Heave [*m*]	Heave (linear)	Roll [*deg*]	Roll (linear)	Pitch [*m*]	Pitch (linear)
stdev	1.33	0.27	0.26	4.04	1.16	0.76	0.87
Max	3.21	0.56	0.55	12.34	3.43	1.64	1.71
Min	-3.19	-0.69	-0.58	-11.20	-3.63	-1.47	-1.71
Stretched	Wave [*m*]	Heave [*m*]	Heave (linear)	Roll [*deg*]	Roll (linear)	Pitch [*m*]	Pitch (linear)
stdev	1.14	0.19	0.18	2.63	1.09	0.49	0.56
Max	3.16	0.46	0.44	6.94	3.47	1.35	1.43
Min	-3.60	-0.45	-0.43	-6.82	-2.94	-1.10	-1.37

Linear approximation to the global responses of ship in irregular waves is evaluated in order to judge the effects of non-linearity on the derived responses. Normally, in the linear seakeeping theory, a constant forward speed is assumed. In the Laidyn method surge motion of ship is evaluated in the time domain taking into account amongst the others propeller action and variations of the wetted surface. Thus in-plane motion of ship is simulated in time-domain along with the other motion components. This results in ship position X_G, Y_G in the Earth-fixed co-ordinate system.   This and the knowledge of transfer functions of the corresponding responses make it possible to evaluate linear approximation of the responses using the expressions

$$z_L(t) = \sum_{i=1}^{N} A_i z_{L0} \cos\left[k_i(X_G \cos\mu - Y_G \sin\mu) - \omega_i t + \delta_i - \gamma_{zi}\right]$$

$$\phi_L(t) = \sum_{i=1}^{N} k_i A_i \phi_{L0} \cos\left[k_i(X_G \cos\mu - Y_G \sin\mu) - \omega_i t + \delta_i - \gamma_{\phi i}\right]$$

$$\theta_L(t) = \sum_{i=1}^{N} k_i A_i \theta_{L0} \cos\left[k_i(X_G \cos\mu - Y_G \sin\mu) - \omega_i t + \delta_i - \gamma_{\theta i}\right]$$

$$(9)$$

where terms with subscripts *L0* depict gain factors of the transfer functions and γ corresponding phase angles The transfer functions were obtained with the software based on linear seakeeping theory (Journee, 1992).

There are no significant differences between the results obtained with different models for Froude-Krylov pressures, except for the roll motion. Time domain simulations with Laidyn give much higher roll angles than the linear frequency domain strip theory. The selected operational conditions that

is a combination of wave period, heading and ship speed yield frequently a resonant roll motion, which is visible in the selected time histories of the responses presented in Figs. 3, 4 and 5. Simulated case is critical for the ship in this particular case because of a kind of focusing effect of waves.

The encounter period of majority of waves is very close to the natural period of roll. As a result a resonant roll motion develops frequently during the simulation period.

The selected records represent maxima of roll motion in irregular waves for each model of Froude-Krylov pressure. Built-up of a roll develops for the wave groups having the encounter period close to the natural period of roll. The same cannot be seen with a fully linear frequency domain analysis. Linear solution relates roll angle to an instantaneous value of wave slope and thus it does not have relation to wave grouping. It is worth noting that roll damping was kept same valued in all models.
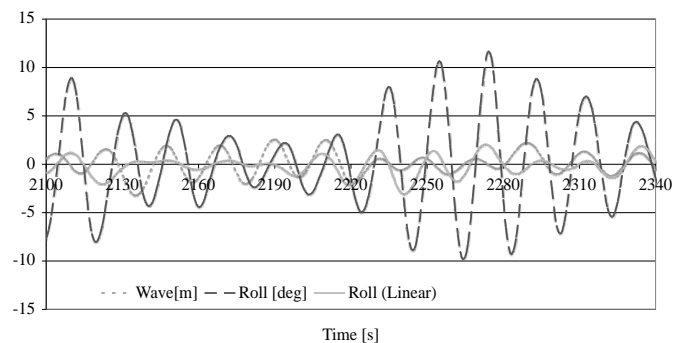

Fig. 3 Built-up of a resonant roll motion in irregular stern quartering waves simulated using a linear Froude-Krylov pressure model.
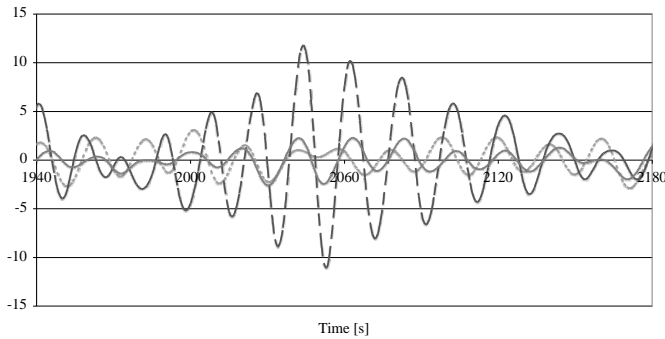
Fig. 4 Built-up of a resonant roll motion in irregular stern quartering waves simulated using the wave pressure model of Faltinsen.
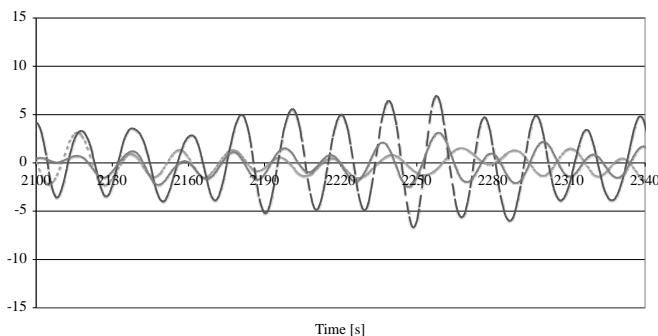

Fig. 5 Built-up of a resonant roll motion in irregular stern quartering waves simulated using stretched wave pressure model.

CONCLUSIONS

It is impossible to draw a conclusion which of the models used in the presented time-domain simulations is best one. For the investigated case, all three yield much higher roll angles than the ones evaluated by the fully linear frequency domain model. A development of roll resonance for the investigated situation is known from the literature (Kluwe and Krüger, 2007) and acknowledged by the Authorities (IMO, 1995, 2006). A further research is needed to validate the method. In particular model tests in both regular and also in irregular quartering waves will provide better validation data for the method.

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