Computer Simulation: A Hybrid Model for Traffic Signal Optimisation

Mohamed Kamal JBIRA* and Munir AHMED**

Abstract—With the increasing number of vehicles in use in our daily life and the rise of traffic congestion problems, many methods and models have been developed for real time optimisation of traffic lights. Nevertheless, most methods which consider real time physical queue sizes of vehicles waiting for green lights overestimate the optimal cycle length for such real traffic control. This paper deals with the development of a generic hybrid model describing both physical traffic flows and control of signalised intersections. The firing times assigned to the transitions of the control part are considered dynamic and are calculated by a simplified optimisation method. This method is based on splitting green times proportionally to the predicted queue sizes through input links for each new cycle time. The proposed model can be easily translated into a control code for implementation in a real time control system.

Keywords—Traffic Congestion, Hybrid Model, Optimisation Method

1. Introduction

1.1 Problem statement

The rapid increase of the number of vehicles within the limited capacities of urban traffic networks, has led to traffic congestion becoming a predominant problem giving rise to transportation delays. The most prevalent interest in managing urban traffic areas and road networks is traffic signal control. Traffic signal control has been recognised as an important means of solving the traffic congestion problem. The ideal objective is to allow all vehicles that approach a traffic signal for crossing an intersection area to receive a green signal and then be left with zero queues everywhere. This ideal condition of multidirectional green waves cannot be achieved in real application, but continues to represent the desirable objective.

A variety of models, methods, and strategies have been developed and applied for controlling urban traffic via signalised intersections. The currently available methods and strategies utilising traffic-light controls may be classified into two categories [1-6]. First: fixed-time strategies, which consider a given time of day and determine the optimal green times and optimal cycle times, based on the historical constant demands over the considered signalised urban area. Second: traffic-response strategies, which are based on constructing real-time control systems with optimum signal settings.

The most used strategies for urban traffic control are fixed-time strategies [3, 4], nevertheless, nowadays; more interest is being given to developing control strategies in which the control

Manuscript received June 25, 2010; accepted December 20, 2010.
Corresponding Author: Munir Ahmed
* IPEIM, University of Monastir, Tunisia (kamel.jbira@ipeim.rnu.tn)
** College of Computer Science and Engineering, Taibah University, Saudi Arabia (mahmed@taibahu.edu.sa)

system is fully responsive. Many traffic-response strategies have been developed and are based on dynamic programming optimisation procedures, which allow continual incremental adjustments in real-time of cycle lengths, splits, and offsets [3]. In general, for both fixed-time and traffic-response strategies, a modelling of the urban traffic network is essential either for simulation purposes or for the implementation requirements of a real time control system. Moreover, it is crucial to consider real parameters in any work of modelling or development of an optimisation method.

1.2 Related work

It is well known that urban traffic networks are considered to be Discrete Event Systems (DESs). Among the modelling tools applied for studying behaviour of a DES, Petri Nets (PNs) provide a simple and clear means for analysing and synthesis. Moreover, PNs are able to capture the precedence relations and interactions among the concurrent and asynchronous events typical of DES [5-7]. Different classes of PNs are applied in traffic control. In [8] authors proved the real PN capabilities for traffic control analysis through a representation of a network of signalised intersections via coloured Petri nets. In [9] authors chose deterministic and stochastic Petri nets as the modelling tools. In [10], a signal timing plan was proposed by timed Petri nets. Recently, [11] proposed an urban traffic network model via coloured timed Petri nets for control purposes.

Many others' works considered urban traffic networks to be hybrid systems and are interested in using Hybrid Petri Nets (HPNs) to model and analyse these systems [12, 13]. Recently, the HPNs models proposed by [14, 15] attempted to state and solve the problem of coordinating several traffic lights with the aim of improving the performance of some classes of special vehicles (public and emergency vehicles). More recently, in [16] authors proposed a new framework for traffic flow control based on an integrated model description by means of a hybrid dynamical system. They transformed traffic flow into a mixed logical dynamical system form to introduce an optimisation technique. The adopted technique enabled them to optimise the control policies for traffic light systems.

1.3 Proposed modelling framework

In the present work, a signalised intersection with its input and output flows is considered to be a hybrid system, thereby including both continuous-time and discrete-event components. A class of HPN is then proposed for modelling purposes. Hence, the vehicle flow behaviour is represented by means of a Continuous Petri Net (CPN), and a Timed Petri Net (TPN) represents the traffic light controller. In section 3.1 a recall of basic definitions of HPNs is introduced. In this modelling framework, we considered one of the important parameters representing the dynamicity of modelled systems i.e. queue sizes of vehicles in input links. This vital parameter requires a real- time estimation based on real observation which can be accomplished by a simple prediction procedure before being used by the model. Firing times assigned to transitions in this model will then be dynamic and estimated proportionally to the predicted queue sizes in input links for each new simulated cycle time.

Moreover, we are interested in this work on signalised intersections, which are distinguished by their specific locations near public institutions such as hospitals, schools, higher colleges, or any other similar establishment. These types of intersections, unlike other signalised intersections are characterised by amazing changes of flow rates in input links according to peak periods per day. For this particular reason, physical queue sizes of vehicles in real time are considered the main parameter for this framework.

This paper is organised as follows. In section 2, we present a discussion of a method optimisation based on considering queue lengths from different directions approaching an intersection. A generic hybrid Petri net model for signalised intersections is described in section 3. In sections 4 and 5 a study case and some simulation results are presented. Finally, we conclude in section 6 with some comments on the proposed modelling framework and a brief description of on-going extensions of this work.

2. FORMULATION OF THE SIGNAL OPTIMISATION PROBLEM

Traffic light plan optimisation is one of the most complex problems as it requires all the control strategies together. In this section, a description of the model formulation for the signal optimisation problem based on observed queue sizes in real time is presented. First, a general signalised intersection is defined, then, information and parameters of the considered signalised intersections are stated, and finally a method for calculation of green durations for one cycle is developed.

2.1 Definition of a signalised intersection

Signalised intersections can be defined as nodes of an urban traffic network connecting adjacent roads from which vehicle flows come and to which vehicle flows go. At each signalised intersection, a multi-phase traffic light rules the vehicle flows by means of the light signal, which can assume the three usual logic values: green, amber, and red.

In general, a generic signalised intersection consists of a set $IL = \{IL_i / i = 1 \text{ to } n\}$ of n incoming links, a set $OL = \{OL_j / j = 1 \text{ to } m\}$ of m outgoing directions, and one crossing area. In addition, each input link or output direction may have one or more lanes. To each input link IL_i, we have a unique, independent queue size Q^i of vehicles waiting for the green signal with an input vehicle flow Φin^i and an output vehicle flow Φout^i. Output vehicle flow represents the flow crossing the intersection, entering from the incoming link i when enabled by the traffic light. In the other hand, for each outgoing direction OL_j we have an output vehicle flow φ^j.

2.2 Parameters of an isolated intersection

In general, models of urban traffic flows are developed from point control of a single intersection to surface control of a whole traffic network. Since the first use of a signalised intersection, many traffic theories have been developed and proposed. In the 1950s, the famous British transportation researcher, F. V. Webster, developed a series of useful traffic theories, which continue to have a very big influence on contemporary traffic analysis [17, 18]. Webster conducted a series of experiments on pre-timed isolated intersection operations. Two traffic signal timing strategies came from his study. One is signal phase splits, for which Webster demonstrated, both theoretically and experimentally, that pre-timed signals should have their critical phases timed for the equal degrees of saturation for a given cycle length to minimise the delay. The other is the calculation of the optimal minimum delay cycle length, for which it was assumed that the effective green times of the phases were in the ratio of their respective flow ratios. This important theory had a very useful impact on traffic design and planning and quite practically minimises the resulting delay at an isolated pre-timed signalised intersection, nevertheless this method is closely related to the delay calculation.

Even in the earlier days when signalised intersection theories were just beginning, there were few control strategies which considered the dynamic of vehicle queue sizes as observed in real time in all input links. For example, the OPAC strategy considered the number of vehicles in a queue multiplied by the interval of time in which the cars were queued as the cost to be optimised [19]. The method of [20] considered the sum of the weighted time delay for each vehicle in the optimisation algorithm.

Recently, several other models and algorithms have been developed to optimise control of traffic signals in real time [21-24]. Most of these traffic-responsive schemes were based on selecting on-line predefined plans from a set of plans generated off-line or optimal sequences of green times using the predicted arrivals of vehicles.

In general, the possible traffic flows for an isolated intersection is related to the number of allowed outgoing directions. Many phases' schemes can be defined for one intersection (A phase of traffic is defined as the flows of vehicles that may proceed through an intersection without conflict). In this work we consider that all input links are enabled to be successive, i.e., flows enabled during each phase are corresponding only to one incoming link, which has right of way and is going towards different enabled directions.

The delay of vehicles passing through an intersection from one direction can be many times longer than those coming from other sides, and then for long (standing) queues, only a small portion from the front of the queue will cross the intersection when the light turns green and the others will be delayed until the next cycle time. This is the prediction if queue sizes prove to make a huge difference and the applied time plan does not afford importance to their lengths.

Basically, a generic signalised intersection may be described by the following information:

- Queue sizes on all incoming links: $\mathbf{Q^i}$ **[veh]** = $\mathbf{[Q^1(t), Q^2(t), \ldots, Q^n(t)]}$;
- Green times corresponding to the incoming links: $\mathbf{G^i}$ **[sec]** = $\mathbf{[G^1(t), G^2(t), \ldots, G^n(t)]}$;
- Input flow rates of different incoming links: $\mathbf{\Phi in^i}$ **[veh/sec]** = $\mathbf{[\Phi in^1(t), \Phi in^2(t), \ldots, \Phi in^n(t)]}$;
- Output flow rates of different incoming links: $\mathbf{\Phi out^i}$ **[veh/sec]** = $\mathbf{[\Phi out^1(t), \Phi out^2(t), \ldots, \Phi out^n(t)]}$;
- Output flow rates of different outgoing directions: $\boldsymbol{\varphi^i}$ **[veh/sec]** = $\boldsymbol{[\varphi^1(t), \varphi^2(t), \ldots, \varphi^m(t)]}$;
- Saturation flow. It is the average flow crossing the stop line of one link when the corresponding stream has right of way;
- Current cycle time **C(t) [sec]**;
- Next cycle time **C(t+1) [sec]**.

2.3 Calculation of green durations

As previously discussed, the green light times for the traffic flow should depend on the queue sizes of vehicles for different input links. Therefore, to optimise systematic performance is to optimise green durations for one cycle time, i.e., to minimise the vehicles delayed in input links.

This work considers predicated queue size of next cycles by taking into account the real number of vehicles measured at the end of actual cycle time plus the preventive number of arriving new vehicles for the next cycle.

Hence, the investigated optimal green lights time required for each input link IL_i during a next cycle is determined according to the prediction of queue size by the input link: $Q^i(t+1)$. The prediction model that calculates the probable queue size of arrival vehicles for an input link IL_i during the next cycle time C (t+1) may be considered as follows:

$$Q^i(t+1) = Q^i(t) + \phi in^i(t+1).\sum_{k=1}^{i-1} G^k(t) \tag{1}$$

t is the time allocation of each cycle time. For our present purposes, we suppose the period of one cycle time has no variation and is equal to a fixed period time T. In the global optimisation of a whole traffic network, this time should be dynamic according to global optimisation criteria. Furthermore, we consider that the duration of amber light is too limited and fixed for each timing plan.

i denotes direction serial number. For example if we have four directions, i values can be 1, 2, 3 and 4, or W, N, E, and S standing for West, North, East and South directions.

To predict this queue size for one input link, we need first to predict traffic flow rates for this link: $\Phi^i(t+1)$. Because each input link has different traffic flow at different times during one cycle, it is impossible to predict exactly the queue size during the next cycle time. The prediction principle which is adopted in this work is to predict traffic flow for the next cycle time by using the real flow values of the actual cycle one (actual time interval). In addition, the considered prediction model will be applied through the following suppositions:

- first: we consider the measurement of queue sizes for different input links to be at the end of the actual cycle time;
- second: we consider the green light times of the next cycle are close/near to these of an actual cycle. Then they can be emerged in the probability calculation: $G^i(t+1)= G^i(t)$.
- third: Output flow rates Φout^i are considered to be confused with the saturation flow for all input links and equal to the average flow crossing the stop line when the corresponding stream has right of way.

Moreover, to predict the traffic flow rate during the next cycle time, we consider the average of the traffic flows based on detected flow information of each input link during actual cycle time: Avg^i . The predicted traffic flow rate during the next cycle time is then considered by the following expression: $\Phi^i(t+1)= Avg^i$

Finally, the predicted optimal green light time required for one incoming direction for next cycle time C(t+1) is calculated using the following expression:

$$G^i(t+1) = \frac{Q^i(t+1)}{\sum_{k=1}^{n} Q^k(t+1)}.C(t) \tag{2}$$

3. MODELLING OF A SIGNALISED INTERSECTION

3.1 Basic definitions of Hybrid Petri nets

Hybrid Petri nets (HPNs) were initially successfully applied for modelling, performance evaluation and the design of manufacturing systems and more recently they have also been successfully used for studying transportation systems. Following is a short introduction of the hybrid formalism; the reader can find a more detailed presentation of HPNs and examples in [25].

In order to set the notation of the hybrid model, let us start with the recall of some formal definitions.

Definition 1: **An *HPN structure* is the 6-tuple HPN = { P, T, *Pre*, *Post*, *h*, Mo}**

Where:

- $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places; P can be split into two subsets P^D and P^C gathering, respectively, the discrete and continuous places;
- $T = \{t_1, t_2, \ldots, t_m\}$ is a finite set of transitions; T can also be split into two subsets T^D and T^C gathering, respectively, the discrete and continuous transitions; the sets P and T are disjointed;
- $Pre(p_i, t_j)$ is the *pre-set* function that assigns a weight to any arc between a transition t_j and its input place p_i:

$$Pre(p_i, t_j): (\mathrm{PxT}) \rightarrow \begin{cases} \Re_0^+, \forall p_i \in P^C \\ \aleph_0^+, \forall p_i \in P^D \end{cases}$$

- $Post(p_i, t_j)$ is the *post set* function that assigns a weight to any arc between a transition t_j and its output place p_i:

$$Post(p_i, t_j): (\mathrm{PxT}) \rightarrow \begin{cases} \Re_0^+, \forall p_i \in P^C \\ \aleph_0^+, \forall p_i \in P^D \end{cases}$$

- h: $P \cup T \rightarrow \{D,C\}$ called ''hybrid function'' indicates for each node whether it is discrete (D) or continuous (C); for example, if p_i is a discrete place, then $h(p_i) = D$ and if t_j is a continuous transition, then $h(t_j) = C$;
- Mo is the initial marking.

Graphically, discrete places are represented by circles and discrete transitions are represented by bars, whereas continuous places are represented by double circles and continuous transitions are represented by boxes. Finally, arcs are represented by arrows.

Places and transitions can have both discrete and continuous inputs according to the following definitions. For each transition t_j, ${}^{D}t_j$ and ${}^{C}t_j$ denote the sets of its input discrete and continuous places, respectively, whereas t_j^D and t_j^C are the sets of its output discrete and continuous places. The whole input and output place sets of transition t_j are defined as $^{\bullet}t_j = {}^{D}t_j \cup {}^{C}t_j$ and $t_i^{\bullet} = {}^{D}t_j \cup {}^{C}t_j$, respectively. On the other hand, for each place p_i, $^{D}p_i$ and $^{C}p_i$ denote the sets of its input discrete and continuous transitions, whereas p_i^D and p_i^C are the sets of its output discrete and continuous transitions. In this case, the whole input and output transition sets of place p_i are defined as $^{\bullet}p_i = {}^{D}P_i \cup {}^{C}P_i$ and $p_i^{\bullet} = {}^{D}p_i \cup {}^{C}p_i$, respectively.

Definition 2: **Marking of an HPN and firing rules of its transitions.**

The marking of an HPN is a function which assigns a nonnegative integer to each discrete place and a nonnegative real number to each continuous place. Two firing rules are applied for the dynamic evolution of an HPN.

R1: Enabling and firing rules of discrete transitions

There are mainly four rules which control the enabling and firing of discrete transitions:

(a) A discrete transition $t_j \in T^D$ is enabled, i.e. it can be fired, if and only if: $\forall p_i \in {}^{\bullet}t_j, M(p_i) \geq Pre(p_i, t_j)$; when a discrete transition has no upstream places, it is called *source transition* and it is always enabled.

(b) A discrete transition t_j starts firing as soon as it is enabled;

(c) Firing of t_j lasts τ_j time units, where τ_j is a non-negative deterministic real number. If $\tau_j > 0$, then t_j is named a timed transition whereas, if $\tau_j = 0$, then t_j is named an immediate transition.

(d) Upon completion of the firing of t_j, the marking of its input and output places change as follows:

$$\begin{cases} M'(p_i) = M(p_i) - Pre(p_i, t_j), & \forall p_i \in {}^{\bullet}t_j \\ M'(P_i) = M(p_i) + Pre(p_i, t_j), & \forall p_i \in t_j^{\bullet} \end{cases}$$

R2: Enabling and firing rules of continuous transitions

Any continuous transition $t_j \in T^C$ of an HPN is characterised by two speed data: (i) its instantaneous firing speed denoted $\upsilon_j \in \Re^+$ which corresponds to the real speed of the marks ‘‘inside’’ the transition, and (ii) its maximal firing speed V_j which represents the upper bound of υ_j. The main rules which control the enabling and firing of continuous transitions are then:

(a) A continuous transition t_j is enabled if every discrete or continuous upstream place p_i ($p_i \in {}^{\bullet}t_j$) meets the following conditions:
- if p_i is a discrete place, then $M(p_i) \geq Pre(p_i, t_j)$;
- if p_i is a continuous place, then either: (i) $M(p_i) \geq 0$ or (ii) p_i is fed, i.e. there is at least one continuous transition $t_k \in {}^{\bullet}p_i$ such that its instantaneous speed $\upsilon_k > 0$.

(b) t_j is said to *be strongly enabled* at time t if all continuous places of ${}^{\bullet}t_j$ verify $M(p_i) \geq 0$ or to be *weakly enabled* otherwise; when a continuous transition has no upstream places, it is also called *source transition* and it is always strongly enabled.

(c) A strongly enabled transition t_j is fired at its maximal firing speed V_j whereas a weakly enabled one can fire with a speed $\upsilon_j = \min(V_j, \upsilon_k)$, where υ_k is the instantaneous speed of each upstream continuous transition t_k of ${}^{\bullet}p_i$ (p_i is fed).

The marking evolution of the continuous places, with respect to the time, can be described easily by means of the *balance equation* given as follows:

for a very short interval τ, the marking evolution of a continuous place p_i, can be described by the differential equation:

$$B(p_i) = M(p_i, \tau) = \sum_{t_j \in {}^\bullet p_i} Post(p_i, t_j).\upsilon_j \ - \ \sum_{Tk \in {}^\bullet Pi} Pre(p_i, t_k).\upsilon_k$$

Definition 3: **Conflicts in HPNs**

The structure of a place having two (or more) output transitions is referred to as a conflict, decision, or choice, depending on applications. If there are conflicts in a model, several behaviours are possible. We consider here the conflicts specific to HPNs by briefly recalling the main rules for solving conflicts between a continuous and a discrete transition (rule 1), and conflicts between two continuous transitions (rules 2 and 3).

Rule 1: If there is a conflict between a discrete transition and a continuous transition, the discrete transition has priority over the continuous transition.

Rule 2: If there is a conflict between several continuous transitions with a common continuous input place p_i which is empty ($M(p_i) = 0$), a solution is admissible if the balance $B(p_i)$ of this place is equal to zero. Examples of solutions are the *priority* of a transition over the other one(s) or a sharing proportional to maximal speeds, etc.

Rule 3: If there is a conflict between several continuous transitions $t_1, \ldots t_j, \ldots, t_p$, with a common discrete input place containing k tokens, any solution such that $\sum_{i=1}^{p} v_j / V = k$ is admissible.

3.2 Hybrid model of a signalised intersection

In this section, a hybrid model of a signalised intersection is designed. Such a model is composed of n modules representing the modelling of two hierarchical levels: **Traffic lights control** and **physical traffic flows**. A Timed Petri net models the traffic lights control and a continuous Petri net models the physical traffic flows. The generic model as given in Figure 1 is built by joining four kinds of elementary modules representing a generic phase of the traffic control, a generic incoming link, the common crossing area, and a generic outgoing direction.

First, for the discrete part of the model, phases enabling right of way for the input link are modelled by three discrete places: p_{Gi} for green phase, p_{Ai} for amber phase and p_{Li} for transitory change to next phase. The red phase is not represented in this generic model, since all incoming flows, which do not find a green or an amber signal are supposed to find a red one. The total time enabling input link IL_i is then modelled by the place p_i. For the marking evolution of this part, a token is removed from input places and put in place p_i and in place p_{Gi} immediately when the enabling time corresponding to upstream link IL_{i-1} is ended. This token is, then removed from p_i and place p_{Li} and then placed in output places by the end of the enabling time corresponding to input link IL_i. The holding of the token in the place p_i is conditioned by the firing time of transitions t_{Gi} and t_{Ai}. The first time is dynamic and estimated for each new cycle time by applying the expression (2) whereas the second time may be fixed to one predefined value.

Regarding the continuous part representing the flow model we have a combination of generic incoming link, the common crossing area and generic outgoing direction sub models. The queue

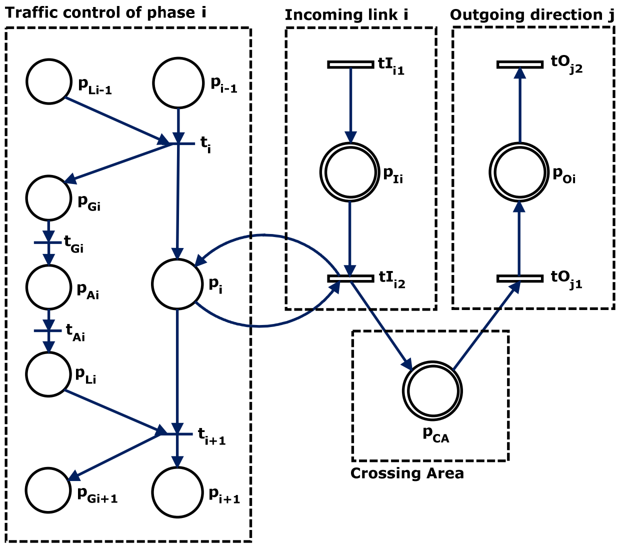

Fig. 1. HPN of a generic module

at incoming link i, the vehicles crossing the intersection area and vehicles leaving the intersection toward direction j are modelled respectively by the continuous places p_{Ii}, p_{CA}, and p_{Oj}. The marking of this part of the model, evolves according to the associated firing speeds of different continuous transitions. In the present case, the firing speeds are considered as follows: Φin^i, Φout^i are respectively firing speeds of t_{Ii1}, t_{Ii2}, and φ^j firing speeds of both t_{Oj1} and t_{Oj2}.

4. STUDY CASE

To prove efficacy of the proposed model, a study case is discussed in this section.

4.1 Traffic intersection Description

We will consider a real case of a signalised intersection located in the north of an urban traffic area at AL-MADINA city in Saudi Arabia. Figure 2, shows this traffic intersection. This intersection includes four directions, with two links (one input link and one output link) for each. All input links are formed of 4 lanes without any inflicted direction which allows the crossing of 4 vehicles in parallel. Indeed, such an assumption is not restrictive and it is supported by experimental observations, showing that lane changing occurrences are rare for this intersection. However, three traffic flows are allowed for each direction: left turns, through and right turns. On each corner of the intersection there is a traffic signal allowing three subsequent phases: green, amber and red.

Where:
IL_i/OL_i: Input/Output Links for direction i (West, South, East or North)
CA: Crossing Area.

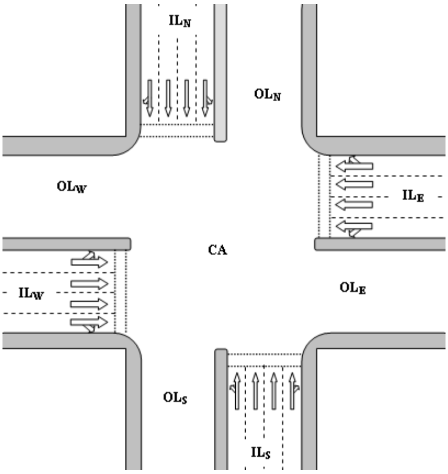

Fig. 2. A real traffic intersection

4.2 HPN model

The HPN model of the described intersection is shown in Figure 3. This model is composed of two sub models:

- A continuous PN, modelling the flows of vehicles entering the intersection, the crossing area, and the flows of vehicles leaving the intersection.
- A discrete PN modelling the control part of traffic lights.

The marking of the continuous model represents the number of vehicles. Whereas the marking of the discrete model represents the different states of control signals of the traffic lights.

Places meanings:

p_i : Input link i is enabled.
p_{Gi} : Green time for input link i.
p_{Ai} : Amber time for input link i.
p_{Ii} : Vehicles are in the queue at input link i
p_{Oi} : Vehicles are leaving the intersection toward direction i
p_{CA} : Vehicles are crossing the intersection area.

Transition meanings:

t_i : Enabling the input link i.
t_{GAi} : Green time ends and amber time starts for input link i.
t_{Ii1} : Vehicles entering in the queue at input link i
t_{Ii2} : Vehicles leaving the queue at input link i
t_{Oi1} : Vehicles leaving the cross area toward direction i
t_{Oi2} : Vehicles leaving the intersection toward direction i

i : correspond to one direction: West, South, East, or North

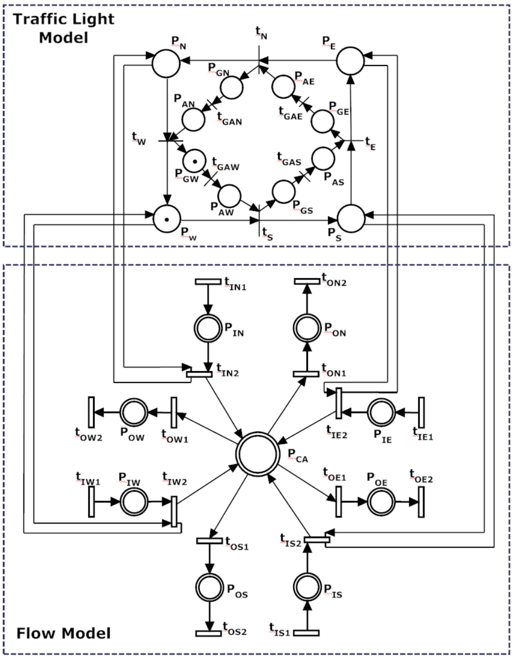

Fig. 3. HPN of the signalised intersection

5. SIMULATION VALIDATING

5.1 System specification

To validate the proposed models, we consider the real case described in the previous section. The considered intersection is currently ruled by a fixed signal-timing plan. The fixed durations of the traffic light phases applied to this intersection are: green phases are 27s and amber phases are 3s long for each. The total of different phases amount to one cycle at 120s long. A real observation and measurement of the dynamic of this intersection for 5 successive cycles during three separated times: 7:15 a.m., 10 a.m. and 2:15 p.m. were conducted on 19 January 2009.

Table 1 shows the input flow rates of successively input links (west, north, east, and south): $\mathbf{\Phi_W}$, $\mathbf{\Phi_N}$, $\mathbf{\Phi_E}$, $\mathbf{\Phi_S}$. These rates are calculated by considering the average rate of the traffic flows during each observed cycle time.

Table 1. Input flow rates

Date:19/01/09	Time 1- 7:15 a.m.				Time 2- 10:00 a.m.				Time 3- 02:00 p.m.			
Cycle	Φ_W	Φ_N	Φ_E	Φ_S	Φ_W	Φ_N	Φ_E	Φ_S	Φ_W	Φ_N	Φ_E	Φ_S
0	0.26	0.11	0.28	0.95	0.54	0.34	0.33	0.45	0.68	0.36	0.91	0.23
1	0.26	0.09	0.28	0.96	0.52	0.38	0.34	0.48	0.67	0.35	0.87	0.22
2	0.27	0.08	0.28	0.96	0.57	0.33	0.38	0.51	0.67	0.30	0.88	0.20
3	0.27	0.10	0.29	0.97	0.60	0.35	0.34	0.52	0.61	0.31	0.87	0.21
4	0.27	0.10	0.28	0.98	0.61	0.34	0.32	0.50	0.60	0.30	0.85	0.24

Table 2. shows initial queues sizes of different input links observed at three separate times

	Q_W	Q_N	Q_E	Q_S
Time 1- 7:15 a.m.	23	7	8	91
Time 2- 10:00 a.m.	49	20	10	2
Time 3- 02:00 p.m.	61	22	27	2

Table 2 shows initial queues sizes of different input links observed at three separate times. The other parameters are assumed as follows:

- Saturation flow rate is determined experimentally according to measured data and reveals an average of 2veh. /sec. for all links.
- Lengths of input links are assumed to be more than 500 meters.

5.2 Simulation parameters

Predicted traffic flow rates for next cycle times are obtained by considering the average of different measured flows for each cycle. And queue sizes for next cycles are estimated by applying the expression (1). As a result, predicted queue sizes for next cycle times are given by table 3.

Finally, by applying expression (2), optimal green durations for next cycles' times are given by table 4.

Table 3. Predicted queue sizes

Cycle	Time 1- 7:15 a.m.				Time 2- 10:00 a.m.				Time 3- 02:00 p.m.			
	$Q^w(t+1)$	$Q^N(t+1)$	$Q^E(t+1)$	$Q^S(t+1)$	$Q^w(t+1)$	$Q^N(t+1)$	$Q^E(t+1)$	$Q^S(t+1)$	$Q^w(t+1)$	$Q^N(t+1)$	$Q^E(t+1)$	$Q^S(t+1)$
1	23	8	16	125	49	48	45	58	61	44	104	28
2	27	10	31	61	51	33	34	43	71	34	74	26
3	27	9	27	63	47	32	34	46	68	33	99	22
4	26	11	27	58	50	34	32	44	65	34	88	29
5	26	11	27	61	51	33	34	42	65	31	90	32

Table 4. Optimal green durations

Cycle	Time 1- 7:15 am				Time 2- 10:00 am				Time 3- 02:00 pm			
	$G^w(t+1)$	$G^N(t+1)$	$G^E(t+1)$	$G^S(t+1)$	$G^w(t+1)$	$G^N(t+1)$	$G^E(t+1)$	$G^S(t+1)$	$G^w(t+1)$	$G^N(t+1)$	$G^E(t+1)$	$G^S(t+1)$
1	16.16	5.87	11.12	86.85	29.27	28.82	26.87	35.04	30.72	22.44	52.72	14.12
2	25.24	9.43	28.48	56.85	37.84	24.82	25.18	32.16	41.61	19.88	43.30	15.20
3	25.66	8.82	25.82	59.70	35.46	24.36	25.47	34.71	36.63	17.63	53.78	11.96
4	25.72	10.90	26.74	56.63	38.31	25.82	24.41	33.46	35.87	18.89	48.97	16.27
5	24.77	10.38	25.76	59.09	38.06	24.84	25.42	31.68	35.67	16.96	49.50	17.88

5.3 Simulation results and analysis

The developed model has been implemented and simulated within the Matlab environment. Parameters presented in previous paragraphs are used.

First, regarding the real observation of different queue sizes as given by Figure 4, we can draw the following comments: In the morning (7:15 a.m.), input link IL_S represents the most significant link of the congestion feature. This is due to vehicle traffic coming from the city cen-

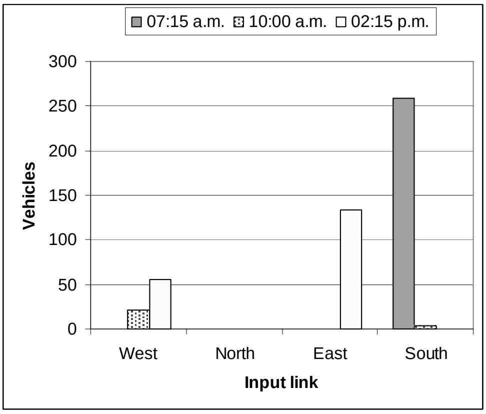

Fig. 4. Average of real number of delayed vehicles

tre and going to destinations west, north and east where a hospital and a university are located. At ten o'clock the traffic flow is normal and no significant congestion is observed in one specific direction. At 2: 15 p.m., successively input link IL_E, then IL_W become the most significant links of the congestion feature. This is due to the time of the exit/leaving of employees and students when most vehicles have one common destination: the city centre.

Let us focus on the most significant links of the congestion feature: IL_S at 7:15 a.m., IL_E and IL_W at 2:15 p.m. Figures 5, 6, and 7 respectively, give real and simulated numbers of vehicles in each input link at the indicated times. According to these graphics, by comparing results, the queue sizes of vehicles waiting for green times may be reduced from 33% to 100 %.

Finally, Table 5 shows the average of lost times in both situations: real data and simulated results. A reduction in lost times for the considered times from 17% to 97% is then indicated. This decrease of lost times confirm that the proposed model is more appropriate to these types of signalised intersections in cases of real time implementation.

It is worth finally stressing that the largest benefits were achieved. In addition, it is concluded this control model achieved an average saving in delayed times and lost times compared with the actual situation using a fixed time plan.

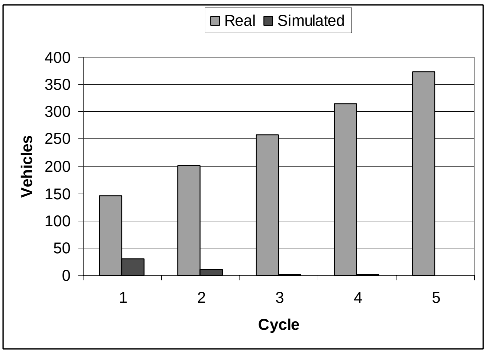

Fig. 5. Real and simulated number of delayed vehicles in south input link ILS corresponding to time 7:15 a.m

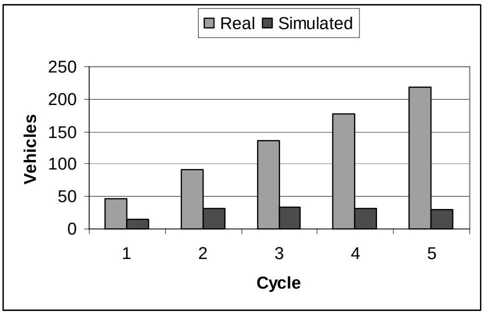

Fig. 6. Real and simulated number of delayed vehicles in east input link ILE corresponding to time 10:00 a.m

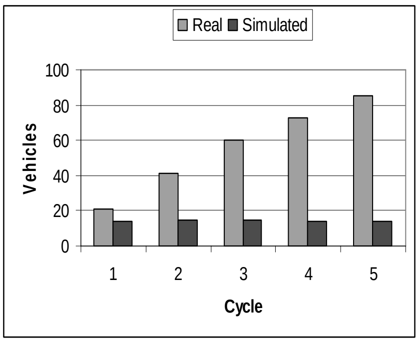

Fig. 7. Real and simulated number of delayed vehicles in west input link ILW corresponding to time 2:15 p.m

Table 5. Averages of lost times in both situations: real data and simulated results

Time	Average of lost times	
	Real data	Simulated results
7:15 a.m.	58	19
10:00 a.m.	23	19
02:00 p.m.	32	1

6. CONCLUSION AND WORK EXTENSIONS

A hybrid model of signalised intersections has been discussed in this paper. Special regard is given to queue sizes of vehicles in input links and is considered a principal parameter for optimisation. This is so because such optimisation concerns the calculation of optimal green durations proportionally to the real time queues sizes in input links. A study case is discussed to prove the efficacy of the proposed model and the simulation results obtained showed it as effective and feasible for real time implementation. Work is in progress considering the dynamicity of the cycle time itself according to the global traffic flow dynamic during peak periods of the day. This will allow for further decreases in delay times of vehicles waiting for green signals when the input flow rates are less than their corresponding saturation flows.

REFERENCES

[1] Diakaki, C., Papageorgiou, M., and Aboudolas, K.,*"A multivariable regulator approach to traffic-responsive network-wide signal control,"* Control Engineering Practice, 10(2), 2002, pp.183-195.

[2] Papageorgiou, M., Diakaki, C., Dinopoulou, V., Kotsialos, A., and Wang, Y., *"Review of road traffic control strategies,"* Proceedings of the IEEE, 91(12), 2003, pp.2043-2067.

[3] Patel, M., and Ranganathan, N., "IDUTC: An intelligent decision making system for urban traffic-control applications," IEEE Transactions on Vehicular Technology, 50(3), 2001, pp.816-829.

[4] Huang, Y. S., and Su, P. J., *"Modelling and analysis of traffic light control systems,"* IET Control Theory and Applications, Vol.3, Issue 3, 2009, pp.340-350.

[5] Peterson, J. L., *"Petri net theory and the modelling of systems,"* Prentice Hall, 1981.

[6] Murata, T., *"Petri Nets: Properties, Analysis and Applications,"* Proceeding of the IEEE, Vol.77, No.4, 1989, pp.541-580.

[7] List, G. F., and Cetin, M., *" Modelling Traffic Signal Control Using Petri Nets,"* IEEE Transactions on Intelligent Transportation Systems, Vol.5, No.3, September, 2004.

[8] Di Cesare, F., Kulp, P., Gile, K., & List, G., *"The applications of Petri nets to the modelling analysis and control of intelligent urban traffic networks,"* Lecture notes in computer science, Vol.815. Berlin: Springer, 1994.

[9] Wang, H., List, G.F., Dicesare, F., *"Modelling and evaluation of traffic signal control using timed Petri nets," IEEE Proceedings of International Conference on Systems*, Man and Cybernetics, Le Touquet, France, 1993, pp.180-185.

[10] Dottily, M., Faint, M. P., Lacobellis, G., *"Validation of an urban traffic network model using coloured timed Petri nets," IEEE proceedings of International Conference on Systems*, Man and Cybernetics, Waikoloa, HI, USA, 2005, pp.1347-1352.

[11] Dotoli, M., and Fanti, M.-P., *"An urban traffic network model via coloured timed Petri nets,"* Control Engineering Practice 14, 2006, pp.1213-1229.

[12] Lee, J., and Song, S., *"Modelling urban transportation systems with hybrid systems: An overview,"* IEEE proceedings of Intelligent Transportation Systems, Vol.2, October, 2003, pp.1269-1272.

[13] Tolba, C., Lefebvre D., Thomas, P., and El Moudni, A., "Continuous and timed Petri nets for the macroscopic and microscopic traffic flow modeling," Simulation Modelling Practice and Theory 13, 2005, pp.407-436.

[14] Di Febbraro*, A.,* Giglio*, D.,* and Sacco, N., *"Urban Traffic Control Structure Based on Hybrid Petri Nets,"* IEEE proceedings of Intelligent Transportation Systems, Vol.5, No.4, December, 2004.

[15] Di Febbraro, A., and Sacco, N., *"On modelling urban transportation networks via hybrid Petri nets,"* Control Engineering Practice, Vol.12, No.10, 2004, pp.1225-1239.

[16] Kim, Y.-W, Kato, T., Okuma, S., Narikiyo, T., *"Traffic Network Control Based on Hybrid Dynamical System Modelling and Mixed Integer Nonlinear Programming With Convexity Analysis,"* IEEE Transactions on Systems, Man and Cybernetics - Part A: Systems and Humans, Vol.38, No.2, 2008.

[17] Webster, F.V. and B.M. Cobbe, *"Traffic Signals,"* Technical Paper 56, Road Research Laboratory, London, 1966.

[18] Webster, F.V., *"Traffic Signal Settings, Road Research,"* Technical Paper No.39, London, Her Majesty's Stationery Office, 1958: reprinted with minor amendments, 1969.

[19] Gartner, N., *"OPAC: A Demand-responsive strategy for traffic signal control,"* U.S. Department Transportation, Washington, DC, Transp. Res. Rec. 906, 1983.

[20] Porche, I., Sampath, M., Sengupta, R., Chen, Y. L., and Lafortune, S., *"A Decentralised Scheme for Real-Time Optimisation of Traffic Signals," IEEE Proceedings of International Conference on Control Applications*, Dearborn, MI, September, 1996, pp.582-589.

[21] Clement, S., and Anderson, J., *"Traffic signal timing determination: the Cable model,"* Genetic Algorithms in Engineering Systems: Innovations and Applications, 1997, pp.63-68.

[22] Dinopoulou, V., Diakaki, C., and M. Papageorgiou, *"Simulation investigation of the traffic-responsive urban control strategy TUC," IEEE Proceeding of Intelligent Transportation Systems Conference*, Dearborn, MI, 2000, pp.458-463.

[23] Gao, H. J., Li, L. X., Liu, R., and Wang, Y., *"Changeable phases signal control of an isolated intersection," IEEE Proceedings of International Conference on Systems*, Man and Cybernetics, Vol.5, 4 pp., October, 2002.

[24] Gang T., Fengying C., and Chunling F., *"Genetic Algorithm and Its Application in the Real time Traffic Signal Optimisation Control," Proceedings of 6th International Conference on ITS Telecommunications*, 2006.

[25] Alla, H., and David, R., *"On hybrid Petri nets,"* Journal of Discrete Event Dynamic Systems, Vol.8, No.1-2, 2001, pp.9-40.

Dr Mohamed Kamal Jbira

Dr. Mohamed Kamel Jbira is an Assistant Professor of Computer Engineering and Control at Taibah University. He earned his PhD in Computer-Aided Manufacturing from ESSTT, Tunisia in 1997; his MSc in Automatic from ENSET, Tunisia in 1991 and BSc in Electrical Engineering from ENSET, Tunisia in 1988. His research interests include Modelling and Simulation Applications, functional analysis of complex systems, urban traffic networks and Computer-Aided Manufacturing.

Professor Dr (Engineer) Munir Ahmed

Professor Dr (Engineer) Munir Ahmed is a professional member of the Institution of Engineering and Technology (MIET), United Kingdom. He is currently pursuing his DProf (Doctorate in Professional Studies) in Computer Communications Engineering Specialisation in Information Security with Middlesex University, London, United Kingdom. He earned his PhD in Digital Communications Systems Engineering from London Institute of Technology, London, United Kingdom in 1997; his MSc in Information Systems Engineering from London South Bank University, United Kingdom in 1994 and BSc in Electrical Engineering from the University of AJK, Kashmir in 1990. Before joining Taibah University, Saudi Arabia as Professor of Computer Networks and Communications Engineering on contractual basis, he was a Professor of Computing and Telecommunications at The American University, London, United Kingdom. He holds permanent positions as Chairman of Advisory Board and Director of Research at London College of Research, Reading, United Kingdom. He has extensive experience in the commercial sector and has held a variety of high-level positions in the industry, including Chief Executive Officer (CEO), Chief Operations Officer (COO), Training Director and Chief Network Architect in the United Kingdom. His current research activities aim to consolidate his skills and extensive commercial experience with various research areas in the field of Computer Networking and Communications Engineering. His particular areas of focus include Wireless Sensor Networks, Mobile Ad-Hoc Networks, Routing Protocols, Network/Information Security and Digital Modulations. Prof. Ahmed is author or co-author of 6 books and more than 30 papers.