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Filterness of Soft Sets

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Abstract

The notions of soft filters, ultra soft filters and bases of a soft filter are introduced and their basic properties are investigated. The adherence and convergence of soft filters in soft topological spaces with related results is also discussed.

Key words : Soft filters, ultra soft filters, base for a soft filter, convergence, adherence.

1. Introduction

The theories such as probability theory [12], fuzzy set theory [14], intuitionists fuzzy set theory [2, 3], vague set theory [5] and rough set theory [9], which can be considered as mathematical tools for dealing with uncertainties, have their inherent difficulties (see [8]). The reason for these difficulties is possibly the inadequacy of parameterization tool of the theories. Molodtsov [8] introduced soft sets as a mathematical tool for dealing with uncertainties which is free from the above-mentioned difficulties. Since the soft set theory offers mathematical tool for dealing with uncertain, fuzzy and not clearly defined objects, it has a rich potential for applications to problems in real life situation. Shabir and Naz [11] applied the soft set theory in topological structures and introduced soft topological spaces. Çağman et al. [4] introduced a topology on a soft set, so-called “soft topology”, and its related properties. They then presented the foundations of the theory of soft topological spaces. This is the starting point for soft mathematical concepts and structures that are based on soft set-theoretic operations. The purpose of present paper is a further attempt to broad the theoretical aspects of soft topological spaces introduced in [11, 4]. We refer to [13] in order to refresh the fundamental concepts in topological space. The rest of this paper is organized as follows. In Section 2 basic notions about soft sets reviewed. Section 3 presents the concept of soft filters and gives that every soft filter on non-null soft set is the intersection of the family of ultra soft filters which include it. The adherence and convergence of soft filters in a soft topology with related results is also discussed.

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2. Preliminaries

Let U be an initial universe of objects and E the set of parameters in relation to objects in U. Let $\mathcal{P}(U)$ denote the power set of U and $X \subseteq E$.

Definition 2.1. [8] A soft set (γ, X) on the universe U is defined by the set of ordered pairs

$$(\gamma, X) = \{(x, \gamma_X(x)) : x \in E, \gamma_X(x) \in \mathcal{P}(U)\},$$

where $\gamma_X : E \to \mathcal{P}(U)$ is a mapping such that $\gamma_X(x) = \emptyset$ if $x \notin X$.

Note that the set of all soft sets over U will denoted by $\mathcal{S}(U, E)$.

Definition 2.2. Let (γ, A) and (γ, B) be two soft sets in $\mathcal{S}(U, E)$. Then:

(1) (γ, A) is called a null soft set, denoted by Φ, if $\gamma_A(x) = \emptyset$ for all $x \in E$;

(2) (γ, A) is called an absolute soft set, denoted by $\overline{A}$, if $\gamma_A(x) = U$ for all $x \in E$;

(3) (γ, A) is called a soft subset of (γ, B), denoted by $(\gamma, A)\widetilde{\subset}(\gamma, B)$, if (i) $A \subseteq B$ and (ii) $\gamma_A(x) \subseteq \gamma_B(x)$ for any $x \in E$;

(4) (γ, A) is called a soft superset of (γ, B), denoted by $(\gamma, A)\widetilde{\supset}(\gamma, B)$, if (γ, B) is a soft subset of (γ, A);

(5) (γ, A) and (γ, B) are called soft equal if $(\gamma, A)\widetilde{\subset}(\gamma, B)$ and $(\gamma, A)\widetilde{\supset}(\gamma, B)$;

(6) the union $(\gamma, A)\widetilde{\cup}(\gamma, B)$ of (γ, A) and (γ, B) is defined by the approximate function $\gamma_{A\cup B}(x) = \gamma_A(x) \cup \gamma_B(x)$ for all $x \in E$;

(7) the intersection $(\gamma, A)\widetilde{\cap}(\gamma, B)$ of (γ, A) and (γ, B) is defined by the approximate function $\gamma_{A\cap B}(x) = \gamma_A(x) \cap \gamma_B(x)$ for all $x \in E$;

(8) the complement $(\gamma, A)^c$ of (γ, A) is defined by the approximate function $\gamma_A^c(x) = U \setminus \gamma_A(x)$ for all $x \in E$.

It should be noted that this definition is different from that of [6]. Note that the set of all soft subsets of (γ, X) will be denoted by $\mathcal{P}(\gamma, X)$. Since some researchers are in some conflict about a null soft set due to its notation, we prefer to use Φ_A instead of Φ for the null soft set of (γ, A) as Ali et al. [1] used.

Proposition 2.3. For three soft sets (γ, A), (γ, B) and (γ, C) in $\mathcal{S}(U, E)$, the following are true

(a) $(\gamma, A)\widetilde{\cap}(\gamma, A) = (\gamma, A)$.

(b) $(\gamma, A)\widetilde{\cup}(\gamma, A) = (\gamma, A)$.

(c) $(\gamma, A)\widetilde{\cap}\ \Phi_A = \Phi$.

(d) $(\gamma, A)\widetilde{\cup}\ \Phi_A = (\gamma, A)$.

(e) $(\gamma, A)\widetilde{\cap}\bar{A} = (\gamma, A)$.

(f) $(\gamma, A)\widetilde{\cup}\bar{A} = \bar{A}$.

(g) $((\gamma, A)\widetilde{\cap}(\gamma, B))^c = (\gamma, A)^c\widetilde{\cup}(\gamma, B)^c$.

(h) $((\gamma, A)\widetilde{\cup}(\gamma, B))^c = (\gamma, A)^c\widetilde{\cap}(\gamma, B)^c$.

(i) $((\gamma, A)\widetilde{\cap}(\gamma, B))\widetilde{\cap}(\gamma, C) = (\gamma, A)\widetilde{\cap}((\gamma, B)\widetilde{\cap}(\gamma, C))$.

(j) $((\gamma, A)\widetilde{\cup}(\gamma, B))\widetilde{\cup}(\gamma, C) = (\gamma, A)\widetilde{\cup}((\gamma, B)\widetilde{\cup}(\gamma, C))$.

(k) $((\gamma, A)\widetilde{\cap}(\gamma, B))\widetilde{\cup}(\gamma, C) = ((\gamma, A)\widetilde{\cup}(\gamma, C))\widetilde{\cap}((\gamma, B)\widetilde{\cup}(\gamma, C))$

(l) $((\gamma, A)\widetilde{\cup}(\gamma, B))\ \widetilde{\cap}\ (\gamma, C) = ((\gamma, A)\widetilde{\cap}(\gamma, C))\widetilde{\cup}((\gamma, B)\widetilde{\cap}(\gamma, C))$.

Definition 2.4. [11] Let (γ, X) be an element of $\mathcal{S}(U, E)$ and τ be a subfamily of $\mathcal{P}(\gamma, X)$. Then τ is called soft topology on (γ, X) if the following conditions are satisfied:

(1) $\Phi_X, (\gamma, X) \in \tau$;

(2) If $(\gamma, A), (\gamma, B) \in \tau$, then $(\gamma, A)\widetilde{\cap}(\gamma, B) \in \tau$;

(3) If $\{(\gamma, A_k) : k \in K\} \subset \tau$, then $\widetilde{\bigcup}_{k\in K}(\gamma, A_k) \in \tau$.

The triple (γ, X, τ) (simply denoted by (X_γ, τ)) is called a soft topological space (simply, soft space) over X. Every member of τ is called τ-open soft set. A soft set is called τ-closed if its complement is τ-open. $\{\Phi_X, (\gamma, X)\}$ and $\mathcal{P}(\gamma, X)$ are two examples for soft topology on X and shall call indiscrete soft topology and discrete soft topology respectively as called in point-set topology. Moreover, $\mathcal{S}(U, E)$ is a soft topology on $\widetilde{U}$.

For two soft topologies τ and τ' on (γ, X), τ is said to be finer than τ' and τ' coarser than τ if $\tau \subseteq \tau'$; thus τ is finer than τ' if and only if every τ'-open soft subset of (γ, X) is τ-open.

Lemma 2.5. [11] Let (X_γ, τ) be a soft topological space and (γ, B) be a soft set in $\mathcal{P}(\gamma, X)$. Then (γ, B) is τ-open if and only if it is a τ-neighborhood of each of its soft subset.

3. Soft filter

Definition 3.1. A soft filter on (γ, X) is a non-empty subfamily $\mathcal{F}$ of $\mathcal{P}(\gamma, X)$ having the following properties:

(1) Every soft subset of $\mathcal{P}(\gamma, X)$ which includes a soft set in $\mathcal{F}$ belongs to $\mathcal{F}$;

(2) The intersection of each finite family of soft sets in $\mathcal{F}$ belongs to $\mathcal{F}$;

(3) All the soft sets in $\mathcal{F}$ are not null soft set.

Let $\mathcal{F}$ be a soft filter on (γ, X). A collection $\mathcal{B}$ of soft subsets of $\mathcal{P}(\gamma, X)$ is called a base for the soft filter $\mathcal{F}$ if (1) $\mathcal{B} \subseteq \mathcal{F}$ and (2) for every soft set (γ, A) in $\mathcal{F}$, there is a soft set (γ, B) in $\mathcal{B}$ such that $(\gamma, B)\widetilde{\subset}(\gamma, A)$; we say also that $\mathcal{B}$ generates $\mathcal{F}$.

Theorem 3.2. Let $\mathcal{F}$ and $\mathcal{G}$ be soft filters on (γ, X). Then a soft set (γ, C) in $\mathcal{P}(\gamma, X)$ belongs to both $\mathcal{F}$ and $\mathcal{G}$ if and only if there are soft sets $(\gamma, A) \in \mathcal{F}$ and $(\gamma, B) \in \mathcal{G}$ such that $(\gamma, C) = (\gamma, A)\widetilde{\cup}(\gamma, B)$.

Proof. Suppose $(\gamma, C) \in \mathcal{F} \cap \mathcal{G}$. Then $(\gamma, C) = (\gamma, C)\widetilde{\cup}(\gamma, C)$, $(\gamma, C) \in \mathcal{F}$ and $(\gamma, C) \in \mathcal{G}$. Conversely, suppose $(\gamma, C) = (\gamma, A)\widetilde{\cup}(\gamma, B)$ where $(\gamma, A) \in \mathcal{F}$ and $(\gamma, B) \in \mathcal{G}$. Then $(\gamma, C)\widetilde{\supset}(\gamma, A)$, so $(\gamma, C) \in \mathcal{F}$, and $(\gamma, C)\widetilde{\supset}(\gamma, B)$, so $(\gamma, C) \in \mathcal{G}$. □

Theorem 3.3. Let $\mathcal{B}$ be a collection of soft sets in $\mathcal{P}(\gamma, X)$. Then $\mathcal{B}$ is a base for a soft filter on (γ, X) if and only if (1) the finite intersection of members of $\mathcal{B}$ includes a member of $\mathcal{B}$ and (2) $\mathcal{B}$ is non-empty and Φ_X does not belong to $\mathcal{B}$.

Proof. Suppose that $\mathcal{B}$ is a base for a soft filter $\mathcal{F}$ on (γ, X). Let $\{(\gamma, B_i) : i = 1, \ldots, n\}$ be a finite family of soft sets in $\mathcal{B}$. Since $\mathcal{B} \subseteq \mathcal{F}$, it follows that $\widetilde{\bigcap}_{i=1}^{n}(\gamma, B_i) \in \mathcal{F}$ and so $\widetilde{\bigcap}_{i=1}^{n}(\gamma, B_i)$ includes a soft set in $\mathcal{B}$. Since $\mathcal{F}$ is non-empty and every soft set in $\mathcal{F}$ includes a soft set in $\mathcal{B}$, it follows that $\mathcal{B}$ is non-empty. Since $\Phi_X \notin \mathcal{F}$ and $\mathcal{B} \subseteq \mathcal{F}$, we have $\Phi_X \notin \mathcal{B}$.

Conversely, suppose the conditions are satisfied. Let $\mathcal{F} = \{(\gamma, A) \in \mathcal{P}(\gamma, X) : (\gamma, A)$ includes a soft set in $\mathcal{B}\}$. Then $\mathcal{F}$ is a soft filter on (γ, X) with base $\mathcal{B}$. □

Let $\mathbf{A}$ be a collection of soft subsets of (γ, X); let $\mathbf{A}'$ be the collection of intersections of all finite families of soft sets in $\mathbf{A}$. If $\mathbf{A}'$ does not contain the null soft set Φ, then it satisfies the conditions of Theorem 3.3 and hence is a base for a soft filter $\mathcal{F}$ on (γ, X). We call $\mathcal{F}$ the soft filter generated by $\mathbf{A}$.

Theorem 3.4. Let $\mathcal{F}$ and $\mathcal{G}$ be soft filters on (γ, X). Suppose that for every pair of soft subsets $(\gamma, A), (\gamma, B)$ of (γ, X) in $\mathcal{F} \cup \mathcal{G}$, we have $(\gamma, A)\widetilde{\cap}(\gamma, B) \neq \Phi_X$. Then the soft filter generated by $\mathcal{F}\cup\mathcal{G}$ consists of all soft sets of the form $(\gamma, C)\widetilde{\cap}(\gamma, D)$ where $(\gamma, C) \in \mathcal{F}$ and $(\gamma, D) \in \mathcal{G}$.

Proof. Let $\mathcal{H}$ be the soft filter generated by $\mathcal{F} \cup \mathcal{G}$. Let $\mathcal{S}$ be the set of intersections of all finite families of soft sets from $\mathcal{F} \cup \mathcal{G}$. Let $(\gamma, A) \in \mathcal{H}$. Then (γ, A) includes a soft set in $\mathcal{S}$. Every soft set in $\mathcal{S}$ has the form $(\gamma, C)\widetilde{\cap}(\gamma, D)$ where $(\gamma, C) \in \mathcal{F}$ and $(\gamma, D) \in \mathcal{G}$. If $(\gamma, A)\widetilde{\supset}(\gamma, C)\widetilde{\cap}(\gamma, D)$ where $(\gamma, C) \in \mathcal{F}$ and $(\gamma, D) \in \mathcal{G}$, then it follows that we have

$$\begin{aligned}(\gamma, A) &= (\gamma, A)\widetilde{\cup}((\gamma, C)\widetilde{\cap}(\gamma, D)) \\ &= ((\gamma, A)\widetilde{\cup}(\gamma, C))\widetilde{\cap}((\gamma, A)\widetilde{\cup}(\gamma, D)).\end{aligned}$$

Since $(\gamma, A)\widetilde{\cup}(\gamma, C)\widetilde{\supset}(\gamma, C)$, $(\gamma, A)\widetilde{\cup}(\gamma, D)\widetilde{\supset}(\gamma, D)$, we have $(\gamma, A)\widetilde{\cup}(\gamma, C) \in \mathcal{F}$ and $(\gamma, A)\widetilde{\cup}(\gamma, D) \in \mathcal{G}$. So $(\gamma, A) \in \mathcal{S}$. Thus $\mathcal{H} = \mathcal{S}$, as required. □

Theorem 3.5. The set of all soft filters on a non-null soft set (γ, X) is inductively ordered by inclusion.

Proof. Let $\mathbf{F} = \{\mathcal{F} : \mathcal{F}$ is a soft filter on $(\gamma, X)\}$ be totally ordered by inclusion $\subseteq$. Let $\mathbf{A}$ be the union of $\mathbf{F}$. Let $\{(\gamma, A_i) : i \in I\}$ be a finite family of soft sets in $\mathbf{A}$. For each $i \in I$, there is a soft filter $\mathcal{F}_i$ in $\mathbf{F}$ such that $(\gamma, A_i) \in \mathcal{F}_i$. Since $\mathbf{F}$ is $\subseteq$-totally ordered, there is an index j in I such that $(\gamma, A_i) \in \mathcal{F}_j$ for all $i \in I$. Hence $\widetilde{\cap}_{i\in I}(\gamma, A_i) \neq \Phi_X$. By Theorem 3.3, $\mathbf{A}$ generates a soft filter $\mathcal{F}$ on (γ, X) which is clearly the $\subseteq$-supremum of $\mathbf{F}$. □

It follows from Theorem 3.5 by the application of Zorn's Lemma that the collection of soft filters on a non-null soft set (γ, X) has $\subseteq$-maximal elements: these maximal soft filters are called ultra soft filters. It is also easy to show that for every soft filter $\mathcal{F}$ on a soft set (γ, X) there is an ultra soft filter on (γ, X) which includes $\mathcal{F}$.

Theorem 3.6. Let $\mathcal{F}$ be an ultra soft filter on a soft set (γ, X). If (γ, A) and (γ, B) are soft sets in $\mathcal{P}(\gamma, X)$ such that $(\gamma, A)\widetilde{\cup}(\gamma, B) \in \mathcal{F}$, then either $(\gamma, A) \in \mathcal{F}$ or $(\gamma, B) \in \mathcal{F}$.

Proof. Suppose $(\gamma, A) \notin \mathcal{F}$ and $(\gamma, B) \notin \mathcal{F}$. Let $\mathcal{F}' = \{(\gamma, C) \in \mathcal{P}(\gamma, X) : (\gamma, A)\widetilde{\cup}(\gamma, C) \in \mathcal{F}\}$. Then

(i) Let $(\gamma, C) \in \mathcal{F}'$ and $(\gamma, D) \in \mathcal{P}(\gamma, X)$ with $(\gamma, C)\widetilde{\subset}(\gamma, D)$. Since $(\gamma, A)\widetilde{\cup}\ (\gamma, C) \in \mathcal{F}$ and $((\gamma, A)\widetilde{\cup}(\gamma, C))\ \widetilde{\subset}(\gamma, A)\widetilde{\cup}(\gamma, D)$, we have $(\gamma, A)\widetilde{\cup}(\gamma, D) \in \mathcal{F}$. So $(\gamma, D) \in \mathcal{F}'$.

(ii) Let $\{(\gamma, C_i) : i \in I\}$ be a finite family of soft sets in $\mathcal{F}'$. Since $\mathcal{F}$ is a soft filter, $(\gamma, A)\widetilde{\cup}(\widetilde{\cap}_{i\in I}(\gamma, C_i)) = \widetilde{\cap}_{i\in I}((\gamma, A)\widetilde{\cup}(\gamma, C_i)) \in \mathcal{F}$. So $\widetilde{\cap}_{i\in I}(\gamma, C_i) \in \mathcal{F}'$.

(iii) Since $(\gamma, A) \notin \mathcal{F}$, we have $\Phi_X \notin \mathcal{F}'$.

Thus $\mathcal{F}'$ is a soft filter on (γ, X). Clearly, $\mathcal{F}' \supseteq \mathcal{F}$ and $(\gamma, B) \in \mathcal{F}'$ although $(\gamma, B) \notin \mathcal{F}$. So $\mathcal{F}' \supset \mathcal{F}$, which contradicts the fact that $\mathcal{F}$ is an ultra soft filter. □

Theorem 3.7. Let (γ, X) be a non-null soft set and $\mathbf{A}$ be a collection of soft sets in $\mathcal{P}(\gamma, X)$ which generates a soft filter $\mathcal{F}$ on (γ, X). If for every soft set $(\gamma, A) \in \mathcal{P}(\gamma, X)$ we have either $(\gamma, A) \in \mathbf{A}$ or $(\gamma, A)^c \in \mathbf{A}$, then $\mathbf{A}$ is an ultra soft filter on (γ, X).

Proof. Let $\mathcal{F}$ be the soft filter generated by $\mathbf{A}$ and $\mathcal{F}'$ be any ultra soft filter which includes $\mathcal{F}$. Then clearly $\mathcal{F}' \supseteq \mathbf{A}$. Let (γ, A) be any soft set in $\mathcal{F}'$. Then $(\gamma, A)^c \notin \mathbf{A}$, for if $(\gamma, A)^c \in \mathbf{A}$ then $(\gamma, A)^c \in \mathcal{F}'$ and $(\gamma, A)\widetilde{\cap}(\gamma, A)^c = \Phi_X \in \mathcal{F}'$. This is a contradiction since $\mathcal{F}'$ is a soft filter. Hence $(\gamma, A) \in \mathbf{A}$ and so $\mathcal{F}' \subseteq \mathbf{A}$. So $\mathbf{A} = \mathcal{F}'$, an ultra soft filter. □

Theorem 3.8. Every soft filter $\mathcal{F}$ on non-null soft set (γ, X) is the intersection of the family of ultra soft filters which include $\mathcal{F}$.

Proof. Let $(\gamma, A) \in \mathcal{P}(\gamma, X)$ be a soft set which does not belong to $\mathcal{F}$. Then for each soft set (γ, B) in $\mathcal{F}$ we cannot have $(\gamma, B)\widetilde{\subset}(\gamma, A)$ and hence we must have $(\gamma, B)\widetilde{\cap}(\gamma, A)^c \neq \Phi_X$. So $\mathcal{F} \cup \{(\gamma, A)^c\}$ generates a soft filter on (γ, X), which is included in some ultra soft filter $\mathcal{F}_{(\gamma,A)}$. Since $(\gamma, A)^c \in \mathcal{F}_{(\gamma,A)}$ we must have $(\gamma, A) \notin \mathcal{F}_{(\gamma,A)}$. Thus (γ, A) does not belong to the intersection of the set of all ultra soft filters which include $\mathcal{F}$. Hence this intersection is just the soft filter $\mathcal{F}$ itself. □

Now, let (X_γ, τ) be a soft topological space and $\mathcal{F}$ be a soft filter on (γ, X). A soft set (γ, A) in $\mathcal{P}(\gamma, X)$ is said to be a limit or a limit soft set of the soft filter $\mathcal{F}$ and $\mathcal{F}$ is said to converge to (γ, A) or to be convergent to (γ, X) if the τ-neighborhood soft filter $\mathcal{V}_{(\gamma,A)}$ of (γ, A) is included in the soft filter $\mathcal{F}$. If $\mathcal{B}$ is a base for a soft filter on (γ, X) then (γ, A) is a limit of $\mathcal{B}$ and $\mathcal{B}$ converges to (γ, A) if the soft filter generated by $\mathcal{B}$ converges to (γ, A).

Theorem 3.9. Let τ and τ' be soft topologies on a soft set (γ, X). Then τ is finer than τ' if and only if every soft filter $\mathcal{F}$ on (γ, X) which converges to (γ, A) for the soft topology τ also converges to (γ, A) for the soft topology τ'.

Proof. Suppose τ is finer than τ'. Let $\mathcal{F}$ be a soft filter which is τ-convergent to (γ, A). Then $\mathcal{F} \supseteq \mathcal{V}^{\tau}_{(\gamma,A)}$, the τ-neighborhood soft filter of (γ, A). Since τ is finer than τ', every τ'-neighborhood of (γ, A) is a τ-neighborhood. So $\mathcal{F} \supseteq \mathcal{V}^{\tau'}_{(\gamma,A)}$, the τ'-neighborhood soft filter of (γ, A), and hence $\mathcal{F}$ is τ'-convergent to (γ, A).

Conversely, suppose that every soft filter on (γ, X) which is τ-convergent to (γ, A) is also τ'-convergent to (γ, A). Let (γ, G') be any τ'-open soft set and (γ, B') be any soft subset of (γ, G'). Then $(\gamma, G') \in \mathcal{V}^{\tau'}_{(\gamma,B')}$. Since $\mathcal{V}^{\tau}_{(\gamma,B')}$ is τ-convergent to (γ, B'), it follows from our hypothesis that it is τ'-convergent to (γ, B'). Thus $\mathcal{V}^{\tau}_{(\gamma,B')} \supseteq \mathcal{V}^{\tau'}_{(\gamma,B')}$ and in particular $(\gamma, G') \in \mathcal{V}^{\tau}_{(\gamma,B')}$. Thus (γ, G') is a τ-neighborhood of each of its soft subsets and hence by Lemma 2.5, (γ, G') is τ-open. So $\tau' \subseteq \tau$, i.e., τ is finer than τ'. $\square$

Again let (X_γ, τ) be a soft topological space and $\mathcal{F}$ be a soft filter on (γ, X). A soft set (γ, A) in $\mathcal{P}(\gamma, X)$ is said to be an adherent soft set of $\mathcal{F}$ if (γ, A) is an adherent soft set of every soft set in $\mathcal{F}$. The adherence of $\mathcal{F}$, $\mathrm{Adh}(\mathcal{F})$, is the set of all adherent soft sets of $\mathcal{F}$; so $\mathrm{Adh}(\mathcal{F}) = \widetilde{\bigcap}_{(\gamma,C)\in\mathcal{F}}\overline{(\gamma, C)}$. If $\mathcal{B}$ is a base for a soft filter on (γ, X), then (γ, A) is an adherent soft set of $\mathcal{B}$ if it is an adherent soft set of the soft filter generated by $\mathcal{B}$. The adherence of $\mathcal{B}$, $\mathrm{Adh}(\mathcal{B})$, is the set of its adherent soft sets.

Theorem 3.10. Let (X_γ, τ) be a soft topological space and $\mathcal{B}$ be a base for a soft filter on (γ, X). Then $\mathrm{Adh}(\mathcal{B}) = \widetilde{\bigcap}_{(\gamma,A)\in\mathcal{B}}\overline{(\gamma, A)}$.

Proof. Let $\mathcal{F}$ be the soft filter which $\mathcal{B}$ is a base. Then, according to the definition of the adherence of a soft filter base,

$$\mathrm{Adh}(\mathcal{B}) = \mathrm{Adh}(\mathcal{F}) = \widetilde{\cap}_{(\gamma,A)\in\mathcal{F}}\overline{(\gamma, A)}\,\widetilde{\subset}\,\widetilde{\cap}_{(\gamma,A)\in\mathcal{B}}\overline{(\gamma, A)}.$$

Let (γ, B) be any soft set in $\mathcal{F}$. Then there is a soft set (γ, C) in $\mathcal{B}$ such that $(\gamma, C)\widetilde{\subset}(\gamma, B)$ and so $\overline{(\gamma, B)}\widetilde{\supset}\overline{(\gamma, C)}\ \widetilde{\supset}\widetilde{\cap}_{(\gamma,A)\in\mathcal{B}}\overline{(\gamma, A)}$. Thus $\widetilde{\cap}_{(\gamma,A)\in\mathcal{F}}\overline{(\gamma, A)}\widetilde{\supset}\ \widetilde{\cap}_{(\gamma,A)\in\mathcal{B}}\overline{(\gamma, A)}$. Hence $\widetilde{\cap}_{(\gamma,A)\in\mathcal{F}}\overline{(\gamma, A)} = \widetilde{\cap}_{(\gamma,A)\in\mathcal{B}}\overline{(\gamma, A)}$. $\square$

Theorem 3.11. Let (X_γ, τ) be a soft topological space and (γ, B) be a soft set in $\mathcal{P}(\gamma, X)$. Then a soft set (γ, A) in $\mathcal{P}(\gamma, X)$ is adherent to (γ, B) if and only if there is a soft filter $\mathcal{F}$ on (γ, X) such that $(\gamma, B) \in \mathcal{F}$ and $\mathcal{F}$ converges to (γ, A).

Proof. Suppose (γ, A) is adherent to (γ, B). Then every τ-neighborhood (γ, C) of (γ, A) meets (γ, B), i.e., $(\gamma, C)\widetilde{\cap}(\gamma, B) \neq \Phi_X$. Thus $\mathcal{V}_{(\gamma,A)} \cup \{(\gamma, B)\}$, where $\mathcal{V}_{(\gamma,A)}$ is the τ-neighborhood soft filter of (γ, A), generates a soft filter which contains (γ, B) and is τ-convergent to (γ, A).

Conversely, suppose there is a soft filter $\mathcal{F}$ such that $(\gamma, B) \in \mathcal{F}$ and $\mathcal{F}$ is τ-convergent to (γ, A). Let (γ, C) be any τ-neighborhood of (γ, A). Then $(\gamma, C) \in \mathcal{F}$, and since $(\gamma, B) \in \mathcal{F}$ it follows that $(\gamma, B)\widetilde{\cap}(\gamma, C) \neq \Phi_X$. So (γ, A) is adherent to (γ, B). $\square$

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