Empirical Correlations for Penetration Height of Liquid Jet in Uniform Cross Flow - A Review

Soo -Young No†

Key Words: Empirical correlations, penetration height, Uniform cross flow, Liquid to air momentum flux ratio, Normalized downstream distance from the injector

Abstract

The empirical correlations for the prediction of penetration height of liquid jet in crossflow are reviewed and classified in this study. Around thirty different correlations had been proposed by many investigators. It has generally known that the penetration height of a liquid jet in a cross-flow is a function of the liquid to air momentum flux ratio and the normalized downstream distance from the injector. However, several researchers incorporated the Weber number, liquid-to-water or air viscosity ratio, pressure ratio or Reynolds number, temperature ratio in the empirical correlations. The existing correlations can be grouped as correlations in a power-law, logarithmic, and exponential forms, respectively. Correlations in a power-law form can be further classified as three groups such as basic form, Weber number form and other parameters form. It should be pointed out that correlations in a logarithmic form in terms of Weber number or any other parameters could not be found. Universal correlation has still not been established due to the significant discrepancies between various correlations suggested to date. Several of the studies reported the significant discrepancies of predicted values by the existing correlations. The possible reasons for discrepancies will be summarized as measurement technique, assumptions made in defining terms in the liquid to air momentum flux ratio, difficulties in defining the boundaries of the liquid jets, and nozzle/injector geometry. Evaluation of validity for the correlations proposed recently by several investigators is essentially required. Those include eight power-law forms, two logarithmic forms, and one exponential form

1. Introduction

The best known example of a plain-orifice atomizer in the combustion field is the diesel injector and another important applications of plain-orifice atomizer are jet engine afterburners and rocket engines(1). Compared with the free jet issuing into quiescent surroundings in conventional diesel engines, the transverse or jet injected normally into crossflow is commonly used in air-breathing propulsion systems including dilution air jets, fuel/air mixers, turbine blade film cooling systems, ramjet/scramjet fuel injectors, V/STOL aircraft) as well as in rocket engine systems and effluent stacks and plumes(2). In the direct injection of gasoline sprays which is recently common in spark ignition engines, swril and tumble cause a crossflow interaction with the fuel(3). The crossflow situation in agricultural field will be the application of chemicals to crops by flat-fan nozzles mounted on boom sprayers(4,5). Injection of liquid friction modifiers on to the rail surface with air-blast atomizers mounted to the external undercarriage of trains is another application of crossflow(6).

Recently, the jet in cross flow or transverse jet had been reviewed fairly extensively by Karagozian(2) because of widespread application in engineering systems. A comprehensive review of the behavior of liquid jets in high-speed cross flow based on the results of a long term research program was presented by Schetz(7).

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†Dept. of Biosystems Engineering, Chungbuk National University, Cheongju, Korea
E-mail : sooyoung@chungbuk.ac.kr
TEL : (043)261-2583 FAX : (043)271-4413

As one of macroscopic spray characteristics, spray penetration is of prime importance in diesel engines and air-breathing propulsion systems. Therefore, the prediction of diesel spray characteristics has been the subject of several works and intensive investigations are still underway by many researchers(8-10). In addition, it is known that the understanding of the trajectory and breakup of liquid jet in a cross-flow is critical to improve the efficiency and performance of liquid-fueled ramjet and scram jet combustors(1), lean prevaporized and premixed gas turbine technology(11). Even though there are several parameters such as column and surface breakup, penetration height, jet width, droplet size, droplet velocity for the liquid jet in crossflow as shown in Fig. 1, penetration height of liquid jet in a cross flow is one of important parameters that indicates how well the injected liquid can mix with the air. In this article, therefore, correlations for penetration height of liquid jet in cross flow will be reviewed.

A detailed survey of correlations related to liquid jet trajectory and penetration height available in the literature was carried out by in Lin *et al.*(12), Stenzler *et al.*(13), Iyogun *et al.*(14), Masuda and McDonell(15) Raguggcci *et al.*(16), Mashayek *et al.*(17). However, a part of them had mixed up with the theoretical model or numerical model and the other part of them did not cover all of the empirical correlations available in the litera-ture. Discussion will, therfore, be limited to empirical correlations available in the literature after 1990 for the prediction of penetration height of liquid jet in gaesous crossflow in this review. In addition, this review will be limited only liquid jet-in-crossflow studies focussed on a crossflow that has a uniform velocity profile across the cross section. The studies considering non-uniform crossflow such as a swirling crossflow(18-20), crossflow containing a shear layer(21) will not be included. In addition, the studies regarding the controlled liquid jet such as exciting liquid jet(22) will be excluded.

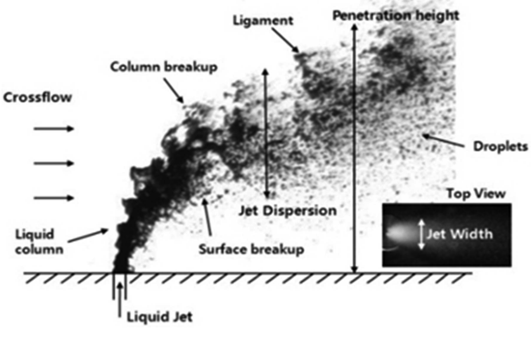

Fig. 1 Typical parameters of liquid jet in cross flow.

2. Empirical Correlations for Penetration Height

In the literature related to liquid jet in the uniform crossflow, several different terms such as spray penetration(13), penetration height(12,13,23), near-field penetration(24), jet penetration(2,25), liquid jet penetration(26), maximum transverse penetration(27), maximum penetration value(28) had been used as synonym. As the penetration height had been used by many researchers and to differentiate spray and liquid penetration of free jet in quiescent surroundings such as in diesel engines, penetration height is selected for discussion in this paper.

The transverse jet is a more complicated flow field than the free jet in quiescent surroundings such as in diesel engines due to its interaction with the cross flow and interaction of the jet with the wall boundary layer. Among the parameters used to characterize this flow field are the jet-to-cross flow momentum flux ratio, jet nozzle diameter, and streamwise distance. The terms “jet-to-cross flow momentum flux ratio”, “liquid to air momentum flux ratio”(29,30), “liquid/air momentum flux ratio”(14) “jet-to-air momentum flux ratio”(31,32), liquid -to-gas momentum ratio(17), momentum flux ratio or momentum ratio(33) have been used as synonym.

Chen *et al.*(34) measured the trajectory(penetration profiles) of water jet in subsonic cross-flow by means of Mie scattering technique and introduced the four early correlations for the comparison. They optimized the constants in the early correlations and found that neither type consistently predicts the entire jet boundary. Therefore, they divided the jet trajectory into three zones, liquid-column zone, ligament zone and droplet zone and used exponential function to represent each region. Finally, they proposed the following correlation in three-term exponential form as the entirely different one with the existing correlations.

$$y/d = 9.91\ q^{0.44}\ [1 - e^{-(x/d)/13.1}]\ [1 + 1.67\ e^{-(x/d)/4.77}]\ [1 + 1.06\ e^{-(x/d)/0.86}] \qquad (1)$$

The breakup processes of liquid jets injected into subsonic air cross flows were experimentally studied by Wu *et al.*(23) and their experimental results for penetration height were compared with the empirical correlations of early studies. It was clear from two studies of Chen *et al.*(34) and Wu *et al.*(23) that penetration height of liquid jet in an air cross flow is governed mainly by liquid to air momentum flux ratio(q), downstream(streamwise) distance(x) and nozzle diameter(d).

However, the following correlations suggested by Inamura *et al.*(25) were not precisely introduced by them. From experimental conditions that consisted of three sizes of nozzle diameter with 0.5, 1.0 and 2.0 mm, jet velocities of 7~26 m/s, and air velocities of 55~140 m/s, they had used a least square fit of data from the observation of side-lighted streak photographs.

For water jet

$$y/d = (1.18 + 0.24\ d)\ q^{0.36}\ \ln\ [1 + (1.56 + 0.48d)x/d] \quad (2)$$

For slurry jet

$$y/d = (1.17 + 0.16\ d)\ q^{0.43}\ \ln\ [1 + (0.75 + 0.95d)x/d] \quad (3)$$

It should be pointed out that this correlation maintain the nozzle diameter d in their expressions, even after normalizing the penetration height(y) and streamwise distance(x) by d. In addition, it seems that this correlation was proposed for positions of $x/d < 15$.

Wu *et al.*(23) found the discrepancies between the measured values and the predicted values from earlier correlations and these were due to the differences between drag coefficients and length scales for liquid/air momentum exchange between the liquid column and the ligament/droplet regimes. Therefore, they developed a power-law for the trajectory correlations as follows.

$$y/d = 1.37\ q^{0.5}\ (x/d)^{0.5} \qquad (4)$$

This correlation can be applicable up to the jet's column breakup(fracture) location.

In their subsequent work(35), they found that the spray dynamics are known to be different in the three spray regimes: the column, the ligament and the droplet regimes. Therefore, while utilizing the correlation of Wu *et al.*(23) for the liquid column region, they suggested a correlation for the spray trajectory in the droplet regime as

$$y/d = 4.3\ q^{0.33}\ (x/d)^{0.33} \qquad (5)$$

The comparison of two correlations, i.e. ones of Wu *et al.*(35) and Inamura *et al.*(25) was conducted by Wu *et al.*(36) in the experimental conditions of 0.5 mm water jets into a subsonic cross flow with q varied from 5 to 60. They found that the predicted result of Wu *et al.*(35) shows a larger penetration than that observed by Inamura *et al.*(25), even though the exponent of q in the correlation of Wu *et al.*(35) agrees with 0.36 obtained by Inamura *et al.*(25). They also concluded that the discrepancy is attributed to the measurement technique between PDPA of Wu *et al.*(36) and photographic method of Inamura *et al.*(25). In addition, the comparison of liquid-jet penetration obtained by measurement, theoretical calculation and these two empirical correlations was performed by Inamura(26). In this study, penetration obtained by two empirical correlations showed the much higher penetration than that of measurement, even though theoretical calculation supplied the relatively close penetration with the measurement. He pointed out that these discrepancies were due to the different estimation of drag coefficient. Even though the drag coefficient of a cylinder is a function of the Reynolds number, the drag coefficient was assumed to be constant as $C_d = 1.0$.

The breakup, penetration and atomization of a plain-orifice jet in an air cross flow were investigated experimentally by Becker and Hassa(24,37) and correlation for the prediction of jet penetration was suggested as

$$y/d = 1.57\ q^{0.36}\ \ln\ [1 + 3.81(x/d)] \quad q = 1{\sim}12 \qquad (6)$$

$$y/d = 1.48\ q^{0.42}\ \ln\ [1 + 3.56(x/d)] \quad q = 1{\sim}40 \qquad (7)$$

According to the agreement of exponent of q between Becker and Hassa (q=1-12)(24,37) and Inamura *et al.*(25) and between Becker and Hassa (q=1-40)(24,37) and Chen *et* *al.*(34), they pointed out that the exponent of q was to depend on the range of q on which the correlation is based. In addition, they found that aerodynamic Weber number and dominant breakup mechanism have no significant effect on the penetration contour(37).

Leong and Hautman(27) had introduced three correlations developed by Wu *et al.*(23), Wu *et al.*(35) and Wotel *et al.*(38), respectively for the comparison of measured and predicted maximum transverse penetration at three different jet-to-cross flow momentum flux ratios. They found that best fit of penetration length was that predicted from a correlation by Wotel *et al.*(38)

Several correlations for the prediction of penetration heights of liquid jets injected into high-speed cross flows were tested by Lin *et al.*(12). In this study, correlations proposed by Wotel *et al.*(38) and Inamura *et al.*(25) were not included. Instead of them, two correlations suggested by Wu *et al.*(23,35) were considered for comparison. Furthermore, they had developed several correlations for the penetration heights of pure liquid jet in subsonic and supersonic cross flows and aerated-liquid jets in subsonic cross flows based on experimental data measured by shadowgraph images and PDPA. They concluded that the use of correlation derived from PDPA measurements is highly recommended as

$$y/d = 3.17\ q^{0.33}(x/d)^{0.40} \qquad (8)$$

An attempt had been made by Carvaliere *et al.*(39) to verify the validity of correlations developed based on liquid to air momentum flux ratio only and proposed by Wu *et al.*(23) in power-law form, Becker and Hassa(24, 37) in logarithmic form and Chen *et al.*(34) in exponential form., respectively. According to a diagram of the residual average error as a function of q, they found the substantial failure of three correlations in reproducing the experimentally observed liquid jet trajectories over a relatively wide range of liquid –to–air momentum flux ratio.

By using a least squares fit of data from experimental conditions that consisted of nozzle diameter of 1.0 mm, $q = 2.54\sim15.88$ and air stream temperature of 298~500 K, Lee *et al.*(40) had modified the correlation of Inamura *et al.*(25) for the penetration height of liquid jet in a dump-type ramjet combustor.

$$y/d = (1.2+0.4\ d)\ q^{0.36} \ln\,[1+(1.56 + 0.48d)(x/d)] \qquad (9)$$

To author's knowledge, among the first who considered the influence of aerodynamic Weber number and liquid viscosity in the correlation for penetration height prediction are Stenzler *et al.*(28)

$$y/d = 2.63\ q^{0.442}(x/d)^{0.39}\ We^{-0.088}(\mu_l/\mu_w)^{-0.027} \qquad (10)$$

Based on the experiments on spray characteristics of three liquids jet in subsonic cross flow, Tambe *et al.*(41) proposed the following correlation in logarithmic form.

$$y/d = 1.55\ q^{0.53} \ln\,[1 + 1.66(x/d)] \qquad (11)$$

The test of five correlations, i.e. one power-law form(23), one exponential form(34), and three logarithmic forms revealed that the best results were obtained with the logarithmic form. In addition, they concluded that jet penetration increases with q and d, but is independent of Weber number.

In their continued study, a correlation including the effect of temperature was suggested by Lakhamraju and Jeng(29) in the following form.

$$y/d = 1.8444\ q^{0.546} \ln\,[1+3.324(x/d)](T_a/T_o)^{-0.117} \qquad (12)$$

where T_o is the ambient temperature (=294 K) and T_a the airstream temperature. They concluded that an increase in airstream temperature leads to a decrease in the penetration height of liquid jet. By choosing the ambient pressure, Weber number and q as the parameters of significance, empirical correlations for the prediction of the upper and the lower boundaries of the spray were offered by the same research group(30). After examination of the three different forms of correlation to develop a correlation which best fits their experimental results, they found that best results were obtained from the correlation in power-law form. Two correlations for the upper and lower boundaries of the spray were proposed as follows.

$$y/d = 4.95\ q^{0.424}\ [(x/d) + 0.5)]^{0.279}\ We^{-0.076}(p/p_o)^{-0.051}$$

for upper boundary (13)

$$y/d = 4.26\ q^{0.408}\ [(x/d\ ) - 0.5)]^{0.349}\ We^{-0.30}\ (p/p_o)^{0.111}$$

for lower boundary (14)

where p_o is the atmospheric pressure as the reference value. They reported that an increase in the ambient pressure results in a slight decrease in the penetration height of liquid jet.

Two correlations were developed by Stenzler *et al.*(13) to predict the penetration heights of the liquid jets in cross flow, including both heated and unheated air conditions as follows.

$$y/d = 3.354\ q^{0.42}\ (x/d)^{0.391}\ \mathrm{We}^{-0.088}\ (\mu_l/\mu_w)^{-0.027} \quad \text{for all conditions} \qquad (15)$$

$$y/d = 3.688\ q^{0.43}\ (x/d)^{0.384}\ \mathrm{We}^{-0.110}(\mu_l/\mu_w)^{-0.108} \quad \text{for unheated air conditions} \qquad (16)$$

The jet penetration of a recessed liquid jet into a cross flow at elevated temperature (350~475 K) and pressure (0.38~0.65 MPa) has been characterized experimentally by Masuda *et al.*(42) and Masuda and McDonell(15). Initially, Masuda *et al.*(48) suggested the following empirical correlation.

$$y/d = 0.92\ q^{0.5}\ (x/d)^{0.33} \quad 2<q<30 \qquad (17)$$

In their continued work, Masuda and McDonell(15) modified their previous correlation by considering the aerodynamic Weber number and viscosity effect, similar with the correlation of Stenzler *et al.*(28) as follows.

$$y/d = 15.0\ q^{0.5}(x/d)^{0.33}\mathrm{We}^{-0.41}(\mu_l/\mu_w)^{-0.02}\ 2<q<50 \qquad (18)$$

Iyogun *et al.*(14) had discussed fairly extensively the six existing correlations for the predictions for the prediction of penetration length and suggested their correlation as follows.

$$y/d = 1.997\ q^{0.444}\ (x/d)^{0.444} \qquad (19)$$

Ahn *et al.*(43) obtained the following empirical correlation, similar with one suggested by Wu *et al.*(23) and tested it to orifice internal flow such as cavitation flow and hydraulic flip flow.

$$y/d = 1.297\ q^{0.491}\ (x/d)^{0.509} \qquad (20)$$

They found that their correlation was effective only for cavitation flow and the diameter and the liquid/air momentum flux ratio obtained from effective velocity and effective area of the liquid jet was more appropriate in hydraulic flow flows.

The effect of liquid viscosity on the penetration height and trajectory of a jet in a low subsonic cross flow was investigated experimentally by Birouk *et al.*(44) and empirical correlations of jet trajectory were proposed to account for the combined effect of viscosity, momentum flux ratio, and nozzle diameter, not including Weber number.

$$y/d = 1.997\ q^{0.444}\ (x/d)^{0.444} \quad x/d < 12 \qquad (21)$$

$$y/d = 1.627\ q^{0.47}\ (x/d)^{0.46}\ (\mu_l/\mu_w)^{0.079} \quad x/d > 12 \qquad (22)$$

They found that close to the nozzle exit, the effect of liquid viscosity on penetration height was not significant. However, far from the nozzle exit, the penetration height of liquid jet increased initially as the liquid viscosity increased, but a further increase in viscosity reduced the penetration.

Ragucci *et al.*(11) have performed experimental study of kerosene and water jets in cross flow at elevated pressure for taking into account the effect of air density, using a shadowgraphic technique at liquid-to-air momentum flux ratios of 5~197, pressures of 1 and 2 MPa, and room temperature. They found that the normalized jet trajectory was not a function of liquid properties and the correlation of Becker and Hassa(24) performs better than the correlation of Wu *et al.*(23). They proposed the following correlation with the inclusion of aerodynamic Weber number, showing better agreement with the experimental data than two existing correlations mentioned in the above.

$$y/d = 2.698\ q^{0.441}\ (x/d)^{0.367}\ \mathrm{We}^{-0.069} \qquad (23)$$

Ragucci *et al.*(45) extended their experiment on penetration height of Jet A-1 fuel and water jet in cross flow at elevated temperature. The other experimental conditions were exactly same with those in the previous study(11). In this study, by considering the possible influence of an increase in air viscosity due to the increase of ambient temperature up to 600 K, they suggested the following correlation for penetration height of liquid jet in cross flow at elevated temperature and pressure.

$$y/d = 2.28q^{0.422}(x/d)0.367\ \mathrm{We}^{-0.015}\ (\mu_l/\mu_a)^{0.186} \qquad (24)$$

It should be pointed out that liquid viscosity(μ_l) was normalized with air viscosity(μ_a) instead of water viscosity(μ_w) in the correlation of Birouk *et al.*(44).

To take into account the effect of an increase in air viscosity due to the increase of ambient temperature instead of normalized viscosity(μ_l/μ_a), gas Reynolods number was newly introduced into the correlation for the penetration height of kerosene and water jets in cross flow by Bellofiore *et al.*(46) and an empirical correlation of trajectory of liquid jet was suggested as follows.

$$y/d = 0.909\, q^{0.476}\, (x/d)^{0.35}\, \mathrm{Re}_g^{0.135}\, \mathrm{We}^{-0.128} \quad (25)$$

The experimental conditions consisted of air pressures of 1.0 and 2.0 MPa, air temperatures of 300 and 600 K, air velocity ranged between 20 and 60 m/s, liquid velocity ranged between 15 and 55 m/s. These data resulted in the liquid to air momentum flux ratio q = 12.1~81.0, gas Reynolds number Re_g = 4840~26066, and aerodynamic Weber number We = 10.4~350.7.

It should be noted that Stenzler *et al.*(13), Ragucci *et al.*(45) and Birouk *et al.*(44) had considered the liquid to water or air viscosity ratio in their correlations. However, Wu *et al.*(23) mentioned that drag coefficients were found to exhibit a weak dependence on liquid viscosity when they developed their correlation. Eq. (4). Therefore, it is required to test the effect of liquid viscosity. In addition, Nejad and Schetz(47) measured the penetration heights for liquid jets having the different viscosities and surface tensions. Increase in liquid viscosity showed a higher penetration height, but no significant effect of surface tension on penetration heights was observed, although decrease in surface tension increased both the wave length and wave amplitude which intensifies the process of jet breakup.

Flow visualization was carried out by Elshamy *et al.*(22) on the liquid jets in a cross flow using Mie-scattering technique to develop the following correlations for the outer boundaries of spray plume.

$$y/d = 12.63q^{0.446}\left[1-e^{-\left(\frac{x}{d}+0.5\right)/10.46}\right]\times \left[1+1.42e^{-\left(\frac{x}{d}+0.5\right)/4.14}\right]\times\left[1+e^{-\left(\frac{x}{d}+0.5\right)/1.39}\right]\times We^{-0.141}\left(\frac{p}{p_0}\right)^{-0.05} \quad (26)$$

It should be noted that in the above correlation cross flow Weber number and pressure ratio is incorporated in addition to the basic multi-zone form of Chen *et a*l's correlation(34).

Freitag and Hassa(48) extended their existing data set with penetration data(24) for three different nozzle diameters. They found the following correlation in power-law form for the penetration height of liquid jet

$$y/d = 3\, q^{0.4}\, (x/d)^{0.27} \quad (27)$$

However, they tried again to correlate their data in logarithmic form and eventually found a little better correlation with higher correlation coefficient that the above correlation as follows.

$$y/d = 1.6\, q^{0.4} \ln\left[1+3.81(x/d)\right] \quad (28)$$

Correlations of Wu *et al.*(23), Tambe *et al.*(41), Becker and Hassa(24), and Ragucci *et al.*(45) had compared with the experimental trajectory of water jets injected into subsonic air cross flow with elevated temperature and pressure by Amighi *et al.*(49). Based on the assumption that correlating the spray trajectory merely with the momentum ratio *q*, which presents equal contributions to cross low and jet velocities, is inadequate, they suggested the following correlations for spray center-line and windward trajectories with *q* and channel and jet Reynolods numbers.

$$y/d = 0.167\, q^{0.31}\, (x/d)^{0.37}\, \mathrm{Re}_{ch}^{0.11}\, \mathrm{Re}_j^{0.15} \quad \text{for windward trajectory} \quad (29)$$

$$y/d = 0.191\, q^{0.30}\, (x/d)^{0.43}\, \mathrm{Re}_{ch}^{0.12}\, \mathrm{Re}_j^{0.14} \quad \text{for centerline trajectory} \quad (30)$$

3. Results and Discussion

Many correlations have been developed to predict the penetration height of liquid jet in a cross-flow. However, a universal correlation has still not been established due to the significant discrepancies between various correlations suggested to date.

It has generally known that the trajectory of a liquid jet in a cross-flow (*y*/*d*) is a function of the liquid to air momentum flux ratio(q) and the normalized downstream distance from the injector (x/d). The existing correlations can be grouped as correlations in a power-law, logarithmic, and exponential forms, respectively.

$$y/d = A\, q^{B}\, (x/d)^{C} \tag{31}$$

$$y/d = A\, q^{B} \ln (1 + C\, x/d) \tag{32}$$

$$y/d = A\, q^{B} [1-\exp(C\, x/d)]\, [1 + D \exp (E\, x/d)]\, [1+ F \exp(G\, x/d)] \tag{33}$$

where A, B, C, D, E, F and G are constants, respectively.

However, several researchers incorporated the Weber number, liquid-to-water or air viscosity ratio, pressure ratio or Reynolds number, temperature ratio in the above power-law functions.

Around 20 correlations in power-law functional form and 8 correlations in logarithmic functional form were available in the literature.

Furthermore, correlations in a power-law form can be classified as three groups, i.e. correlations with basic form Eq. (31), correlations incorporated with the Weber number (Weber number form) or correlations incorporated with other parameters (other parameters form) as follows.

$$y/d = A\, q^{B}\, (x/d)^{C}\, We^{D}\, (Z)^{E} \tag{34}$$

$$y/d = A\, q^{B}\, (x/d)^{C}\, (Z_1)^{D}\, (Z_2)^{E} \tag{35}$$

where Z denotes the viscosity ratio, pressure ratio or jet Reynolds number depending on the empirical correlation. According to the suggested correlation, Z_1 can also be the viscosity ratio or channel Reynolds number, and Z_2 represents 1.0 or jet Reynolds number.

The constants of correlations grouped as basic power-

Table 1 Basic power-law form : $y/d = Aq^{B}(x/d)^{C}$

Investigators	A	B	C	Remarks
Wotel *et al.* (1991)[38]	1.19	0.45	0.45	
Wu *et al.* (1997a)[23]	1.37	0.5	0.5	$x/d < 10$
Wu *et al.* (1997b)[35]	4.30	0.33	0.33	$x/d > 10$
Lin *et al.* (2002)[12]	3.17	0.33	0.40	
Ahn *et al.* (2006)[43]	1.297	0.491	0.509	
Masuda and McDonell (2006)[15]	0.92	0.50	0.33	$2 < q < 30$
Iyogun *et al.* (2006)[14]	1.997	0.444	0.444	Birouk *et al.*(2007)[44]

Table 2 Weber number form: $y/d = Aq^{B}(x/d)^{C}We^{D}(Z)^{E}$

Investigators	A	B	C	D	E	Remarks
Stenzler *et al.* (2003)[28]	2.63	0.442	0.39	-0.088	-0.027	$Z = \frac{\mu_e}{\mu_w}$
Elshamy & Jeng (2005)[30]	4.95	0.424	0.279	-0.076	-0.051	$Z = \frac{p}{p_o}$
Stenzler *et al.* (2006)[13]	3.354	0.442	0.391	-0.088	-0.027	$Z = \frac{\mu_e}{\mu_w}$
Masuda and McDonell (2006)[15]	15.0	0.5	0.33	-0.41	-0.027	$Z = \frac{\mu_e}{\mu_w}$ $2<q<50$
Ragucci *et al.* (2007a)[11]	2.698	0.441	0.367	-0.069	1.0	$Z = 1$
Ragucci *et al.* (2007b)[45]	2.28	0.422	0.367	-0.015	0.186	$Z = \frac{\mu_e}{\mu_w}$
Bellofiore *et al.* (2007)[46]	0.909	0.476	0.35	-0.128	0.135	$Z = Re$

Table 3 Other parameter form $y/d = Aq^B(x/d)^C Z_1^D (Z_2)^E$

Investigators	A	B	C	D	E	Remarks
Birouk *et al*. (2007)[44]	1.672	0.47	0.46	0.079	0	$Z_1 = \frac{\mu_e}{\mu_w}, Z_2 = 1$ $x/d > 2$
Amighi *et al.* (2009)[49]	0.167	0.31	0.37	0.11	0.15	$Z_1 = Re_{ch}, Z_2 = Re_j$ for winward

Table 4 $y/d = Aq^B \ln[1 + C(x/d)]$

Investigators	A	B	C	Remarks
Inamura *et al.* (1991)[25]	1.18+0.24d	0.36	1.56+0.48d	For water jet
	1.17+0.16d	0.43	0.75+0.95d	For slurry jet
Becker and Hassa (1999)[37]	1.48	0.42	3.56	q=1~40
Becker and Hassa (2002)[24]	1.57	0.36	3.56	q=1~12
Lee *et al.* (2003)[40]	1.2+ 0.4d	0.36	1.56+ 0.48d	
Tambe *et al.* (2005)[41]	1.55	0.53	1.66	
Lakhamraju and Jeng (2005)[29]	1.8444	0.546	1.324	
Freitag and Hassa (2008)[48]	1.6	0.4	3.81	q=3~24

law form are listed in Table 1. Even though the constant A shows widely scattered value, the constants B and C can be categorized as three groups: i.e. 0.33, 0.45 and 0.50. In Table 2, which Weber number form is listed, the constants B and C shows the similar values. Most correlations are incorporated with Weber number and viscosity ratio, and their effect on the penetration height is not significant. Two correlations belong to other parameters form as shown in Table 3.

Table 4 shows the constants of logarithmic form. Most correlations show the similar effect on *q* on penetration height of liquid jet. It should be noted that correlation in logarithmic form in terms of Weber number could not be found. In addition, no parameters such as viscosity ratio, pressure ratio and Reynolds number are considered in the logarithmic form.

Universal correlation has still not been established due to the significant discrepancies between various correlations suggested to date. Several of the studies reported the significant discrepancies of predicted values by the existing correlations. The possible reasons for discrepancies will be summarized as measurement technique, assumptions made in defining terms in the liquid to air momentum flux ratio, difficulties in defining the boundaries of the liquid jets, and different nozzle/injector geometry.

The penetration height increases with an increase in injector diameter or liquid to air momentum flux ratio(41). Increasing gas velocity or orifice diameter, which increases the Weber number, will reduce the penetration heights. According to the experimental results from shadowgraph(30), as increasing the air density by increasing the ambient pressure, the jet penetration slightly decreased. Increase in ambient temperature resulted in decrease in liquid jet penetration. As seen in the correlations in logarithmic form, penetration height is independent of Weber number and viscosity ratio. The important physical properties of the liquid such as viscosity and surface tension have been found to effect negligibly on penetration height(36-47).

Evaluation of validity for the correlations proposed by several investigators is essentially required. Those include eight power-law forms proposed by Elshamy and Jeng(30), Iyogun *et al.*(14), Stenzler *et al.*(13), Masuda and McDonell(15), Ragucci *et al.*(16), Birouk *et al.*(44), Bellofiore *et al.*(46), Amighi *et al.*(49), two logarithmic forms suggested by Lakhamraju and Jeng(29) and Freitag and Hassa(48), and one exponential form developed by Elshamy *et al.*(22)

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Nomenclature

d	nozzle diameter
q	liquid-to-air momentum flux ratio ($\rho_l V_l^2 / \rho_g V_g^2$)
Re	Reynolds number ($\rho V d/\mu$)
p	pressure
We	aerodynamic (=crossflow) Weber number ($\rho_g V_l^2 d/\sigma$)
x	cross stream(=downstream) distance
y	streamwise distance
μ	viscosity
ρ	density
σ	surface tension

Subscripts

a	air
ch	channel
g	gas
j	jet
l	liquid
w	water

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