Reduced-State MLSD Based on Volterra Kernels for Square-Law Detected Multipath Channels

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Received May 6, 2011; revised June 10, 2011; accepted November 28, 2011; published December 31, 2011

Abstract

We propose a novel reduced-state maximum-likelihood sequence detection (MLSD) structure using the Viterbi algorithm based on the second-order Volterra kernel modeling nonlinear distortion due to square law detection of multipath channels commonly occurring in chromatic dispersion (CD) or polarization mode dispersion (PMD) in optical communication systems. While all existing MLSD methods for square-law detection receivers are based on direct computation of branch metrics, the proposed algorithm provides an efficient and structured way to implement reduced-state MLSD with almost the same complexity of a MLSD for linear channels. As a result, the proposed algorithm reduces the number of parameters to be estimated and the complexity of computation.

Keywords: Inter symbol interference equalization, MLSD, reduced-state MLSD, square law detection, Volterra series

This work was supported by Basic Science Research Program through the NRF funded by the MEST (NRF-2010-0025437) and BK21.

DOI: 10.3837/tiis.2011.12.003

1. Introduction

Digital equalization of a linear channel has been throughly investigated in wirelss or wire communicaiton systems. However, in optical communication systems or magnetic recoding channels nonlinear channels characterized by a multipath channel followed by a square law detection device have not been fully studied yet. For example, in optical communication systems mitigation of chromatic dispersion (CD) and polarization mode dispersion (PMD) in the electronic domain recently has been gaining importance. This distortion can be modeld as a sqaure law detection of multipath signals. Much work has been carried out in applying various equalization techniques utilized in wireless communication systems to optical communication systems [1][2][3][4][5][6][7][8]. The fundamental limitaion is that the most studies in wireless communication have focused on linear equalization, while the distortin due to sqaure law detection and multipath is nonlinear. Maximum-likelihood sequence detection (MLSD) schemes based on the Viterbi algorithm [9] have been proven to be a powerful and practical means of mitigating linear distiortions such as CD and PMD distortion after square law device. However, the drawback of MLSD is that its complexity grows exponentially as the channel length increases. To reduce the complexity of MLSD in wireless communication systems, a decision feedback equalizer (DFE) is combined with a Viterbi algorithm based on the channel model [10]. In optical communications systems, due to the nonlinearity of the photo detector, a Viterbi algorithm based on a linear channel model is not feasible. Hence, existing reduced-state MLSD techniques for optical systems rely on a brute force truncation [7] or complicated direct computation of branch metrics for all the reduced states reflecting the state reduction [8].

In this paper, using the second-order Volterra kernel channel model for the nonliear channel due to square law detection device as studied for mitiation of PMD and CD distortion in direct detection optical communication systems developed in [11], we develop a reduced-state Viterbi algorithm based on Volterra kernel. The Volterra kernel approach effectively model the multipath structure hidden in square law device effect. Hence, the structure of the proposed reduced-state MLSD is simple and efficient, similar to the reduced-state MLSD [10] for the linear channels. Utilizing the earlier detections the proposed method reconstruct channel states predicted by Volterra kernel model. Furtheremore, for channel estimation of the Volterra kernel the exising efficient channel estimation method in [11] can be directly applied. Hence, both in terms of channel estimation and MLSD equalization the complexity can be substantially reduced.

In section 2, we introduce the second order Volterra kernel for modeling the nolinear distortion due to square law detection and multipath. Assuming CD and PMD distortion can be approximated linear in the optical domain, this model can be applied for CD and PMD distiortion in optical communication system [1]. In section 3, we propose an efficient reduced-state MLSD structure based on the Volterr channel model. The proposed algorithm is a generalization of reduced MLSD algorithm utilizing DFE. The difference is in Volterra kernel model the pre-made decisions are mixed with future values governed by state vectors. Section 4 shows simulation results that confirm the performance of the proposed MLSD algorithm. The simulations are mainly conducted with optical communication systems where CD and PMD are one of the most imporant exapmes of the nonlinear distortions due to square law detection of multipath.

2. System Model

Let us consider the system model shown in **Fig. 1**.

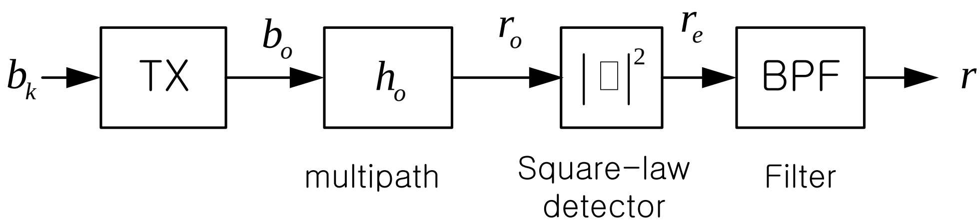


Fig. 1. System model

A bit sequence $\{b_k\}$ of bit rate T is modulated to a standard non-return-to-zero (NRZ) ON-OFF keying (OOK) signal and transmitted through a multipath h_o. The received signal is detected by a square law detector. This channel model represents common distortions in a standard single-mode fiber (SMF) suffering from a linear distortion such as CD or the first order PMD when the dominant noise component is the amplified spontaneous emission (ASE) noise, which is modeled as an additive white Gaussian noise (AWGN). The transmitter output with a pule shaping filter $p(t)$ can be written in the following form:

$$b_o(t) = \sum_{n=-\infty}^{\infty} p(t-nT)b_n \tag{1}$$

The multipath output is then given as

$$r_o(t) = \sum_{n=-\infty}^{\infty} h_p(t-nT)b_n + n(t) \tag{2}$$

where $n(t)$ denotes AWGN and h_o denotes the overall pulse shaping and channel response

$$h_o = h(t) * p(t) = \int_{-\infty}^{\infty} h(\tau)p(t-\tau)d\tau \tag{3}$$

Then the square law detector output is given as

$$r_e(t) = \left| \sum_{n=-\infty}^{\infty} h_p(t-nT)b_n + n(t) \right|^2 = s_e(t) + n_e(t) \ , \tag{4}$$

where $s_e(t)$ represents the signal component and $n_e(t)$ the noise component. Note that the signal component $s_e(t)$ in Eq. (4) can be expressed as

$$s_e(t) = \sum_{n \le m = -\infty}^{\infty} h(t - nT, t - mT) b_n b_m \tag{5}$$

for a real valued two dimensional response, $h(t,s)$, given by

$$h(t,s) = \begin{cases} 2\,\mathrm{Re}\left[h_e(t) h_e^*(s)\right] & t < s \\ |h_e(t)|^2 & t = s \end{cases}, \tag{6}$$

and the noise component is given as

$$n_e(t) = 2\sum_{n=-\infty}^{\infty} \mathrm{Re}\left[h(t - nT) b_n n^*(t)\right] + \left|n(t)\right|^2 \tag{7}$$

The noise component from the square law detector is no more Gaussian and mixed with the signal. The noise is not white and consequently degrades the performance of MLSD detector. The signal component of the electronic filter output, denoted by $s(t)$, is given as $s(t) = h_e(t) * s_e(t)$, where * is the convolution operation. Since convolution is a linear operation, the sampled version of $s(t)$ can be expressed in the form of

$$s_k = s(kT) = \sum_{n \le m} c(n-k, m-k) b_n b_m \tag{8}$$

for a second order Volterra kernel $c(n,m)$, where

$$c(n,m) = \int_{-\infty}^{\infty} h_e(\tau) h(\tau - nT, \tau - mT) d\tau \tag{9}$$

Denoting the sampled noise n_k, the receiver output is written as

$$r_k = \sum_{n \le m} c(n-k, m-k) b_n b_m + n_k \tag{10}$$

Note that in Eq. (10) can be expressed as a square lower-triangular matrix, using finite length truncation if needed. The diagonal elements $c(n,n)$ correspond to the linear portion of the nonlinear channel, since for binary sequence $b_n = b_n^2$. We refer to the size of this lower –triangular matrix as the length of the nonlinear channel, which is the same as the length of the diagonal elements $c(n,n)$ [11].

3. Background and Related Works

Given a received sample r_k , the branch metric of the Viterbi algorithm of 2^M (usually $M = N_c - 1$ for perfect equalization in linear channel cases) states MLSD is given from the log-likelihood function for the received sample r_k and a $M+1$ bit sequence $\mathbf{b}_k = [b_k, b_{k-1}, \cdots, b_{k-M}]$, where $[b_k, \cdots, b_{k-M-1}]$ is determined from the previous state and b_{k-M} is determined by the state transition.

$$\lambda(r_k, \mathbf{b}_k) = \ln p(r_k \mid \mathbf{b}_k) \tag{11}$$

For example, as illustrated in **Fig. 2**, when previous state s_L assigning a length M binary vector $[b_k, \cdots, b_{k-M-1}]$ of which decimal value is L , the transition is determined by the bit ‘0’ or ‘1’. The Viterbi MLSD algorithm computes the branch metrics for all previous 2^M states for each 2^M new states. The branch metrics are computed and compared to obtain the best survivor.

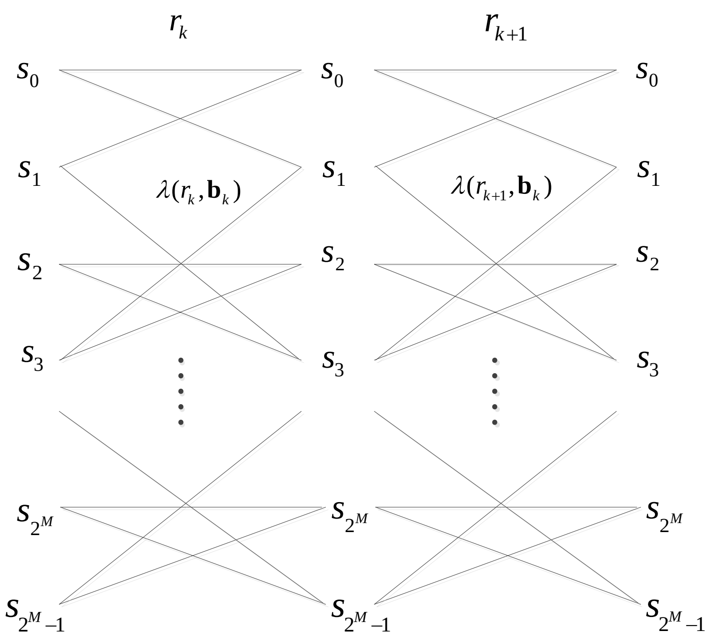


Fig. 2. State transition diagram

Contrast to the linear channel case, the noise in this system model is not Gaussian. Consequently, the log likelihood function becomes complicated and closed form expression is not feasible. A straightforward way to implement Viterbi algorithm is to find the $\lambda(r_k, \mathbf{b}_k)$ functions for all combination of $\mathbf{b}_k$, which is 2^M , as proposed in [8].

$$\lambda(r_k, \mathbf{b}_k) = f_{\mathbf{b}_k}(r_k) \tag{12}$$

However, this straightforward approach requires heavy computational load. To reduce computational load there have been several other approaches to approximate the branch metric.

Using the fact that the noise can be modeled by a χ^2 distribution, the branch metric can be approximated as

$$\lambda(r_k, \mathbf{b}_k) \approx 2\sqrt{r_k s_k} - s_k - \frac{N_0}{2}(\nu - \frac{1}{2}) \ln s_k \tag{13}$$

where s_k is the signal component estimated from $\mathbf{b}_k$ and N_0 and ν are constants related with noise [4]. Using the Volterra kernel model $c(n,m)$, the s_k based on $\mathbf{b}_k$ can be given as the following assuming that we have full knowledge of $c(n,m)$ (by a least mean square (LMS) estimation considered in [12], for example),

$$s_k = \sum_{0 \le n \le m}^{N_c} c(n,m) b_{k-n} b_{k-m} \tag{14}$$

Consequently, we have a closed form branch metric

$$\lambda(r_k, \mathbf{b}_k) \approx 2\sqrt{r_k \sum_{0 \le n \le m}^{N_c} c(n,m) b_{k-n} b_{k-m}} - \sum_{0 \le n \le m}^{N_c} c(n,m) b_{k-n} b_{k-m} - \frac{N_0}{2}(\nu - \frac{1}{2}) \ln \sum_{0 \le n \le m}^{N_c} c(n,m) b_{k-n} b_{k-m} \tag{15}$$

This metric can be further simplified to the Euclidean metric in the presence of post-detection nonlinear processing [13], where the χ^2 noise is transformed to near Gaussian using $|\square|^q$ operation

$$\lambda(r_k, \mathbf{b}_k) \approx (r_k - s_k)^2 = \left(r_k - \sum_{n \le m}^{N_c} c(n,m) b_{k-n} b_{k-m} \right)^2, \tag{16}$$

which reduces the complexity significantly. Note that Eq. (9) is similar to the conventional MLSD branch metric for linear channels

$$\lambda(r_k, \mathbf{b}_k) \approx \left(r_k - \sum_{n=1}^{N_c} c(n) b_{k-n} \right)^2 \tag{17}$$

The difference is in the number of channel coefficients; instead of N_c coefficients in (17) we have $N_c(N_c+1)/2$ coefficients in (16).

4. Proposed Algorithm

Now we introduce reduced-state MLSD with 2^{M-L} states. The goal is to reduce the number of states from 2^M to 2^{M-L}, but to keep the estimate s_k as accurate as possible. Consider a state vector $\mathbf{b}_k = [b_k, b_{k-1}, \cdots, b_{k-M}]$ which has M variables and consequently 2^M choices when each f variable is drawn from 0 or 1. A straightforward way to reduce the number of states is to redefine the branch metric as a function of the received sample r_k and the reduced state vector $\mathbf{b}_k^{M-L} = [b_k, b_{k-1}, \cdots, b_{k-M-L}]$, which is 2^{M-L}, as proposed in [8].

$$\lambda(r_k, \mathbf{b}_k^{M-L}) = f_{\mathbf{b}_k^{M-L}}(r_k) \tag{18}$$

However, this method also requires heavy computational load. To reduce the state systematically, we fix a certain portion of the state vector with decisions and consider structure as illustrated in **Fig. 3**.

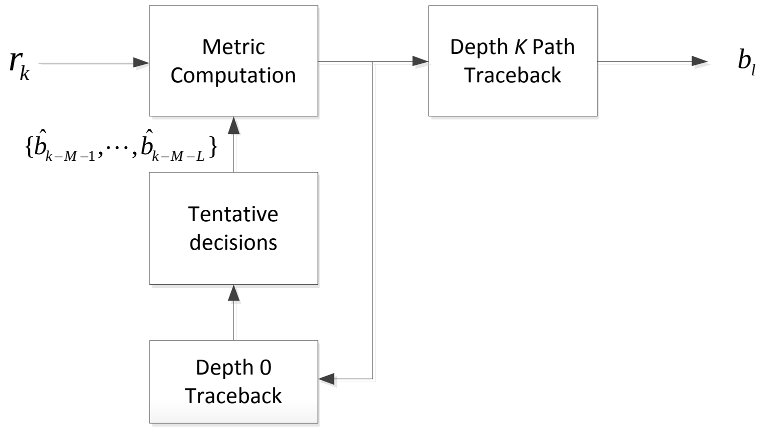


Fig. 3. Block diagram of the proposed algorithm

Let $\{\hat{b}_{k-M-1}, \cdots, \hat{b}_{k-M-L}\}$ denote L consecutive tentative decisions of this reduced-state MLSD equalizer. The tentative decision is obtained by decoding depth 0 trace-back of the trellis metrics. The final decisions are made by tracing back the trellis metrics once survivor trellis paths are converged, but, in practice, with the decoding depth K . Hence, the tentative decisions are not correct always, which will degrade the performance of the proposed reduced MLSD algorithm.

We define a new state vector $\tilde{\mathbf{b}}_k$ as

$$\tilde{\mathbf{b}}_k = [b_k, b_{k-1}, \cdots b_{k-M+L}, \tilde{b}_{k-M+L+1}, \cdots, \tilde{b}_{k-M}] \tag{19}$$

Note that the effective number of free variables in $\tilde{\mathbf{b}}_k$ is $M-L$ .When $\mathbf{b}_k^L = [b_k, b_{k-1}, \cdots, b_{k-L}]$ denotes a length L bit sequence from k ,

$$\tilde{\mathbf{b}}_k = [\mathbf{b}_k^{M-L}, \hat{\mathbf{b}}_{k-M+L+1}^L] \tag{20}$$

Now we define

$$\lambda(r_k, \mathbf{b}_k^L) = \lambda(r_k, \tilde{\mathbf{b}}_k) = \left(r_k - \sum_{0 \le n \le m}^{N_c} c(n,m) \tilde{b}_{k-n} \tilde{b}_{k-m} \right)^2 \tag{21}$$

where

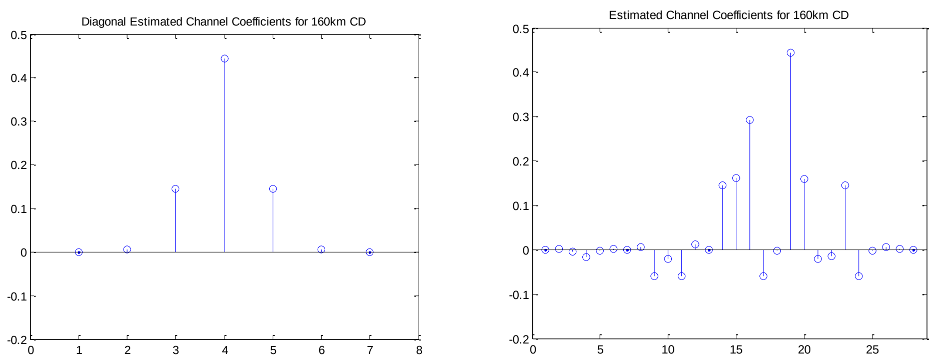
$$\tilde{b}_{k-n} = \begin{cases} b_{k-n} & 0 \le n \le M-L \\ \hat{b}_{k-n} & M-L+1 < n \le M \end{cases} \tag{22}$$

Note that the branch metric in Eq. (21) is explicit and structured unlike that of reduced-state MLSD equalizers in [8]. Without this Volterra kernel model, 2^{M-L} metrics functions for each 2^{M-L} state and L past bits should be separately determined. Again, when the channel c is linear, (21) reduces down to the reduced-state MLSD using decision feedback (DFE) branch metric in [10]. Due to the tentative decisions, the performance degradation of the reduced MLSD is inevitable. However, the benefit from the reduction of state is more significant in many practical applications.

5. Simulation Results

We consider direct detection optical communication links as the main application of the proposed algorithm. The optical standard single-mode fiber usually generates multipath effect (CD and PMD) as the length of the fiber increases and the signal is detected by square-law detector.

Now consider a 10-Gb/s NRZ OOK system through a standard single-mode fiber (SMF) with the dispersion parameter 17 ps/km/nm. The simulation focuses on the CD effect, although the first order PMD produces a similar result. **Fig. 4** shows the estimated channel coefficients for a 160km CD when the channel length 7 is used. **Fig. 4-(a)** plots the diagonal coefficients $c(n,n)$ and **Fig. 4-(b)** plots all $28 = 7\times(7+1)/2$ coefficients. **Fig. 4** shows that the 160km CD produces about 3 effective channel taps (hence $N_c = 3$ and $M = N_c - 1 = 2$), which suggests a $2^2 = 4$ state MLSD is sufficient.



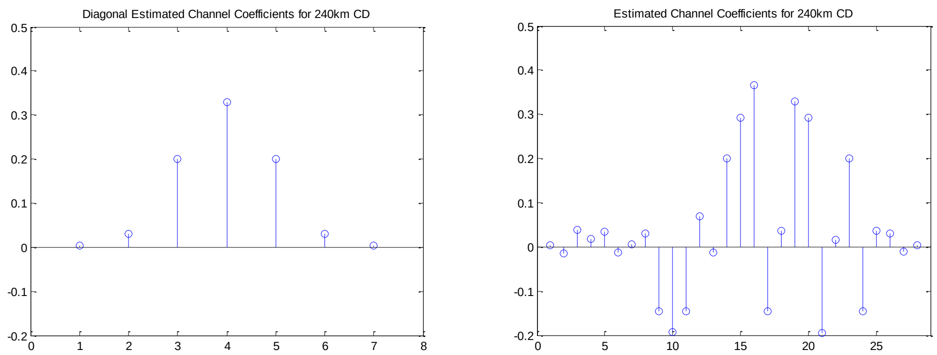
(a) Diagonal elements ($c(n,n)$) **(b)** Channel coefficients

Fig. 4. Channel estimation for 160km CD

Fig. 5 plots the estimated channel coefficients for a 240km CD. **Fig. 5** indicates that 240km CD produces about 5 effective channel taps, which requires a $2^4 = 16$ state MLSD. On the contrary, 280km CD generates about 7 effective channel taps, as **Fig. 6** shows, and $2^6 = 64$ states MLSD is needed to perform perfect equalization.

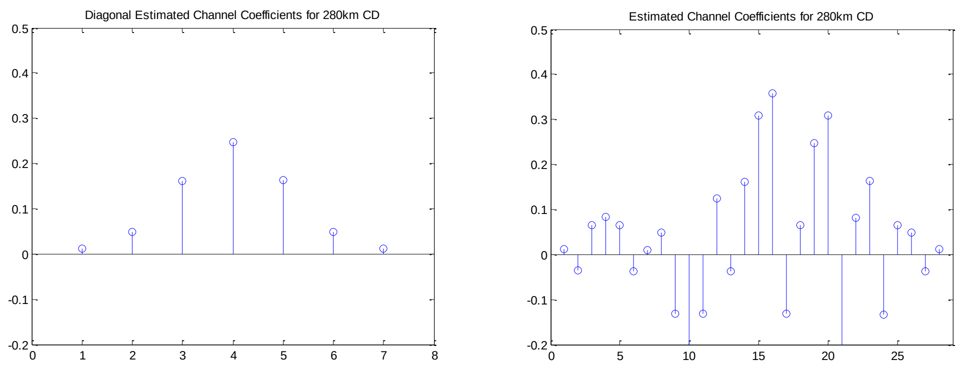
Table 1 summarizes bit error rate (BER) performance of the 8 state, the 64 state, and the reduced from 64 state to 8 state MLSD equalizers for various CD lengths under a low optical signal-to-noise ratio (OSNR [1]) (30dB). The 8-state MLSD starts generating bit errors from

240km CD, while the reduced 8-state MLSD successfully equalizes CD distortions.



(a) Diagonal elements ($c(n,n)$) **(b)** Channel coefficients

Fig. 5. Channel estimation for 240km CD



(a) Diagonal elements ($c(n,n)$) **(b)** Channel coefficients

Fig. 6. Channel estimation for 280km

Table 1. BER performance of the MLSD equalizers

MLSD type	0 (km)	40 (km)	80 (km)	120 (km)	160 (km)	200 (km)	240 (km)	280 (km)
8-State	0	0	0	0	0	0	0.0004	0.0084
8←64	0	0	0	0	0	0	0	0
64-State	0	0	0	0	0	0	0	0

where 0 denotes the BER below the detection capability (2×10^{-9})

Fig. 7 plots the OSNR penalty for the target BER 5×10^{-4} (the OSNR variation to keep the target BER) for three MLSD equalizers, the 8-state, the 64-state and the reduced 8-state MLSD (from 64-state). For the target BER of 5×10^{-4}, the OSNR is relatively low. Hence, the reduced-state MLSD suffers from error propagation due to possible false decisions. However, the OSNR penalty gap between the 8-state and reduced 8-state MLSD equalizers are less than 2dB up to 200 km and the reduced-state MLSD exhibits superior performance above 200 km.

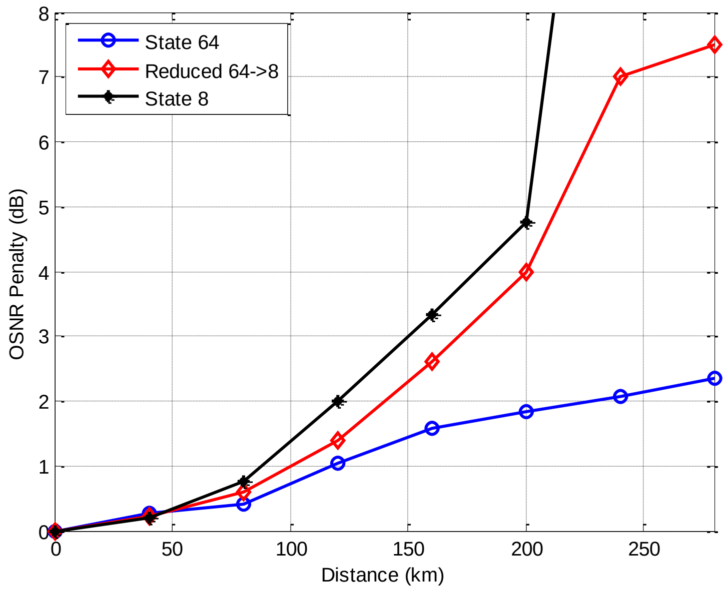


Fig. 7. OSNR penalty of MLSD equalizers

6. Conclusion

In this paper, we proposed a simplified reduced-state MLSD structure for optical receivers in the electronic domain based on the nonlinear channel model using the second-order Volterra kernel. Using this channel model, we achieved a simple and efficient branch metric computation for a reduced MLSD equalizer having about the same complexity of a reduced-state MLSD for linear channels. Simulation results show that the proposed reduced-state MLSD may suffer from error propagation under a low OSNR, but give superior performance in comparison with conventional MLSD having the same number of states under a moderate OSNR.

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