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Contour Integral Method for Crack Detection

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Abstract In this paper, a new approach to detect surface cracks from a noisy thermal image in the infrared thermography is presented using an holomorphic characteristic of temperature field in a thin plate under steady-state thermal condition. The holomorphic function for 2-D heat flow field in the plate was derived from Cauchy Riemann conditions to define a contour integral that varies according to the existence and strength of a singularity in the domain of integration. The contour integral at each point of thermal image eliminated the temperature variation due to heat conduction and suppressed the noise, so that its image emphasized and highlighted the singularity such as crack. This feature of holomorphic function was also investigated numerically using a simple thermal field in the thin plate satisfying the Laplace equation. The simulation results showed that the integral image selected and detected the crack embedded artificially in the plate very well in a noisy environment.

Keywords: Infrared Thermography, Surface Crack, Contour Integral, Conduction Heat Flow, Noise Reduction

1. Introduction

Various infrared thermography techniques (IRT) have been developed to detect surface cracks combining the traditional thermography with active heat sources including laser and pulsed eddy current during the last decade.

Sakagami[1,2] introduced a new induction IRT technique termed "singular method" for crack identification. In the singular method, surface crack was identified from the singular electro-thermal field generated around the crack tips, and a concentrated temperature rise was observed in the vicinity of crack tips. When a periodically modulated electric current was applied to the cracked sample, the intensity of the singular current field oscillated in the same frequency. This cyclic change of the singular electro-thermal field resulted in the cyclically changing temperature distribution, which could be imaged by lock-in mode using the reference signal of electric current.

Almond[3], Nezelmann[4], and Zenzinger[5] calculated the current distribution induced by external coil to investigate current flow near the surface of specimen, current density around a crack, how much heat was produced and diffused at the location of the crack. They could detect the cracks by the direct observation of heating process due to the concentrated current density and modification of heat diffusion.

Also Holland[6] visualized a surface crack by employing a vibration-induced frictional heating of the crack in titanium. A laser scanning technique combined with thermography was developed and proved to be very successful to find surface cracks by Li[7]. The method used a laser heat source such as Nd:YAG laser

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for local excitation on the specimen, where the resulting surface temperature was recorded with infrared camera. The beam was focused on the test sample by using an optical scanner to generate a required lateral heat flow. The time resolved temperature distribution was recorded with a high-speed infrared camera. Based on this data, crack-caused anisotropies in the lateral heat flow was able to be detected and exploited to characterize the cracks.

Another approach to detect surface crack is the forced diffusion thermography proposed by Lesniak[8]. It projects a pattern of dynamic heat to force the flow across the crack thereby optimizing measureable thermal gradient. The basic idea is that the heat flow is impeded by the crack creating a gradient in the thermal image, which clearly defines the crack. Since the temperature gradient caused by emissivity variance from the surface could be misconstrued as cracks, two opposing heat flows were applied to the specimen to separate the flaw image from noise (emissivity variance). However, the derivative of thermal image also magnified the noise from many sources so much that the gradient image did not always provide better information than raw thermal image.

This paper proposes an image processing technique for thermal gradient method to define and locate surface cracks from noisy thermal image obtained in thin plate. This method used the holomorphic function of temperature field in the plate under steady-state thermal condition to enhance the crack profile out of noisy image.

It was derived for 2-D heat flow in a thin plate from Cauchy-Riemann conditions, and applied to define a contour integral that varied according to the existence and strength of a singularity in the domain of integration. A theoretical formulation for the holomorphic function was made and implemented to set up a novel numerical algorithm for data processing. An artificial crack-like singularity was created in the plate model to investigate the performance of the proposed method.

2. Heat Equation in Plate

Heat conduction in a thin plate is governed by the two dimensional heat conduction (diffusion) equation in absence of heat source in the domain of interest[8-10],

$$\frac{\partial T(x,y,t)}{\partial t} = \alpha \cdot \nabla^2 T(x,y,t) = \alpha\left[\frac{\partial^2 T(x,y,t)}{\partial x^2} + \frac{\partial^2 T(x,y,t)}{\partial y^2}\right] \qquad (1)$$

$$\alpha = \frac{k}{\rho c_p}(\text{thermal diffusivity})$$

where T is the temperature, k is the thermal conductivity, α is the thermal diffusivity, ρ and c_p are the density and heat capacity at constant pressure. In the time-independent case, $\frac{\partial T}{\partial t} = 0$, the heat equation of eqn. (1) reduces to

$$\nabla^2 T(x,y,t) = \left[\frac{\partial^2 T(x,y,t)}{\partial x^2} + \frac{\partial^2 T(x,y,t)}{\partial y^2}\right] = 0 \qquad (2)$$

This Laplace equation of eqn. (2) is satisfied wherever in the domain when a steady state is reached without heat source. Exact solution to Eqn. (2) is already known as holomorphic function (or analytic function) in complex analysis. If the temperature distribution $T(x,y)$ is the real-valued function of a complex function $f(x+iy)$, i.e., $f(x+iy) = T(x,y) + i\Theta(x,y)$, where $\Theta(x,y)$ is the conjugate harmonic function of $T(x,y)$, then $T(x,y)$ automatically satisfies the Laplace equation given in eqn. (2) only if the function $f(x+iy)$ is analytic. The Cauchy-Riemann equations corresponding to the function $f(x+iy)$ state

$$\frac{\partial T}{\partial x} = \frac{\partial \Theta}{\partial y}, \quad \frac{\partial T}{\partial y} = -\frac{\partial \Theta}{\partial x} \tag{3}$$

Thus, the increment of conjugate harmonic function $\Theta(x,y)$ is expressed in terms of $T(x,y)$ by

$$d\Theta(x,y) = -T_y(x,y)dx + T_x(x,y)dy \tag{4}$$

Let $f(Z) = f(x+iy)$ is a holomorphic function defined over a domain D. Then

$$\int_C f'(z)dz = df(z) = 0 \tag{5}$$

This integrability condition of eqn. (5) implies that the value of the contour integral surrounding the domain S is zero as shown in Fig. 1. This condition is always valid locally unless the line C does not loop around a singularity.

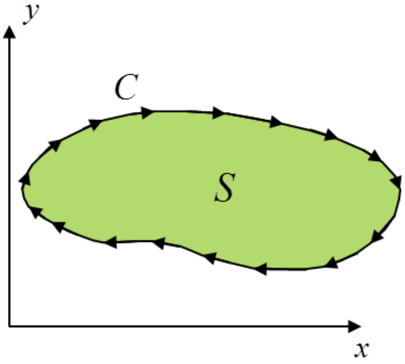


Fig. 1 Contour integral along C

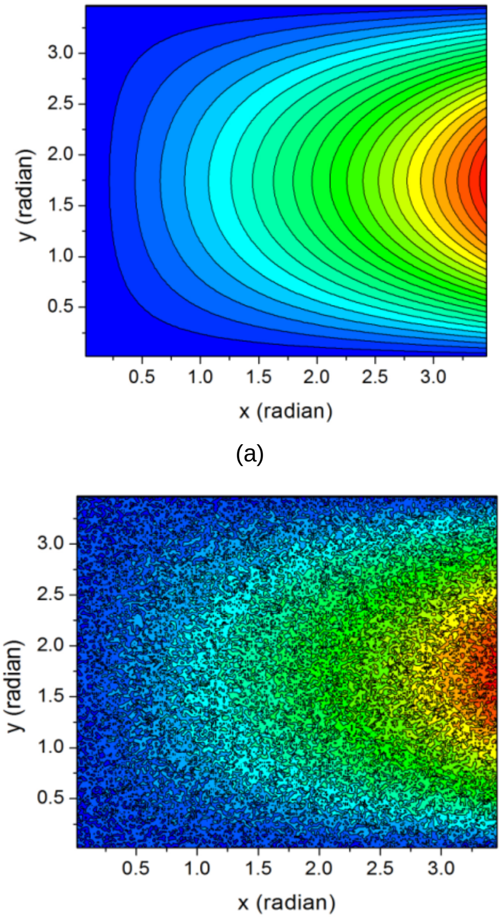
Then, from eqn. (5) and Stokes' theorem,

$$\int_C d\Theta = 0 \tag{6}$$

3. Numerical Simulation for Crack Detection

In order to investigate the characteristics of eqn. (6) in a two-dimensional plate of steady state, we start with a simplest solution to Laplace equation in 2-D, which is,

$$T(x,y) = \sin y \cdot \sinh x \tag{7}$$



(a)

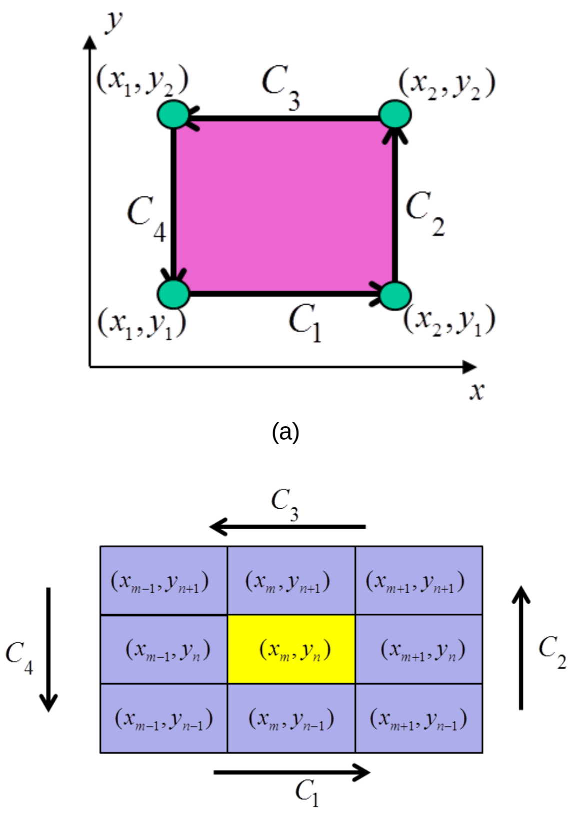
(b)

Fig. 2 Temperature distribution of (a) $T(x,y) = \sin y \cdot \sinh x$, (b) noise-corrupted $T(x,y)$

This temperature field is depicted in Fig. 2(a) naturally satisfying eqn. (6), where red color presents a relatively hotter temperature while the blue does a lower temperature. Now an artificial random noise signal is added to the temperature field $T(x,y)$ of eqn. (7) to simulate an emissivity variance of surface in experiment, which is represented in Fig 2(b). The maximum value of random noise is set to the median of $T(x,y)$.

In order to implement the contour integral of eqn. (6) in the simulation, the integral domain D is set as the entire area of Fig. 2(a) and the closed contour C of integration is set to be a square composed of four perpendicular lines, C_1, C_2, C_3, C_4 like Fig. 3(a). Therefore, the line integral over C in eqn. (6) is conducted along the line C_1 first, C_2 secondly, C_3 and C_4 consecutively in counter-clock-wise direction. Specific finite difference model employed in the simulation for the square-type contour is described in detail in terms of 2-D position (x, y) in Fig. 3(b). In Fig. 3(b), the integers, m and n, represent the locations of each pixel in the temperature field, $T(x,y)$ of Fig. 2. From the eqn. (6), if there is no singularity (crack) in the point (x_m, y_n) in Fig. 3(b), the integral value around the contour $C_1 - C_2 - C_3 - C_4$ must be zero. Otherwise, it takes a nonzero-value, whose amplitude is dependent on the strength of singularity.

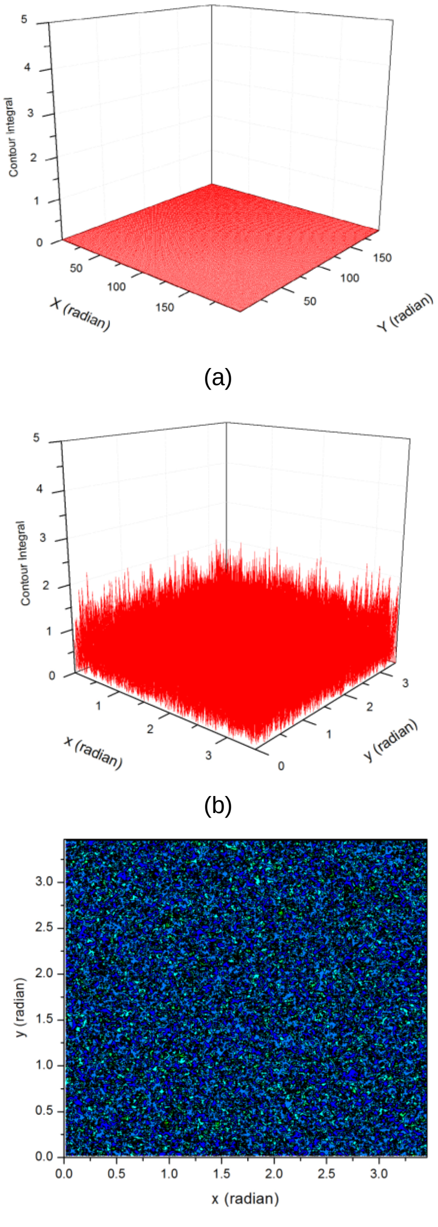
An integral image obtained by calculating eqn. (6) from FDM model of Fig. 3(b) at every point of uncorrupted image in Fig. 2(a) is represented in trimetric view in Fig. 4(a), where the integral values are very small (approximately zero) at all point (x, y) in the plate because it has no singularity in the domain. The corrupted temperature distribution in Fig. 2(b) processed in the same manner using eqn. (6) is shown in Fig. 4(b). Its plane view is plotted again in Fig. 4(c) to show a random distribution of integral value over the region without any particular pattern or tendency of temperature.



(a)

(b)

Fig. 3 Contour integral along $C_1 - C_2 - C_3 - C_4$, (a) Analytic model, (b) FDM model

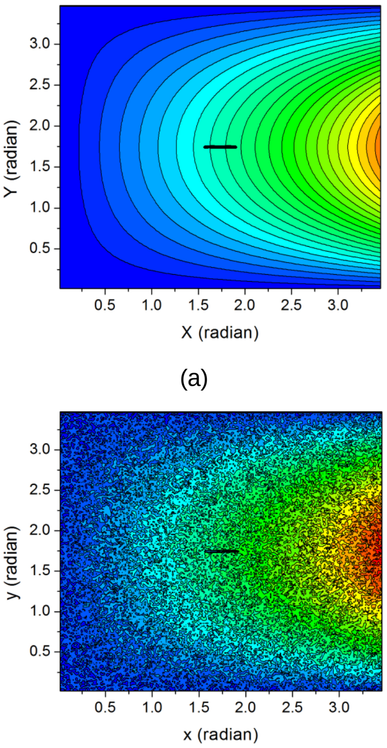


(a)

(b)

(c)

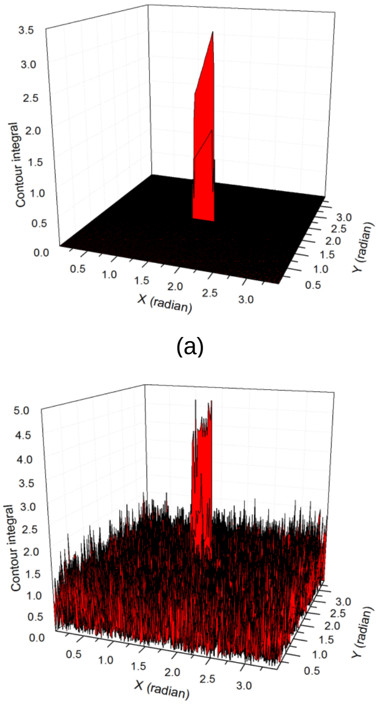
Fig. 4 Contour integral of, (a) Temperature field $T(x,y)=\sin y \cdot \sinh x$, (b) Noise-corrupted $T(x,y)$, (c) The plan view of (b)



(a)

(b)

Fig. 5 A singular crack imbedded in (a) A field $T(x,y)=\sin y\cdot\sinh x$, (b) Noise-corrupted $T(x,y)$



(a)

(b)

Fig. 6 Contour integral image of the crack for, (a) Temperature field $T(x,y)$, (b) Noise-corrupted $T(x,y)$

Now a slit of singularity (0.3 in length) is imbedded in the temperature field $T(x,y)$ of Fig. 2 to simulate an effect of crack in a plate on the integral of eqn. (6) as shown in Fig. 5. Fig. 5(a) is the temperature distribution of a plate with no noise, and Fig. 5(b) is that of a plate with the same noise as in Fig. 4(b). The same integral calculations over the temperature images of Fig. 5(a) and 5(b) are performed and represented in Fig. 6, where the singular area (crack) is revealed well and seen prominently in the center. From these figures, the contour integral given in eqn. (6) has selected the singularity very well as it suppresses the temperature variance due to heat conduction and noise.

4. Conclusions

A novel numerical algorithm in thermal gradient method of infrared thermography was proposed to detect surface crack in thin plate of steady-state condition using Cauchy-Riemann equations. A potential function, which is the conjugate harmonic function of temperature field, was derived and proved that its contour integral could be used as an indicator of singularity. The contour integral became zero over a domain where there does not exist any singularity, while it gave a non-zero value when the domain has any singular points like crack.

This characteristic of the integral was verified numerically for a simple plate of steady-state heat conduction using the algorithm introduced in the paper. The temperature field of the model severely corrupted by random noise was integrated along a closed square loop at each point of the temperature field to extract the location and shape of crack. A good enhancement in crack image was achieved through this contour integral. The contour integral reduced not only the variation of temperature produced by conductive heat flow, but also the level of noise amplification. In order to investigate how the integral technique works with a singularity, an artificial singular slit-like crack was put in the plate model and processed through the proposed method, the crack was selected excellently even when a relatively high level of noise was present.

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