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A Numerical Model for Prediction of Residual Stress Using Rayleigh Waves

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Abstract In this work, a numerical model is proposed for the relation between the magnitudes and the depth residual stress with the velocity of Rayleigh wave. Three cases, stress-free, uniform stress and layered stress, are investigated for the change tendency of the Rayleigh wave speed. Using the simulated signal with variation of residual stress magnitude and depth, investigation of the parameters for fitting residual stress and velocity change are performed. The speed change of Rayleigh wave shows a linear relation with the magnitude and an exponential relation with the depth of residual stress. The combination of these two effects could be used for the depth profile evaluation of the residual stress.

Keywords: Numerical Model, Residual Stress, Depth Profile, Rayleigh Wave

1. Introduction

To meet the requirement of structural integrality assessment, it is necessary for us to take into account of the total effects of both applied stresses and residual stresses. Residual stresses are defined as internal stresses remaining in a component as a mechanical equilibrium state when no external forces and moments are applied[1]. Tensile residual stresses can lead to premature crack and ultimate failure, while the compressive ones will prevent the generation and propagation of initial micro-cracks. For this reason, exact information about the residual stresses levels within components is of significance[1].

On the other hand, residual stresses could be introduced during both manufacturing and post-processing, which means that the residual stress is a cumulative result of manufacturing, assembling, repairing and other operating processes. This characteristic makes residual stresses much more difficult to be determined than applied stresses. Therefore despite that a lot of researches have been conducted on stress analysis, the residual stresses determination is still relatively primitive and usually conservative [2].

Usually residual stresses could not be measured directly, instead the engineers obtain the stress from measurements of the displacement, strain or other secondary physical quantity, such as the speed of ultrasonic wave or magnetic signature related to the stress. A great variety of techniques, including mechanical methods and physical methods, have been developed for the residual stress evaluation in different cases. The penetration depth and resolution ability comparisons of different methods for residual stress measurement are shown in Fig. 1[1].

Mechanical methods, usually requiring material removal, are applied world widely but semi-destructive. These techniques generally rely

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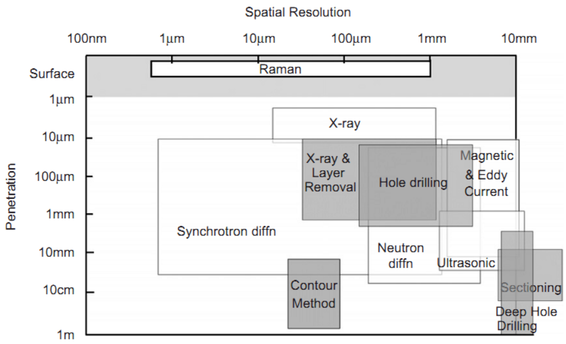


Fig. 1 The comparisons of the different techniques

on derivations of the original stress from the displacement or strain after completely or partially stress relaxation. Different techniques have different application ranges. For instance, the contour method is able to provide area maps of residual stress, while deep hole-drilling can provide depth profiles and sectioning only provides single stress measurements[2].

In physical techniques, the residual stress determination is usually based on the measurement of some physical parameters and deducing the stress from certain relation. Diffraction techniques such as X-ray diffraction and Neutron diffraction have been developed very well, on the basis of the elastic strain measurement of specific atomic lattice planes radiated by certain high energy rays. The magnetic method is confined for ferromagnetic materials, investigating the interaction effect between magnetization and elastic strain. Ultrasonic methods are based on the acousto-elasticity, the relation of the velocity variation and residual stress states. However, in these techniques, the interpretations and reconstruction of the stress field from signals accumulating over the magnetic or acoustic paths are rather difficult[1,2].

Ultrasonic techniques for residual stress measurement have the advantages of nondestructive and relatively easy to conduct the experiment. Therefore many efforts have been tried to develop the ultrasonic measurement system for residual stress evaluation. However, as the accumulation along the wave path and other structural effects, the explanation of the signal from the general stress states is very cumbersome[3]. Finite element method is effective and convenient in the simulation of some simplified models to investigate the influence of different factors. In such a way, it is much better for us to understanding the exact relations in acousto-elasticity. In order to take the residual stress depth profile into considera-tion, the depth and the magnitude of residual stress should be investigated separately at first.

In this paper, some theoretical information will be firstly introduced for the ultrasonic method for residual stress measurement. Next, finite element models for the Rayleigh wave with stress-free, uniform and layered stress are put forward. Then, the results are discussed including expressions relating the residual stress field to the predicted change in ultrasonic velocity in terms of stress magnitude and depth. Finally, conclusions are drawn and future work is put forward.

2. Ultrasonic Testing for Residual Stress Measurement

The evaluation of third-order-elastic constants (TOE) is an interesting topic attracting many researchers, thus stimulates the development of acousto-elasticity. Based on Murnaghan's theory of nonlinear elasticity, Hughes and Kelly proposed the theory of acousto-elasticity, and they just focused on the relation of ultrasonic velocities and the elastic strains in an isotropic solid[4]. In 1958, Bergman and Shahbender observed the birefringent phenomenon in acoustics, similar to the change of light speed in a stressed transparent body[5]. Since then, the acouto-elasticity phenomenon for stress analysis has been investigated more and more.

Acousto-elastic effect of Rayleigh surface wave in isotropic materials has been investigated by Hirao et al.[6] and they showed the dispersion of a Rayleigh wave related to the product of the wave number and the characteristic depth with the variation of stress. Duquennoy et al. also did some investigations on residual stresses in orthotropic material using Rayleigh waves[7].

Three different states of a body are adopted to make this problem clear by many researchers [3,6,7]. We call a body in an original state free of stress and strain, the natural state or undeformed configuration. However, in reality such a state almost never exists in a real material because there are always residual stresses or applied stresses due to loading or manufacturing processes. The state within certain initial stress level is called the initial state or predeformed configuration. When an ultrasonic wave is superimposed on the body, the resultant state with further deformation is called the final state or final deformed configuration. Fig. 2 shows coordinates of a single material point for each of the three states[7].

The positions of a material point in the natural, initial and final states are expressed by the vectors ξ, **X** and **x**, respectively, the superscript labels 0, i, f are corresponding to the physical variables in natural, initial and final state respectively. $u=u^f-u^i$ is the infinitesimal displacement due to the propagation of the wave in a prestressed state, and the governing equation is shown as:

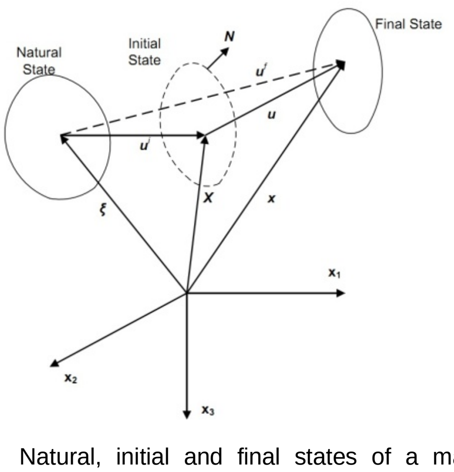


Fig. 2 Natural, initial and final states of a material point[7]

$$\frac{\partial^2 u_K}{\partial X_J \partial X_L}\left(R_{IJKL}+\delta_{IK}t^i_{JL}\right)=\rho^i\frac{\partial^2 u_I}{\partial t^2} \quad (1)$$

where,

$$\rho^i=\rho^0\left(1-\frac{\partial u^i_M}{\partial X_M}\right)$$

$$R_{IJKL}=C_{IJKL}(1-\frac{\partial u^i_M}{\partial X_M})+C_{IJKLMN}\frac{\partial u^i_M}{\partial X_N}+C_{MJKL}\frac{\partial u^i_I}{\partial X_M}$$
$$+C_{IMKL}\frac{\partial u^i_J}{\partial X_M}+C_{IJML}\frac{\partial u^i_K}{\partial X_M}+C_{IJKM}\frac{\partial u^i_L}{\partial X_M}$$

u^i and u^f are displacement vectors at the initial and the finial states respectively; t^i is the Cauchy stress tensor in the initial system; ρ^0 and ρ^i are the densities in the natural and initial states respectively; R_{IJKL} is the coefficient depend on both the material constants and the initial state; C_{IJKL}, C_{IJKLMN} are second- and third-order elastic constants(I, J, K, L, M, N=1,2,3); δ is Kronecker delta.

As the speed change of Rayleigh wave due to stress is very small, let us consider the relative variation of velocity and the expression is shown as:

$$\frac{\delta V}{V_0}=\frac{V-V_0}{V_0} \quad (2)$$

δV is the change of the velocity, V and V_0 are the velocity in the stress-free and pre-stress cases respectively.

From M. Duennoy's work, they can get the relative variation of the velocity in terms of initial tresses[7]:

$$\frac{\delta V}{V_0}=A_1 t^i_{11}+A_3 t^i_{33} \quad (3)$$

A_1 and A_3 are the acousto-elastic coefficients, and their analytical forms are given in the literature[7]. From this expression, we can see the change of velocity of Rayleigh wave is linearly depending on the principle stress t_{11} and t_{33}.

Usually the variation of the Rayleigh wave speed is about 0.1%, which means precise enough equipments are required for practical applications. Furthermore, in engineering applications of the theory of elasticity, other effects such as temperature, micro-inhomogeneity and texture of a material, small though, cannot be ignored in the application of acousto-elasticity [3]. And also stress-induced anisotropy in a material is comparable to or even less than the slight anisotropy introduced by drawing, rolling, or other materials processes. It is for this reason that the ultrasonic method of measuring residual stresses has been limited and more investigations are necessary. More general expressions of residual stress based on the acousto-elasticity and more precise equipments are the critical issues for the development of the ultrasonic method for residual stress measurement.

3. Finite Element Modeling for Rayleigh Wave

A two dimension model is set up to verify the feasibility of Rayleigh wave for residual stress determination and compared with the literature[7]. The size of this model is an iron plate of 10 mm×2 mm mounted with a Lucite wedge, see Fig. 3. The model is two dimensional with a plane strain assumption.

Linear plane strain elements are used with reduced integration (ABAQUS element type CPE4R). The three edges (left, right and bottom) use the infinite element CINPE4 in order to eliminate the boundary reflection. The top surface is free boundary, and coupling with the wedge bottom. The models are solved by ABAQUS/Explicit.

The properties of the iron and Lucite are listed in Table 1.

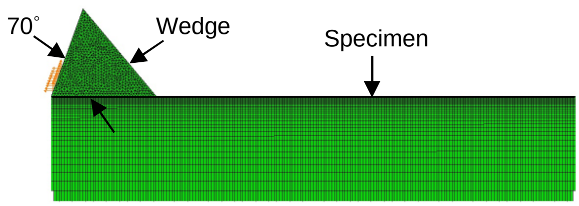


Fig. 3 The setup of the model

Table 1 The material constants for the model

Parameters	Density (g/cm^3)	Young's Modulus (MPa)	Poisson ratio	Compression wave speed(m/s)
Specimen (iron)	7.69	210000	0.3	6063
Wedge (Lucite)	1.15	3402	0.4	2700

From these parameters, we can get the approximate speed of Rayleigh wave using the equation which is given by Graff in 1978 [8]:

$$V_0 = \frac{0.862 + 1.14\nu}{1+\nu} V_S = 3000\text{m/s} \tag{4}$$

V_S is the shear wave speed in the iron, and ν is the Poisson ratio.

According to the Snell's Law, we can use these parameters to calculate the critical angle of the Rayleigh wave.

$$\theta_R = \sin^{-1}\left(\frac{V_P^i}{V_0}\right) = 64.16^0 \tag{5}$$

V_P^i is the compression wave speed of the wedge. In this model, an angle of 70° is set for the wedge to generate the Rayleigh wave.

A boundary condition with displacement perturbation is introduced on the surface of the wedge, as shown in Fig. 3. The profile amplitude of the displacement is a sine wave function with 5 MHz, which is shown in Fig. 4. The magnitude is set to 1×10^{-5} mm.

Since the interest is the propagation behavior of the wave remote from the input and the distance can be exactly obtained in the model, the test point is set at the end of the specimen. As the Rayleigh surface wave main propagates along the surface, we need to use much finer mesh size for the surface. But for the lower region, the result is not so important, so we use much coarser mesh to reduce the computation time.

Here we take 3 cases into consideration:

1) At first we use the stress-free model to get the behavior of Rayleigh wave propagation along the surface and then calculate the initial speed of the wave, verifying the accuracy of this model.
2) Then a uniform profile with different magnitude stresses is introduced for the whole specimen. The tensile stresses are set as 400 MPa, 800 MPa and 1200 MPa respectively.
3) At last, in order to investigate the depth effect of residual stress separately, the magnitude of the tensile residual stresses are fixed at 1000 MPa, while the depths are set with different layers, as shown in Fig. 5.

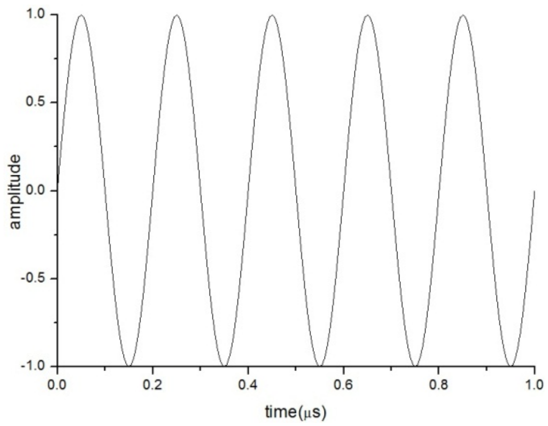


Fig. 4 The input signal for this model

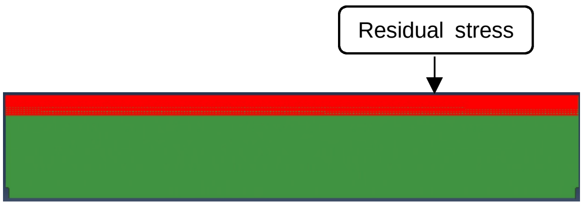


Fig. 5 The setup of different depth of residual stress layer (16 meshed layers of top surface) for the modeling

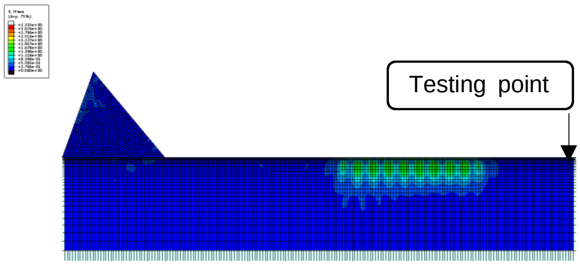


Fig. 6 The Von Mises contour for Rayleigh wave propagation

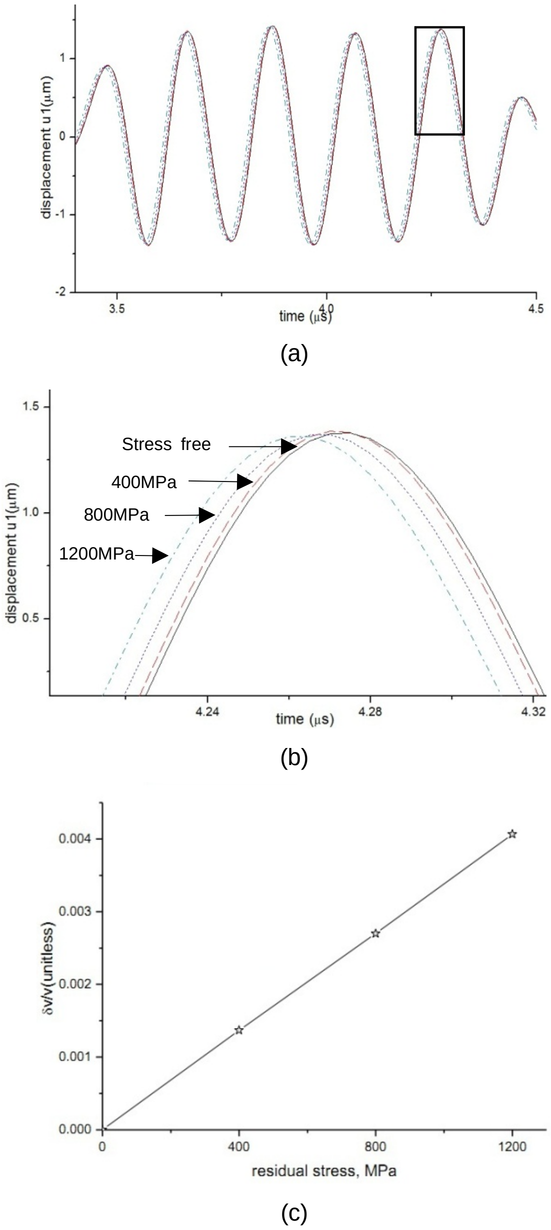


Fig. 7 The results for the uniform stress case: (a) The shift of displacement profiles; (b) The magnified profiles for the rectangular in Fig. 7(a); (c) The relation of the relative change of velocity to the magnitude of stress

4. Results and Discussions

Fig. 6 shows the predicted von Mises stress at certain time for an unstressed model. The incident pulse is loaded on the left side of the wedge, around Rayleigh angle, and most of the energy propagates along the surface of the specimen. The Rayleigh wave propagation behavior is observed along the surface of the specimen.

The velocity of the Rayleigh wave on the surface is calculated:

V_0 = 2965.87m/s,

which is within the margin for error, ensuring the reliability of the model.

For the case 2, when uniform 400 MPa, 800 MPa, 1200 MPa tensile stress are introduced, we can get different arriving time shift of the Rayleigh wave, which is shown in Fig. 7(a). Fig. 7(c) shows a plot of $\delta V/V_0$ in the case that the tensile residual stress magnitudes vary. This demonstrates that there is a clear correlation between the residual stress (0-1200 MPa) and the surface acoustic signal.

The coefficient of proportion is estimated to be 3.416×10^{-6}/MPa. This is consistent with the order of magnitude value given in the literature [9] of 10^{-6}/MPa.

Fig. 8 shows the shift of arriving time and the plot of $\delta V/V_0$ versus the depth of the residual stress field at the magnitude of 1000 MPa. The different depths of residual stress are introduced layer by layer, shown in Table 2.

From the result, we can see as the depth of residual stress increasing, the change of Rayleigh wave speed reach a stable level at about 0.6 mm. This depth is approximately equal to the wavelength of Rayleigh wave when the frequency is 5 MHz, that is λ=V/f=0.6 mm. In turn, this phenomenon demonstrates that it is impossible for Rayleigh wave to investigate residual stress beyond depth of one wavelength.

Table 2 The cases for different depths of residual stress

cases	Depth of residual stress layers
I	0.046396mm
II	0.115012mm
III	0.218222mm
IV	0.372926mm
V	0.602753mm
VI	0.945625mm

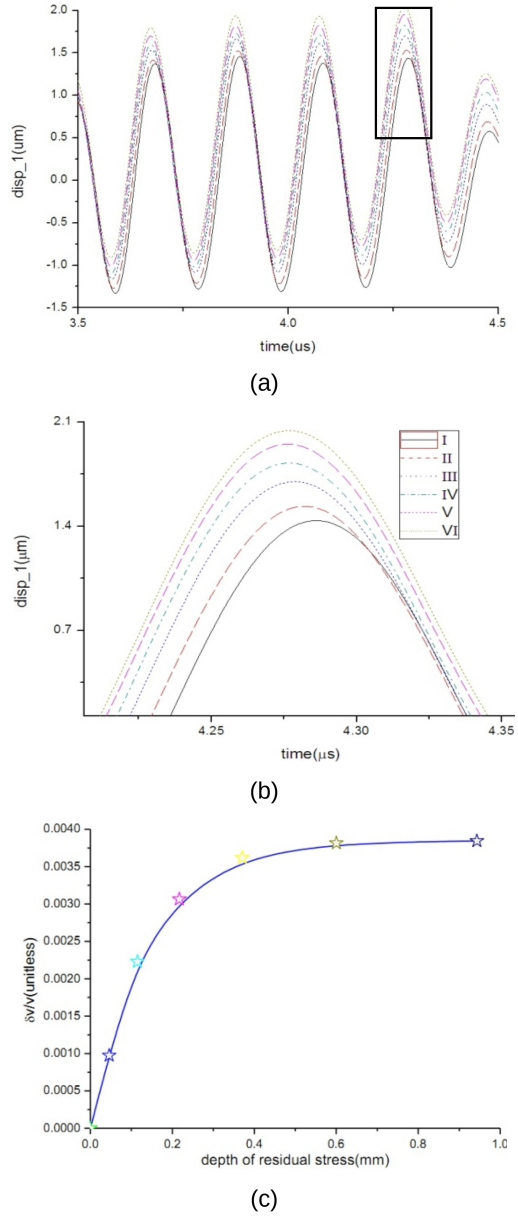


Fig. 8 The results for different depth with stress of 1000MPa: (a) The shift of displacement profiles; (b) The magnified profiles for the rectangular in Fig. 8(a); (c) The relation of the relative change of velocity to the magnitude of stress

For this reason, with different incident frequencies, different responses of different depth of residual stresses could be obtained. From this point, this method is potential to build a model for the depth profile evaluation of residual stress distribution.

For this case, a function can be used to fit these data. From the work of M. Hirao[6], they adopt an equation as

$$\frac{\Delta V}{V_0} = \left(\beta_0 + \frac{\beta_1}{kx_3}\right)e_{11} \tag{6}$$

k is the wave number, x_3 is the depth, β_0 and β_1 can be written in terms of material elastic constants, e_{11} is the strain along the x_1 direction. Of course, the elastic strain could be converted into corresponding stress. The estimate β_0 is not equal to β_1. However, the problem happens when the depth of residual stress is zero.

From the work of R.M. Sanderson[9], they assume an equation as

$$\Delta V / V_0 = \beta - 1/(\alpha x_3 + 1/\beta) \tag{7}$$

α and β are the constants for a known stress magnitude. The expression can satisfy the initial point, i.e. when the depth of residual stress is zero; the change of velocity is zero. This equation satisfies the singularity when x_3=0, that is to say the stress-free state.

As the energy of the Rayleigh wave attenuates along the depth with an exponential form, so we can assume another meaningful function to fit the data. With the data analysis, we use the following equation to get the fitting result.

$$\delta V / V_0 = A\left(e^{Bx_3 - 1} - 1\right) \tag{8}$$

x_3 is the depth of the residual stress; A and B are constants for the certain stress magnitude. With the data and this fitting function, this function is fitting well with the data and we get the parameters from the analysis and also the fitting plot, shown as Fig. 9. The fitting parameters are A=-0.00387 and B=-7.717239 respectively. The adjusted coefficient of determination R^2=0.99851, and the standard errors for A and B are 4.04535×10^{-5} and 0.25447 respectively.

It is necessary for us to look more into the relation of the constants A and B with the stress magnitudes. When we change the residual stress magnitudes as 1000 MPa, 800 MPa, 600 MPa and 400 MPa, the depth profiles change like Fig. 10.

Using the equation (8) to fit these data, we get the parameters list in Table 3.

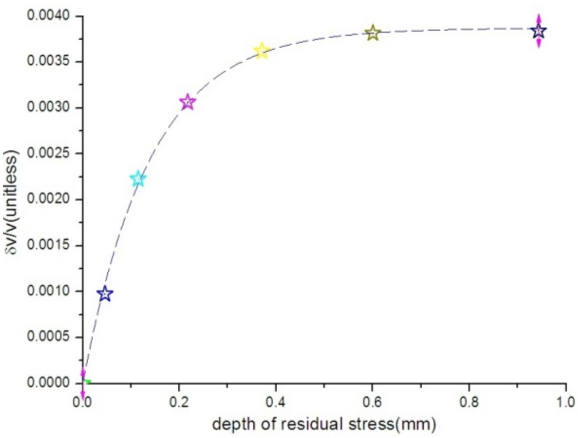


Fig. 9 The new plot with defined fitting function

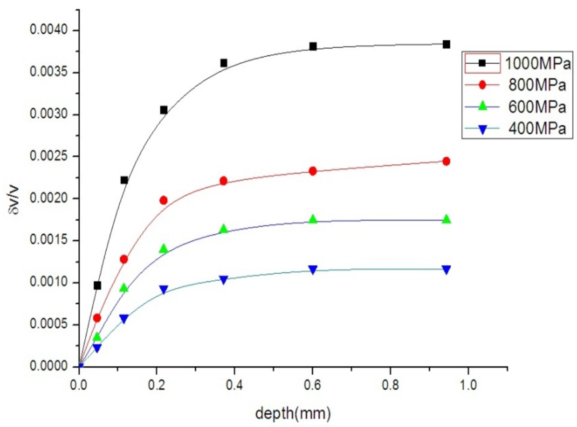
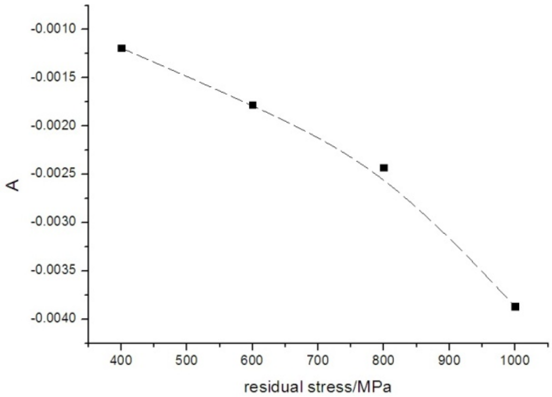


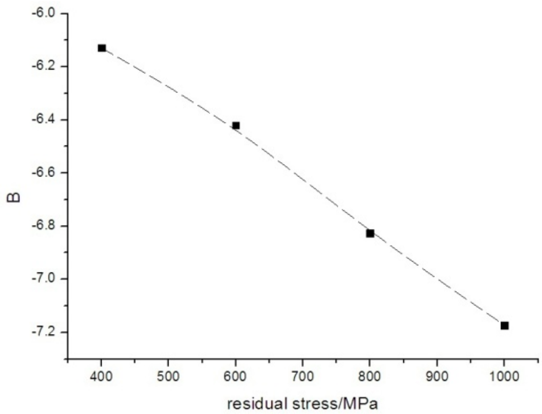
Fig. 10 The speed change of different magnitude and depth

Table 3 The calculated parameters of equation (8)

	A	B
400MPa	-0.00119	-6.12866
600MPa	-0.00178	-6.41989
800MPa	-0.00243	-6.82658
1000MPa	-0.00387	-7.17293



(a)



(b)

Fig. 11 The changes of constants with different magnitudes of residual stress: (a) for A; (b) for B

From these data, we plot them into Fig. 11. Initially, we can see certain relationship between A and B with residual stress both in magnitude and depth. Both these two parameters change almost linearly with the magnitude of the residual stress. For the accurate defining the constants, more simulations and theoretical studies are required. Residual stress distribution could be taken as multilayer structures and for each minimum layer the residual stress assumes to be the same. So the speed variation due to each layer forms the increment for every Δx_3. Finally considering magnitude and depth of residual stress, the total effects could be expressed as,

$$\delta V / V_0 = \sum A(\sigma)B(\sigma)\,\mathrm{e}^{B(\sigma)x_3}\,\Delta x_3 \qquad (9)$$

For the real case, the residual stress σ is always various in depth, but some researches have used a polynomial function to represent the residual stress depth profile in hard turning[10], so it is also possible for us to introduce a function $\sigma\left(x_3\right) = a + bx_3 + cx_3^{\,2} + dx_3^{\,3} + \mathrm{K}$, (a, b, c, d…are the constants for a certain machining condition) for shot peened residual stress. If we approximate the depth profile with a third order polynomial, there are three different known values needed for the evaluation for residual stress. As different frequencies mean different depths for the Rayleigh wave, it is possible for us to get different values for the same specimen, and then inverse the depth profile of the residual stress.

5. Conclusions

A numerical model of the Rayleigh wave for the residual stress theoretical prediction is shown in this paper. The focus is on the velocity change of Rayleigh wave as the residual stress magnitude and depth change. From the numerical results, a linear relation is shown between the velocity change and the magnitude of residual stress. For different depth, the change tendency has been observed and a curve fitting method is proposed for data analysis. Also the investigation of these parameters A and B, both also are related with the residual stress, has been conducted. The comprehensive effect residual stress on the velocity variation of Rayleigh wave is proposed at last. As different frequencies are corresponding to different detect depth, it is possible for us to get the residual stress profile with several different frequencies. Further research will be focused on the effects of different frequencies and trying to inverse the depth profile of residual stress.

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