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Model-Based Prediction of Pulsed Eddy Current Testing Signals from Stratified Conductive Structures

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Abstract Excitation and propagation of electromagnetic field of a cylindrical coil above an arbitrary number of conductive plates for pulsed eddy current testing(PECT) are very complex problems due to their complicated physical properties. In this paper, analytical modeling of PECT is established by Fourier series based on truncated region eigenfunction expansion(TREE) method for a single air-cored coil above stratified conductive structures(SCS) to investigate their integrity. From the presented expression of PECT, the coil impedance due to SCS is calculated based on analytical approach using the generalized reflection coefficient in series form. Then the multilayered structures manufactured by non-ferromagnetic (STS301L) and ferromagnetic materials (SS400) are investigated by the developed PECT model. Good prediction of analytical model of PECT not only contributes to the development of an efficient solver but also can be applied to optimize the conditions of experimental setup in PECT.

Keywords: Pulsed Eddy Current Testing, Stratified Conductive Structures, Analytical Modeling, Interlayer Corrosion

1. Introduction

Stratified conductive structures play a significant role in high speed train, aircraft and nuclear industries with various purposes[1]. However, it is widely recognized that the metal thinning and interlayer gap variation are of common occurrence in SCS caused by various hidden corrosion are complex and urgent tasks to tackle in various industries. So real time evaluation of SCS is extremely important for product quality assurance, accident reduction, especially for safety-critical fields[2]. nondestructive evaluation(NDE) techniques to determine interlayer gap variation of SCS are needed in practice. Among the NDT methods, pulsed eddy current testing has emerged as a promising option which can be applied in tackling difficult inspection tasks in various multilayered conductive structures for its practical advantages such as high sensitivity, rapid scanning, deep penetration and contactless inspection. Over other eddy current testing methods, such as harmonic eddy current testing(HECT) and swept frequency eddy current testing(SFECT), PECT method has wider application prospects in the quantitative detection of surface and subsurface defects and thickness measurement of SCS[2].

Modeling technique has been extensively used in prediction of PEC signal. It is well known that numerical modeling, based on finite element technique, has vast contributions and developments for PECT modeling of multilayered conductive structures. However, the accurate numerical simulation of PECT usually requires large computational efforts and quite

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time-consuming[4]. Analytical approach to PEC nondestructive evaluation is generally accepted as a very effective way to understand the underlying physical process of probe response[5]. Unfortunately, very few researchers are involved in the study of analytical approaches to PEC evaluation of the stratified conductive layers. Recently, the extended truncated region eigenfunction expansion (ETREE) method[6] was applied to calculate the magnetic field for stratified conductive structures and the problem region covers a truncated problem region, thus leading to series form solutions with matrix coefficient. Fan et al,[7] followed an original method to investigate PECT and developed a novel model based on the reflection and transmission theory of the electromagnetic waves for multilayered conductive structures, which is different from matrix coefficient.

Series form solution in truncated region does not suffer from determining the limitation of integration so that the convergence is much easier to control[1]. So analytical modeling of PECT is developed using the generalized reflection coefficient for a single coil above stratified conductive structures in series form. Furthermore, our developed model can be used to understand the essence of PECT technique, design a novel probe for experiment and develop the algorithms for inverse problem.

This paper is organized as follows: Section 1 introduces multilayered conductive structures and reviews the modeling techniques in PECT. In section 2, analytical model of a single air cored coil developed for PECT for stratified conductive structures. In section 3, the analytical modeling of coil impedance of eddy current testing is introduced, and also numerical calculated results based on the developed analytical model is carried out to predict the PEC signal in section 4. The last section is devoted to conclusion.

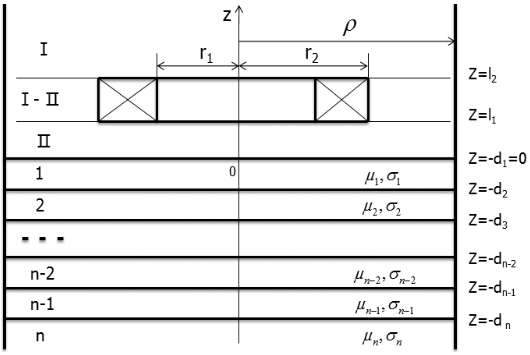

Fig. 1 Single probe above stratified conducting plates in truncated region

2. Analytical Model for Pulsed Eddy Current Testing

The pulsed eddy current technique, uses a pulse of voltage or current to excite the probes, has advantages of using a pulse function that contains a continuum of frequencies[8]. The pulse exciting voltage which is difficult to express in differential equations of magnetic vector potential, but it can be expressed in a series of sinusoidal frequencies by Fourier transformation. Then, the theory of the impedance calculations of a coil which is excited by a single sinusoidal frequency can be used. After that, the response of the coil can be summed from all the components.

To simulate qualitative signals sensitive to change the thickness of specimens, the schematic diagram of the model is established as illustrated in Fig. 1. All these components are added and the voltage response of a pulse is obtained. The detailed process is given as follows[8].

In our model, the coil is excited by a rectangular pulse of duration d during the pulse period T. And the amplitude of voltage is V_0 as shown in Fig. 2.

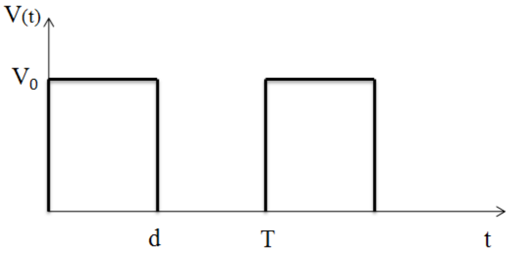

Fig. 2 Rectangular pulse of voltage excitation

$$V(t)=V_0\left[\frac{d}{T}+\sum_{n=1}^{N}\left(A_n\cos(\omega_n t)+B_n\sin(\omega_n t)\right)\right] \quad (1)$$

Where, the Fourier coefficients A_n and B_n are given by,

$$A_n=\frac{\sin(\omega_n d)}{n\pi} \qquad B_n=\frac{1-\cos(\omega_n d)}{n\pi} \quad (2)$$

The angular frequency and frequency are expressed as $\omega_n = n\pi/T$ and $f_n = \omega_n/(2\pi)$, respectively.

However, Gibbs phenomenon is encountered in practical PEC apparatus[9]. In order to reduce Gibbs phenomenon, herein, we multiply the Fourier series coefficients by a harmonic order-dependent decreasing factor $\phi_n = \mathrm{sinc}(n\pi/N)$ which makes $V(t)$ closer to the exciting voltage as follows

$$V(t)=V_0\left[\frac{d}{T}+\sum_{n=1}^{N}\varphi_n\left(A_n\cos(\omega_n t)+B_n\sin(\omega_n t)\right)\right] \quad (3)$$

Applying the relation $I = V(t)/Z(\omega)$ for each frequency, the voltage response of single coil over multilayered conductive sample is given by,

$$V_L(t) =$$
$$V_0 R_L\cdot\left[\frac{d}{TR_T}+\sum_{n=1}^{N}\varphi_n\left(\frac{A_n\cos\left(\omega_n t-\theta_L(\omega_n)\right)+B_n\sin\left(\omega_n t-\theta_L(\omega_n)\right)}{Mag_L(\omega_n)}\right)\right] \quad (4)$$

R_T is the total resistance in the experimental circuit, including the load resistance R_L, the coil resistance R_C, and the output resistance of the pulse generator R_O. In this case, the complex impedance of the coil and the real resistance of the other components are represented in polar coordinates by magnitude Mag_L and phase θ_L as:

$$Mag_L(\omega_n)=\sqrt{\mathrm{Im}(Z_L(\omega_n))^2+\left[\mathrm{Re}(Z_L(\omega_n))+R_t\right]^2} \quad (5)$$

$$\theta_L(\omega_n)=\tan^{-1}\left(\frac{\mathrm{Im}(Z_L(\omega_n))}{\mathrm{Re}(Z_L(\omega_n))+R_t}\right) \quad (6)$$

From the theoretical study, the advantage of such pulse excitation is that one pulse provides the impedance information in many frequencies. This is very important because each frequency has different eddy current penetrability inside SCS, and thus a response signal can be obtained from different depths. After obtaining results from measurements, the voltage measurements have been identified as important characteristics of the signal giving the indications of the situation of SCS.

The impedance Z_L of a single coil should be calculated by harmonic eddy current model based on the generalized reflection method in series form.

3. Analytical Approach of Probe Impedance

In the Cheng-Dodd and Deeds' model[10], the boundaries of the problem were set to infinity, thus leading to Fourier-Bessel integral-form solutions. Here in our research, integral expressions for the electromagnetic field and the impedance of the eddy current coil are replaced by series expansions, as a result, computation time is considerably reduced, convergence is better controlled and computer implementation is greatly reduced. ETREE method[11] is applied to stimulate the eddy current response to one conductive layer. Yong Li[1] reviewed Cheng-

Dodd and Deeds' model and used ETREE method for computation of the magnetic field for multilayered specimens. Here, From Cheng Dodd-Deeds model, the coil impedance using the generalized reflection coefficient based on reflection and transmission theory of electromagnetic wave for SCS can be derived by Fan et al.[7].

As illustrated in Fig. 1, the schematic diagram of the model is established in an axisymmetric configuration which includes a right cylindrical air-cored coil and layered conductors. The coil parameters of importance are number of turns n, the inner and the outer radius r_1 and r_2, the lift-off of coil and the height of the upper surfaces l_1 and l_2, respectively. The impedance of the coil is given by:

$$Z = Z_0 + \Delta Z \tag{7}$$

The coil impedance in the air is written as:

$$\begin{aligned} Z_0 = {} & \frac{j4\pi\omega\mu_o n^2}{(l_2-l_1)^2(r_2-r_1)^2}\sum_{i=1}^{\infty}\chi^2(a_i r_2, a_i r_1) \\ & \times \frac{\left(a_i(l_2-l_1)+e^{-a_i(l_2-l_1)}-1\right)}{[(a_i\rho)J_0(a_i\rho)]^2 a_i^5} \end{aligned} \tag{8}$$

The impedance change in the coil can be derived:

$$\begin{aligned} \Delta Z = {} & \frac{j2\pi\omega\mu_o n^2}{(l_2-l_1)^2(r_2-r_1)^2}\sum_{i=1}^{\infty}\chi^2(a_i r_2, a_i r_1) \\ & \times \frac{\left(e^{-a_i l_1}-e^{-a_i l_2}\right)^2}{[(a_i\rho)J_0(a_i\rho)]^2 a_i^5} R'_{0i,1i} \end{aligned} \tag{9}$$

where, Z_0 is the impedance generated by an isolated coil and $\triangle Z$ is the impedance change induced due to eddy current in the layered conductor. ω is the angular frequency $(\omega = 2\pi f)$, ρ is the truncated boundary for our model, in our calculation, $\rho = 10r_2$, and μ_0 is the permeability of free space. The eigenvalues a_i are the positive roots of the equation $J_1(a_i h) = 0$. In the arguments of the function χ which is defined as eqn.(10) [12].

$$\begin{aligned} \chi(a_i r_1, a_i r_2) & = \int_{x_1}^{x_2} xJ_1(x)dx \\ & = \left[x_1 J_0(x_1) - 2\sum_{k=0}^{\infty} J_{2k-1}(x_1)\right] - \left[x_2 J_0(x_2) - 2\sum_{k=0}^{\infty} J_{2k-1}(x_2)\right] \end{aligned} \tag{10}$$

For multilayered conductors, we obtain the generalized reflection coefficient[7]:

$$\mathrm{R}'_{\mathrm{ji,(j+1)i}} = \frac{R_{ji,(j+1)i} + R'_{(j+1)i,(j+2)i}e^{-2a_{(j+1)i}(d_{j+2}-d_{j+1})}}{1 + R_{ji,(j+1)i}R'_{(j+1)i,(j+2)i}e^{-2a_{(j+1)i}(d_{j+2}-d_{j+1})}} \tag{11}$$

The reflection coefficients $R_{ji,(j+1)i}$ in eqn. (11) are expressed as

$$R_{ji,(j+1)i} = \frac{\mu_{j+1}a_{ji} - \mu_j a_{(j+1)i}}{\mu_{j+1}a_{ji} + \mu_j a_{(j+1)i}} \tag{12}$$

For n layered conductive structures (n>2), $R'_{ji,(j+1)i} = 0$ that leading to $R'_{(n-1)i,ni} = R_{(n-1)i,ni}$, note that $R'_{ji,(j+1)i}$ can be expressed by $R'_{(j-1)i,ji}$. So the generalized reflection coefficient $R'_{0i,1i}$ could be calculated iteratively. Where, $a_{ji} = \sqrt{a_{0i}^2 + j\omega\sigma_i\mu_i}$, and $d_{j+1} - d_j$, μ_j and σ_j denote the thickness, permeability and conductivity of the layer j, respectively.

4. Simulation and Results

In order to verify the developed model, two kind of multilayered conductive structures are investigated to simulate the voltage change due to the interlayer corrosion in the second layer of multilayered structures. The parameters of the air-cored coil are shown in Fig. 3 and the lift-off of the coil is 0.5 mm.

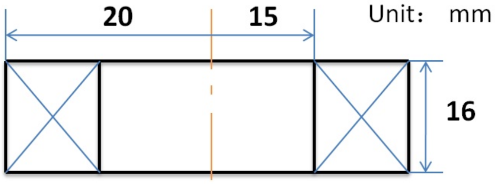

Fig. 3 The structure of PEC probe

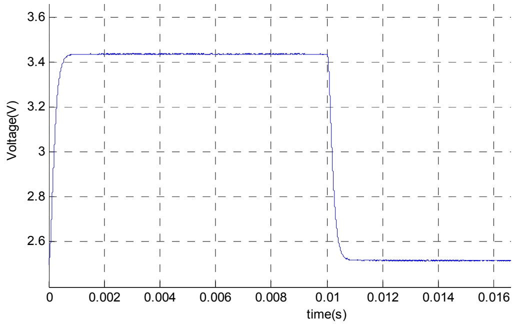

Fig. 4 Temporal PEC responses to two layered specimens

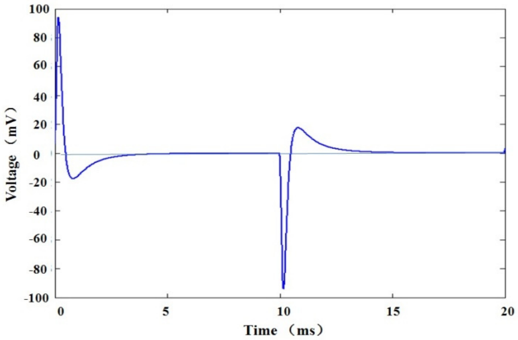

Fig. 5 PECT signal from two layered specimen

The temporal PECT responses to two layered specimens are investigated via numerical results of analytical model in Fig. 4 when the exciting voltage V_0 is 5V. In the process of testing the samples of steel, the signals were achieved from different positions, respectively, while taking the signal of specimen without metal loss as reference signal, the temporal PECT signals from different thickness variations are substracted with reference signal, and then pulsed eddy current signals can be obtained, respectively, which are plotted in Fig. 5.

4.1 Multilayered Conductive Structures of Stainless Steel

The conductivity of stainless steel STS301L is 1.333×10^6 S/m, and the magnetic permeability is μ_0. The structures of two layered stainless steel is shown in Fig. 6. The thickness change of two layered stainless steel is investigated in Fig. 7.

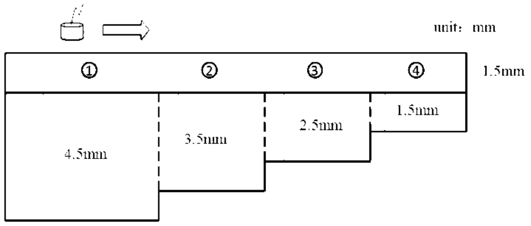

Fig. 6 STS301L+STS301L two layered stainless steel step wedge

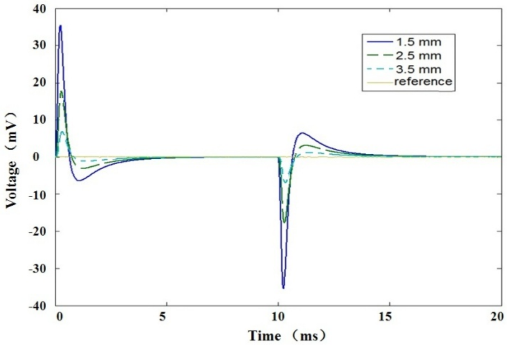

Fig. 7 Simulating result of STS301L+STS301L two layered stainless steel step wedge

Three features of differential PECT signal mainly employed to quantify geometric variaons of SCS are peak amplitude, time to peak amplitude and time to zero crossing. We investigate the peak amplitude to quantify two layered stainless steel plates in Fig. 8.

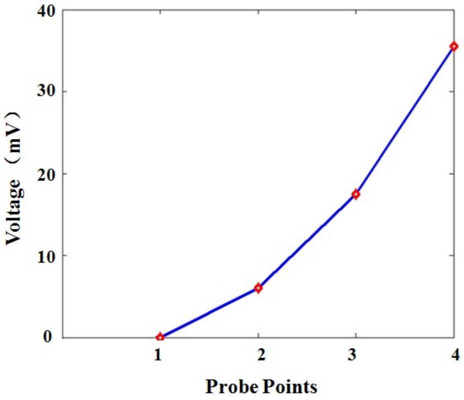

Fig. 8 The peak amplitudes in analytical simulating results of probe points from Fig. 7

4.2 Multilayered Conductive Structures of Mild Steel

For the ferromagnetic materials, most of the pulsed eddy current detecting research is mainly focusing on pulsed eddy current signal processing, the signal processing PECT responses and many coil models to replace eddy current effect theory of PECT for single layer ferromagnetic materials were developed[13,14]. In this chapter, PEC theory for multilayered conductive structure is mainly applied to saturation magnetic treatment of ferromagnetic materials research. The main shortcoming of the theory is a need for the target system to be linear.

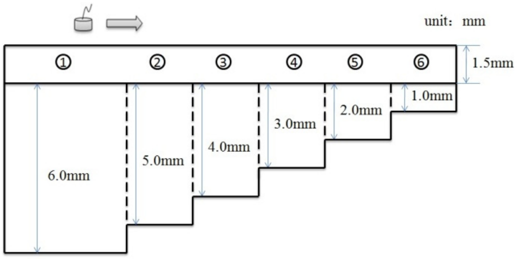

Fig. 9 STS301L+SS400 two layered stainless steel step wedge

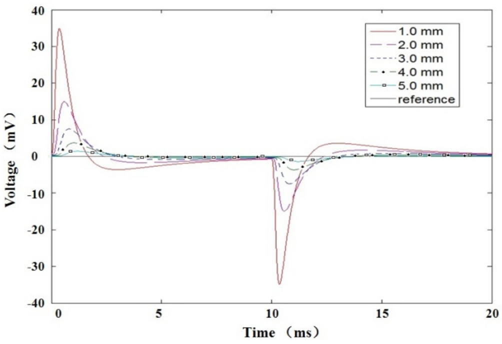

Fig. 10 Simulating result of STS301L+SS400 two layered stainless steel step wedge

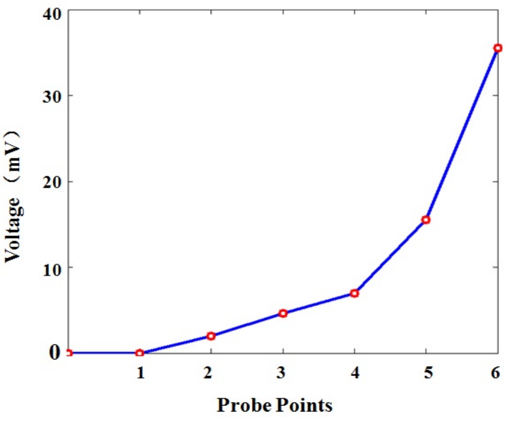

Fig. 11 The peak amplitudes in analytical simulating result of probe points from Fig. 10

Consequently, we may use the proposed PECT theory for ECT systems applied to ferromagnetic materials only in cases where the excitation current is big enough to retain the magnetization curve in the quasi-linear saturation region. The conductivity of mild steel SS400 is 5.8×10^7 S/m, and the magnetic permeability is $5\mu_0$. The structures of two layered steel is shown in Fig. 9, which the first layer is stainless steel(STS301L) and the other is mild steel(SS400). The thickness change of two layered steel is also investigated in Fig. 10 and the peaks of PECT signal are shown in Fig. 11.

5. Conclusions

The purpose of this paper is to study the evaluation of interlayer corrosion in stratified conductive structures using analytical model of PECT in series form. The obtained results show that the model-based prediction of PECT is effective due to its advantages of determination of integration limit, time saving and accuracy control of round-off errors.

Analytical modeling for PECT not only contributes to the development of an analytical solver but also can be applied to optimize the inspection condition in PECT. We have presented analytical modeling development made in order to simulate the PECT response of the probe to thickness measurement in a multilayered structure. Furthermore, work is in process to setup the experiment to verify PECT model of the probe response due to interlayer corrosion in multilayered conductive structures.

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