

# 유한 가동 능력을 고려한 IMES(지능형 다단물류 시스템)을 위한 최적 알고리즘

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## An Optimal Algorithm for the IMES (Intelligent Multi-Echelon Systems) with Finite Operating Capacity

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수리가능 제품은 가격 비싸고 중요성이 크면서 새 제품을 구매하기 어려운 부품을 의미하며, 항공기 또는 선박의 엔진 등을 들 수 있다. 수리가능 제품에 대하여 고장이 발생할 경우 가용성을 유지하기 위하여 즉시 교체하여야 하고, 교체된 부품은 수리에 들어가야 한다. 이러한 문제에 대한 통제 시스템은 시스템의 효율성을 결정하는 중요한 요소이기 때문에, 다양한 시스템 구성에 대하여 많은 연구가 이루어져 왔다.

본 연구에서는 우선 중앙 수리기지과 여러 지역 수리기지를 대상으로 하고 있으며, 물류량의 변화에 따라 중앙 수리기지과 지역 수리기지 간의 효과적인 운영 방안을 고려하고 있다. 각 지역 수리기지는 수리 운영 능력이 제한되어 있는 것으로 가정하고 있으며, 한정된 수의 여분의 제품을 유지하고 있다. 중앙 수리기지는 여분을 제품을 보유하거나 유지하지 않고 오직 수리 작업만을 수행하고 있다. 유한 가동에 대한 효율적인 알고리즘은 이러한 상황에 대하여 운영 능력의 한계를 고려하면서 효율성을 최대화하기 위하여 개발되었다.

지역 수리기지에서는 제품의 고장이 발생하는 속도와 현재 보유한 제품 수량에 따라 효율성이 바뀌고, 시스템이 보유할 수 있는 재고량과 수리할 수 있는 능력의 한계가 제한되어 있다. 대기행렬이론에 기반하여 고장율과 수리율에 의한 비용 함수를 정의하고, 최적해를 구할 수 있는 IMES(지능형 다단물류 시스템)을 개발하였다. 총 예상 재고량에 의한 재고 비용과 품질 비용을 최소화함으로써 최적의 보유 재고수량을 찾아내고, 실제로 사용자가 활용할 수 있는 시간 내에 해를 구할 수 있는 효율성을 갖춘 알고리즘을 개발하였다. 본 연구의 목적은 전체 시스템의 총비용을 최소화하는 것이며, 예제를 통하여 알고리즘을 묘사하고 수치예제를 통한 실험을 실시하여 제안 알고리즘이 정확하고 효율적임을 보였다. 제안된 알고리즘에 의하여 매우 정확하고 효율적인 계산 결과를 얻을 수 있었으며, 이 방법을 통해 신속하고 정확하게 많은 문제를 해결할 수 있을 것으로 생각된다.

**Keywords :** IMES (Intelligent Multi-Echelon Systems), Repairable-Item, Optimal Algorithm

### 1. Introduction

Repairable items are referred to as components, which

are expensive, critically important, and subject to infrequent failures such as engines of a fighter plane or a ship. They should be replaced or repaired immediately, if failed, for

the system to maintain availability. For this reason the policy on the inventory or shortage levels is very important and naturally has been studied for a long time by many researchers. There are two main streams of research in this area. A METRIC model, developed by Sherbrooke [19] assumes infinite repair capacity. In his model, there are many bases and a central depot. A failed item at a base is dispatched to a repair facility and a spare, if available, is plugged in. Otherwise, it is backordered. A repaired item fills the backorder or is stored at a spare inventory point if there is no backorder. Feeney and Sherbrooke [4], Muckstadt [15, 16], Muckstadt and Thomas [17] extended this model. However, as Albright [1] has pointed out, models assuming infinite repair capacity always underestimate the amount of congestion in the system and, consequently, result in fewer spares than are really needed to achieve a specified backorder level.

Another stream of study adopts the finite repair capacity, constant-failure-rate assumptions. The models in this stream are more realistic than the comparable METRIC models. Gross et al. [5] considered a two-echelon (two levels of repair, one level of supply) system and presented an implicit enumeration algorithm to calculate the capacities of the base and depot repair facilities as well as the spares level which together guarantee a specified fill-rate at a minimum cost. Inevitably, the enumeration scheme of the method requires considerable computing times even for relatively small problems. Gross et al. [5, 6] and Albright and Soni [2] analyzed the operating characteristics of a given system with multi-dimensional Markov process. In another paper Albright [1] developed an approximation algorithm with a single type of item stocked and repaired by several bases and a central depot. The proposed methods in this stream concentrate on the analysis of the current status of a given system and, consequently, are impractical to apply to optimization problems.

More recently, Pasandideh et al. [18] and Wang et al. [20] developed the genetic-algorithm for two-echelon systems and Gumus et al. [8] and Hu et al. [9] developed the algorithm using neuro-fuzzy for multi-echelon system. Kim et al. [12, 13, 14] developed an algorithm to determine the optimal inventory level under finite repair capacities. Jung et al. [10] and Kim et al. [11] presented a method to solve a two-echelon system with lateral transshipment. However their assumption that infinite number of items operating at each base is limited to real world. Thus, an efficient algo-

rithm for IMES (Intelligent Multi-Echelon Systems) with finite items is developed to overcome their limitation. In other words, the system has finite number of operating items so the failure rates of items at the bases depend on the current number of items online. Based on the queueing theory and special properties of a cost function, an algorithm is developed to find the amount of spare at each inventory which minimizes the total expected holding plus shortage costs and simultaneously achieve a specified minimum fill-rate.

This article is organized as follows. In Section 2 the model with the required parameters and probability distributions of model are described. In Section 3, the detailed algorithm steps for the model along with an example are presented. In Section 4, computational experiments and the results of the study are summarized. Finally, concluding remarks are presented in Section 5.

## 2. Model Description

### 2.1 Assumptions and Notation

We consider a system with several bases and a central depot and a single type of items. At base  $i$  there can be as many as  $m_i$  operating items at any given time. Each base also maintains extra items as spare. The central depot stocks no spare and only repairs the failed items from bases. Time intervals between failures of an item operating at base  $i$  are exponentially distributed with mean,  $1/\alpha_i$ ,  $i = 1, 2, \dots, I$ . A failed item is replaced immediately by a base spare if one is available. Otherwise, the replacement is backordered until a spare becomes available, i.e., a repair is completed at the base repair center or an item is arrived from the depot. A failed item is evaluated to be depot-repairable with probability  $1 - q_i$  and base-repairable with  $q_i$ .

At the base repair center, item completed a repair is used to fill an outstanding backorder or stored at the spare inventory point. At the depot, item finished a repair is sent to a base which has repair request yet to be filled. When there are several such bases, a first-come, first-serve basis is used to select a base. Since the depot has no inventory, repaired item is transported to a selected base immediately

Notations are as follows;

$\lambda_i$  : failure rate of online item at base  $i$ ,

$\mu_i$  : repair rate of a service channel at base  $i$  repair

center,

- $\mu_d$  : repair rate of a service channel at depot repair center,
- $c_i$  : number of repair channel at base  $i$  repair center,
- $c_d$  : number of repair channel at depot repair center,
- $f_i$  : minimum required fill-rate at base  $i$ ,
- $F_i$  : actual fill-rate at base  $i$ ,
- $n_i$  : number of operating items at base  $i$ ,
- $m_i$  : maximum number of operating items online at base  $i$ ,
- $b_i$  : number of item at base  $i$  repair center,
- $z_i$  : total number of failed items of base  $i$ ,
- $d$  : number of item at depot repair center,
- $d_i$  : number of item at depot repair center owed to base  $i$ ,
- $h_i$  : unit holding cost per unit time of base  $i$ ,
- $e_i$  : unit shortage cost per unit time of base  $i$ ,
- $r_i$  : initial spare inventory level at base  $i$ ,
- $\bar{r}_i$  : minimum inventory level satisfying minimum required fill-rate at base  $i$ ,
- $r_i^*$  : inventory level with the minimum total expected cost and satisfying minimum required fill-rate at base  $i$ .

We make three additional assumptions. First, all failed items can be repaired, so that there are no condemnations. Second, no lateral supply between bases is allowed. Finally, travel times from the bases to the depot and vice versa are ignored. A system with travel times is more general than the considered system and might require a more complex approach to find a solution.

## 2.2 Probability Distribution of Items at the Base Repair Center Given the Number of Items at the Depot

The failure rate of base  $i$  depends upon the number of operating item at the base. Thus the failure rate is  $n_i\alpha_i$ . Since then umber of operating item  $n_i = o_i + r_i - b_i - d_i$ , failure rate of the bases as follows.

$$\lambda(b_i|d_i) = \begin{cases} m_i q_i \alpha_i & (0 \leq b_i \leq r_i - d_i) \\ [m_i + r_i - (b_i + d_i)] q_i \alpha_i & (r_i - d_i < b_i \leq m_i + r_i - d_i) \\ 0 & (b_i > m_i + r_i - d_i) \end{cases} \quad (1)$$

Let there be  $c_i$  service channels at the repair center of base  $i$  item. The repair times at each channel are assumed to be exponential with mean  $1/\mu_i$ , then there pair rate at

base  $i$ ,  $\mu(b_i)$ , is given by equation (2).

$$\mu(b_i) = \begin{cases} b_i \mu_i & (b_i < c_i) \\ c_i \mu_i & (b_i \geq c_i) \end{cases} \quad (2)$$

Since we assume finite population, the base repair center can be modeled as an M/M/ $c_i$ / $\infty$ / $m_i$  queueing model.

The probability distribution that there are  $b_i$  items at the base repair center  $i$ , given value of  $k_i$ , is given by the equation (3)-(5) when  $k_i$  is less than or equal to  $r_i$  and other case is given by equation (6)-(8).

- i)  $d_i \leq r_i$   
if  $1 \leq b_i < c_i$ ,

$$P(b_i|d_i) = \frac{m_i^{b_i}}{b_i!} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} P(0|d_i) \quad (3)$$

as if  $c_i \leq b_i \leq m_i + r_i - d_i$ ,

$$P(b_i|d_i) = \frac{m_i^{r_i - d_i} m_i! / c_i!}{[m_i + r_i - d_i - b_i]! c_i^{b_i - c_i}} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} P(0|d_i) \quad (4)$$

where

$$p(0|d_i) = \left[ \sum_{b_i=0}^{c_i-1} \frac{m_i^{b_i}}{b_i!} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} + \sum_{b_i=c_i}^{r_i+m_i-d_i} \frac{m_i^{r_i-d_i} m_i!}{[m_i + r_i - d_i - b_i]! c_i^{b_i - c_i} c_i!} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} \right]^{-1} \quad (5)$$

- ii)  $d_i > r_i$   
if  $1 \leq b_i < c_i$ ,

$$P(b_i|d_i) = \frac{(m_i + r_i - d_i)!}{[m_i + r_i - d_i - b_i]! b_i!} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} P(0|d_i) \quad (6)$$

as if  $c_i \leq b_i \leq m_i + r_i - d_i$ ,

$$P(b_i|d_i) = \frac{(m_i + r_i - d_i)! / c_i!}{[m_i + r_i - d_i - b_i]! c_i^{b_i - c_i}} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} P(0|d_i) \quad (7)$$

where

$$p(0|d_i) = \left[ \sum_{b_i=0}^{c_i-1} \frac{(m_i + r_i - d_i)!}{(m_i + r_i - b_i - d_i)!} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} + \sum_{b_i=c_i}^{m_i+r_i-d_i} \frac{(m_i + r_i - d_i)!}{[m_i + r_i - b_i - d_i]! c_i^{b_i - c_i} c_i!} \left( \frac{q_i \alpha_i}{\mu_i} \right)^{b_i} \right]^{-1} \quad (8)$$

Note that the above probability distributions do not exist unless the steady-state condition, i.e.,  $p_i = m_i q_i \alpha_i / c_i \mu_i < 1$ , is satisfied.

### 2.3 Probability Distribution at the Depot Repair Center

We approximate the queueing system at the depot as a one-dimensional birth-death model. To do this, we need input rate  $\lambda(d)$  and repair rate  $\mu(d)$  for each state  $d$  of the depot. The repair rate  $\mu(d)$  is in (9).

$$\mu(d) = \begin{cases} d\mu_d, & 0 \leq d \leq c_d \\ c_d\mu_d, & c_d \leq d \end{cases} \quad (9)$$

The input rate  $\mu(d)$  is dependent upon the current states of each base. We estimate expected value of the number of operating item at each base for a given number of item at the depot by iterative procedure. Then we calculate the input rate  $\mu(d)$  using the estimated value of  $E(n_i|d)$  as in (10).

$$\lambda(d) = \sum_i E(n_i|d) \alpha_i (1 - q_i) \quad (10)$$

That is, the arrival rate can be approximated by the sum of the arrival rate from each base setting the number of operating item as its expected value. The iterative procedure starts by assigning an arbitrary value to  $E(n_i|d)$ . Then we use the following relationships to estimate  $E(n_i|d)$  again.

$$\begin{aligned} E(n_i|d) &= \sum_i^{r_i+m_i} P(n_i > n_i|d) \quad (11) \\ &= \sum_{n_i=0}^{r_i+m_i} \sum_{b_i} \sum_{d_i} P(m_i > n_i|d_i, b_i, d) P(b_i|d, d_i) P(d_i|d) \\ &= \sum_{n_i=0}^{r_i+m_i} \sum_{b_i} \sum_{d_i} P(r_i + m_i - b_i - d_i > n_i|d_i, b_i, d) P(b_i|d, d_i) P(d_i|d) \end{aligned}$$

Note that, when  $b_i + d_i \geq r_i$ ,  $n_i = r_i + m_i - b_i - d_i$ , i.e., the number of operating item at base  $i$  = initial spare inventory level + the maximum number of operating item - number at the base repair center - number at the depot repair center. when  $b_i + d_i \geq r_i$ ,  $n_i = m_i$ .

$P(b_i|d, d_i) = P(b_i|d_i)$  can be found in the equation (3)-(8). We can find the conditional distribution  $P(d_i|d, b_i)$  as a binomial distribution with parameters  $d$  and  $q_i$ , truncated at

the minimum and maximum values of  $d_i$ . The maximum value is  $\max(0, d - \sum_{j \neq i} (r_j + m_j))$  and the minimum value is  $\min(d, r_i + m_i)$

$$\text{Let } \theta_i = E(n_i) \alpha_i q_i / \sum_i E(n_i) \alpha_i q_i \quad (12)$$

Thus  $q_i$  approximates the probability that a given item at the depot is from the base  $i$ .

$$\begin{aligned} \text{Then } P(d_i|d) &= \binom{d}{d_i} \theta_i^{d_i} (1 - \theta_i)^{d - d_i}, \quad (13) \\ \max(0, d - \sum_{j \neq i} (r_j + m_j)) &\leq d_i \leq \min(d, r_i + m_i) \end{aligned}$$

$P(d_i|d)$  in (13) should be normalized to make its sum unity.  $\sum_{b_i} P(d_i \leq r_i + m_i - b_i - n_i | b_i, d_i, d)$  can be calculated by  $P(d_i|d)$  in (13). Using the equation (11)-(13), we are able to find a new value of  $E(n_i|d)$ . This new value of  $E(n_i|d)$  is used for initiating the next iteration. The iteration stops when two successive values of  $E(n_i|d)$  are sufficiently close. Once we find an appropriate value of  $E(n_i|d)$ , we then find  $P(d)$  from a one-dimensional birth-death model using  $\lambda(d)$ ,  $\mu(d)$  as input and output rates.

### 2.4 Probability Distribution at the Base Repair Center

To find  $P(b_i)$ , we derive  $P(d_i)$  as in (14).

$$P(d_i) = \sum_{d=0}^M P(d_i|d) P(d) \quad (14)$$

where  $M = \sum (r_i + m_i)$ , we find  $P(b_i)$  as in (15) using  $P(d_i)$ .

$$P(b_i) = \sum_{d=0}^{r_i+m_i} P(b_i|d_i) P(d_i) \quad (15)$$

### 2.5 Probability Distribution of Total Failed Items

We define a total failed item as sum of items in base repair center and in the depot repair center. Its probability distribution is obtained by convolution of the two probability distributions as in equation (16).

$$P(z_i) = \sum_{b_i=0}^{r_i+m_i} P(d_i = z_i - b_i)P(b_i) \quad (16)$$

### 2.6 Minimum Fill-Rate

The problem addressed here is to find an initial spare inventory level at each base to satisfy the minimum required fill-rate of each base with minimum total expected cost. The actual fill-rate,  $F_i$  which is the probability that a failed item is replaced by a spare, should be larger than or equal to the minimum required fill-rate  $f_{ij}$ . The relationship can be expressed as in equation (17).

$$F_i = \Pr\{z_i < r_i\} = \sum_{z_i=0}^{r_i-1} P(z_i) \geq f_{ij} \quad (17)$$

### 2.7 Cost Function

If the total failed items of base  $i$ ,  $z_i$ , is larger than the spare inventory level  $r_i$ , then the shortage cost  $e_i$  is incurred for each backorder. On the other hand, The holding cost  $h_i$  is charged on all the spares in base  $i$ . When we assume linear holding and shortage costs, the total expected cost of the system, which is the sum of the shortage and holding costs of the bases, can be obtained by equation (18).

$$TC(S) = \sum_{i=1}^I TC_i(r_i) = \sum_{i=1}^I \left\{ h_i r_i + e_i \sum_{z_i=r_i+1}^{m_i+r_i} (z_i - r_i) P(z_i) \right\} \quad (18)$$

## 3. The Algorithm

Step 1 : Verify that the following steady-state conditions are satisfied.

$$p_d = \sum_i m_i \alpha_i (1 - q_i) / c_d \mu_d < 1 \text{ and}$$

$$p_d = m_i q_i \alpha_i / c_i \mu_i < 1 \text{ for } i = 1, 2, \dots, I$$

If the conditions are met, go to Step 1. Otherwise, stop since the system can not reach steady state.

Step 2 : Find the minimum cost initial spare inventory level for each base when there is infinite number of operating capacity item.

Step 3 : Find probability distribution for the depot.

Step 4 : Find probability distribution for each base.

Step 5 :

Step 5.1 : For each base with  $s_i \geq 1$ , calculate

$$\Delta TC(S | \delta s_i) = TC(S | s_i - 1) - TC(S | s_i).$$

Step 5.2 : If  $\Delta TC(S | \delta s_i) \geq 0$  or  $s_i = 0$  for all  $i$ , go

to Step 6. Otherwise let  $i^* = \max_i \Delta TC(S | \delta s_i)$

and  $S_i \leftarrow S_i \leftarrow -1$ ,  $m \leftarrow m + 1$ .

Step 5.3 : For base  $i^*$ , recalculate probability for  $s_i - 1$ .

Go to Step 4.

Step 6 : The  $s_i$ 's are solution of the algorithm. Expected total cost of each base is  $TC_i(s_i)$  and expected total cost of system is  $TC(S)$ .

In step 2, we find the minimum cost spare level for the infinite number of operating item case and use the spare level as a starting point for the subsequent search. Detailed discussion for finding the values can be found in Gross et al. [7]. Steps 3 and 4 are to calculate the probability distributions previously defined. In step 5, we find a local minimum point of the total cost function using iterative procedure. During each iteration we depict the base, which gives the largest decrease in the total cost function by unit decrease of its spare level, and decrease the spare level by one unit. Thus the procedure in these steps is similar to the steepest descent method. If no more cost decrease is possible, then we stop and generate the current point as a solution to the problem.

We illustrate the algorithm by the following example. Consider a multi-echelon inventory system with two bases, which operate only one type of item and a depot. The relevant data is described in <Table 1>.

<Table 1> Data Example for the Illustration of the Algorithm

	$\lambda_{ij}$	$\alpha_{ij}$	$c_{ij}$	$\mu_{ij}$	$M_{ij}$	$h_{ij}$	$e_{ij}$
Base 1	3.57	0.47	4	10.73	10	29	204
Base 2	2.85	0.58	5	10.29	10	33	203
Depot			5	17.11			

The solution of the example, inventory levels satisfying the minimum fill-rate at minimum cost, is summarized in the third column of <Table 2>. The spare level and the cost are decreased as the minimum fill-rate is decreased until the minimum point is reached. But if the inventory

level arrives at the minimum point, it remains there despite a further decrease in the minimum fill-rate.

<Table 2> Output of the Algorithm for the Example

Minimum fill rate	Actual fill rate	Optimal spare level	Optimal cost
0.95	0.96(0.96)	8(7)	238.90(235.36)
0.90	0.90(0.91)	7(6)	219.16(209.76)
0.85	0.90(0.91)	7(6)	219.16(209.76)
0.80	0.81(0.81)	6(5)	209.39(194.05)
0.75	0.81(0.81)	6(5)	209.39(194.05)
0.70	0.81(0.81)	6(5)	209.39(194.05)
0.65	0.81(0.81)	6(5)	209.39(194.05)
0.60	0.81(0.81)	6(5)	209.39(194.05)

Note) \*Entry in parenthesis is for the base 2.

### 4. Computational Experiments

The algorithm could be very effective by limiting the search region using the properties described above. An extensive computational experiments for the proposed algorithm is performed in order to test the accuracy and efficiency of the proposed algorithm. For this purpose we compare the minimum total expected cost and actual fill-rate calculated by the algorithm with those obtained from a enumeration technique. An enumeration technique is to find the minimum point of the cost function from lower bound to upper bound

and calculate the minimum inventory level for each type of items at the base satisfying the specified minimum fill-rate. The lower and upper bound for  $s_{ij}$  is 0 and  $M_{ij}$  in the enumeration technique. The proposed algorithm is written in Visual Studio 2008 C++. The experiments are performed on a Pentium IV 2.6 GHz dual CPU based IBM compatible PC system.

The following <Table 3> and <Table 4>, show the input data and experimental results for the cases where there are 5 bases.

<Table 3> Data for Base 5 with 3 Types of Item

	$\lambda_{ij}$	$\alpha_{ij}$	$c_{ij}$	$\mu_{ij}$	$M_{ij}$	$h_{ij}$	$e_{ij}$	$f_{ij}$
Base 1	1.54	0.59	5	10.04	5	31	202	0.61
	1.88	0.48	5	6.25	9	31	208	0.62
	2.12	0.43	6	11.54	6	31	211	0.68
Base 2	1.73	0.51	5	7.73	7	32	198	0.62
	1.59	0.47	4	4.72	8	31	208	0.62
	1.04	0.43	6	6.79	7	28	195	0.64
Base 3	0.73	0.57	3	9.82	6	29	199	0.68
	2.13	0.56	5	7.47	8	34	198	0.69
	2.16	0.55	3	10.98	5	31	198	0.63
Base 4	1.56	0.53	5	9.76	9	30	209	0.65
	2.07	0.48	4	4.42	6	31	207	0.64
	2.16	0.40	6	7.45	7	29	194	0.66
Base 5	2.15	0.60	3	6.17	8	32	208	0.62
	1.44	0.43	6	6.14	5	28	198	0.64
	1.01	0.57	5	7.93	8	30	179	0.70
Depot	-	-	5	16.58	-	-	-	-
	-	-	6	16.96	-	-	-	-
	-	-	7	16.92	-	-	-	-

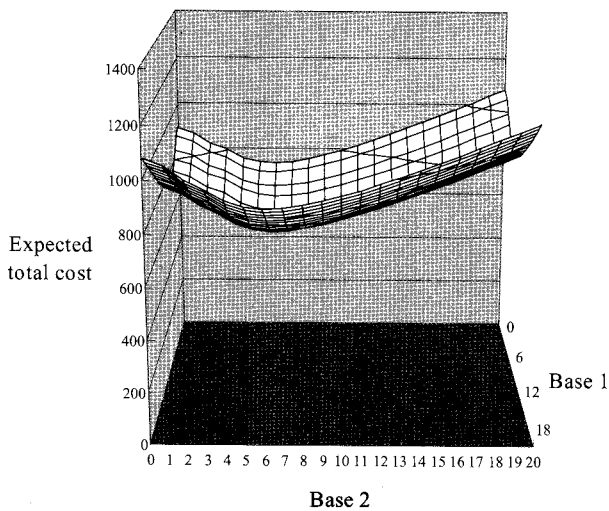
<Table 4> Result for base 5 with 3 types of item

	Cost		fill-rate		Computing time (sec)	Iterations
	Algorithm	Enumeration	Algorithm	Enumeration		
Base 1	75.04	75.04	0.80	0.80	0.04(0.12)	4(8)
	151.30	151.30	0.79	0.79		
	105.87	105.87	0.85	0.85		
Base 2	114.13	114.13	0.81	0.81	0.31(1.55)	4(22)
	140.21	140.21	0.85	0.85		
	73.79	73.79	0.75	0.75		
Base 3	60.84	60.84	0.92	0.92	0.96(6.59)	5(33)
	155.40	155.40	0.83	0.83		
	87.69	87.69	0.70	0.70		
Base 4	111.18	111.18	0.79	0.79	2.57(20.17)	7(47)
	140.41	140.41	0.84	0.84		
	122.31	122.31	0.71	0.71		
Base 5	183.48	183.48	0.72	0.72	4.65(49.78)	7(65)
	76.04	76.04	0.74	0.74		
	78.03	78.03	0.74	0.74		
Depot	-	-	16.58	-	-	-
	-	-	16.96	-		
	-	-	16.92	-		

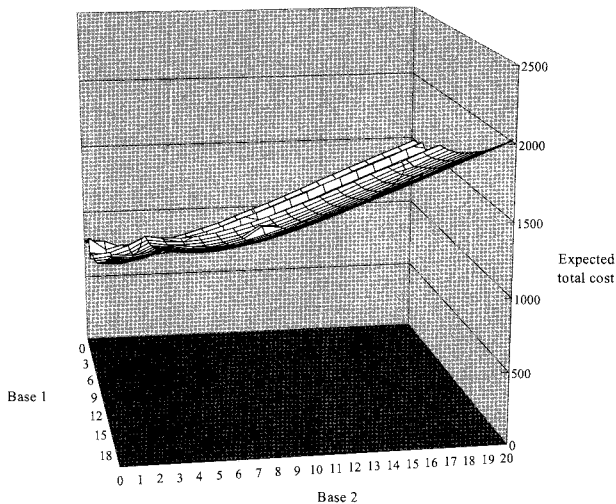
Note) \*Entry in parenthesis is for the enumeration technique.

For the sensitivity analysis of cost function, show <Figure 1> and <Figure 2> that calculate cost function for IMES with identical failure rates and different failure rates. And show <Figure 3>, that is cost function when minimum fill rate is set to 0.6.

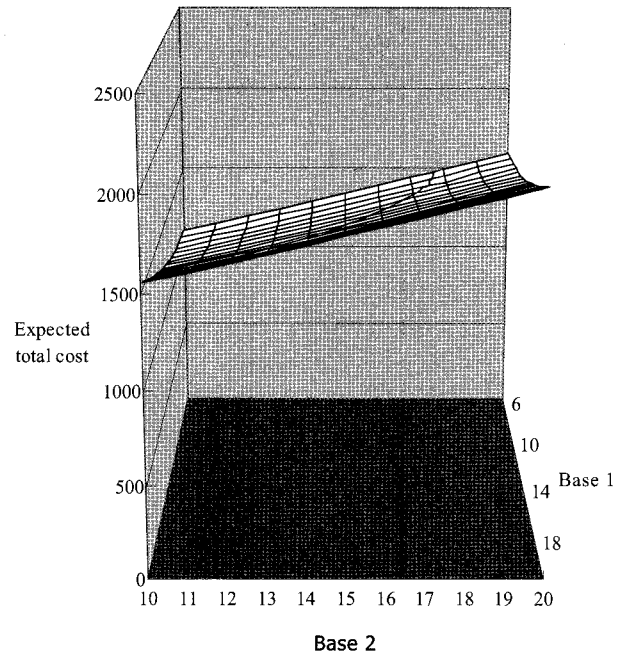
For the quality of solution, all results of the proposed algorithm are exactly matched to those obtained from the enumeration technique. The number of iterations and the CPU time for proposed algorithm range from 4~7 and from 0.04 to 4.65 seconds respectively as the number of base is increased from 1 to 5. On the other hand, 8~65 iterations and 0.12~49.78 seconds were needed to calculate the same measures by using enumeration technique.



<Figure 1> Cost Function for a System with Identical Failure Rates



<Figure 2> Cost Function for a System with Different Failure Rates



<Figure 3> Cost Function when Minimum Fill Rate is Set to 0.6

### 5. Concluding Remarks

In this paper we developed an algorithm for IMES (Intelligent Multi-Echelon Systems) by using the queueing theory and cost function analysis to calculate the spare inventory level which satisfies a predetermined minimum fill-rate and minimizes the total expected cost. In comparison with enumeration results, the proposed algorithm is quite accurate and computationally efficient. With this approach we are able to solve large problems quickly and accurately. For further study, one can relax the assumption of no travel times from the bases to the depot and vice versa. In addition, this model can be extended to the more general case where the spares in a base can be transferred to another if it has no spare to replace the failed item.

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