# Proportional-Fair Downlink Resource Allocation in OFDMA-Based Relay Networks

Chang Liu, Xiaowei Qin, Sihai Zhang, and Wuyang Zhou

*Abstract:* **In this paper, we consider resource allocation with proportional fairness in the downlink orthogonal frequency division multiple access relay networks, in which relay nodes operate in decode-and-forward mode. A joint optimization problem is formulated for relay selection, subcarrier assignment and power allocation. Since the formulated primal problem is nondeterministic polynomial time-complete, we make continuous relaxation and solve the dual problem by Lagrangian dual decomposition method. A near-optimal solution is obtained using Karush-Kuhn-Tucker conditions. Simulation results show that the proposed algorithm provides superior system throughput and much better fairness among users comparing with a heuristic algorithm.**

*Index Terms:* **Dual decomposition, orthogonal frequency division multiple access (OFDMA), proportional fairness (PF), relay network, resource allocation.**

## I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) has been regarded as a promising candidate for the future fourth generation (4G) wireless communication system, due to its high spectral efficiency and significant potential to mitigate the problem of frequency-selective fading. In an OFDMA system, multiuser diversity can be fully exploited by resource allocation, including subcarrier assignment and power allocation schemes. Therefore, resource allocation in the OFDMA systems has drawn much attention in recent years [1], [2].

As it may increase the coverage of communication systems and enhance the link reliability, cooperative relaying has been an attractive technique in OFDMA networks. Meanwhile, relay selection is a meaningful aspect for resource allocation in OFDMA multi-relay networks, as adaptive relay selection scheme can improve the performance of cooperative relaying by achieving the enhanced diversity and multiplexing gain. Recently, there have been many research interests focused on improving the system throughput by resource allocation in OFDMA-based relay networks [3]–[6]. A suboptimal algorithm is proposed to maximize the downlink capacity in [3]. The joint subcarrier assignment and relay selection subproblem is solved with the assumption of equal power allocation, and then the power allocation suboptimal is solved by an iterative method. And in [4], an evolutional-based algorithm is proposed for solving the relay selection and subcarrier assignment problem in uplink multi-relay networks, while the power allocation problem is not considered. To further improve the system throughput, joint solutions for relay selection, subcarrier assignment and power allocation are proposed in [5] and [6], and near-optimal results are obtained using convex optimization. In detail, the authors present a centralized utility maximization framework for a cooperative cellular network without additional relay nodes in [5], where each node may act as a source/destination or relay simultaneously. Reference [6] aims at maximizing the downlink capacity with minimal rate requirements from users.

However, most existing research on resource allocation in OFDMA-based relay networks focuses on maximizing the system throughput, and fairness among multiple users has been seldom considered. Since proportional fairness (PF) can achieve the tradeoff between system throughput and fairness, it is widely adopted to maintain the transmission rate fairness among users in conventional cellular networks [7]–[10]. While in relay networks, few researches have paid attention to the PF resource allocation due to the complexity. In [11], PF based subcarrier allocation is discussed in the network with only one relay and using equal power allocation. Nevertheless, in practical relay networks, relay selection is often required due to the existence of multiple relays. Moreover, adaptive power allocation can further improve the performance.

In this paper, we investigate the PF based relay selection, subcarrier assignment and power allocation problem in downlink OFDMA-based relay networks, and formulate the problem as a joint optimization problem. Since the problem cannot be solved directly, we make continuous relaxation and use Lagrangian dual decomposition method to solve it. A near-optimal solution of the primal problem is obtained by solving the subproblems and the master problem, respectively. By comparing the proposed algorithm with a heuristic algorithm, which is based on greedy subcarrier assignment and equal power allocation, simulation results show that the proposed algorithm can improve the system throughput with much better fairness.

The rest of this paper is organized as follows. Section II provides the system model and formulates the resource allocation problem. Section III analyzes the optimization problem and proposes a near-optimal algorithm. Simulation results are given and discussed in Section IV. Finally, Section V draws conclusions.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first describe the system model of OFDMA-based relay network, and then formulate the problem

Manuscript received June 5, 2010; approved for publication by Hong-Chuan Yang, Division II Editor, March 3, 2011.

This work was supported by the China High-Tech 863 Plan under grant 2009AA011506, the National Major Special Projects in Science and Technology of China under grant 2009ZX03003-009, 2010ZX03003-001, 2010ZX03005-003 and 2011ZX03003-003-04, National key technology R&D program under grant 2008BAH30B12, National Basic Research Program of China under grant 2007CB310602, and China-Finland International S&T Cooperation Program under grant 2010DFB10570.

The authors are with the wireless information network laboratory (WINLAB), Department of Electrical Engineering and Information Science, University of Science and Technology of China, Hefei, Anhui, 230027, P. R. China, email: changool@mail.ustc.edu.cn, {qinxw, shzhang, wyzhou}@ustc.edu.cn.

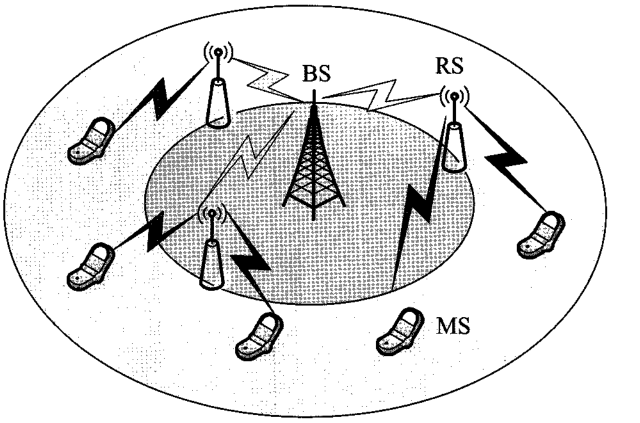



Fig. 1. The system model of downlink OFDMA-based relay network.

of resource allocation with proportional fairness.

### A. OFDMA-Based Relay Networks

We consider an OFDMA relay-assisted cellular network as shown in Fig. 1. The overall bandwidth $W$ is divided into $N$ subcarriers for OFDM transmission. A base station (BS) is located at the center while $K$ fixed relay stations (RS) at the inner boundary, operating in decode-and-forward (DF) mode. $M$ mobile stations (MS) are separated between the inner and the outer boundaries. It is assumed that there is no direct transmission between the BS and the MSs due to heavy blockage and long distance [6], [11]. We consider a two-subframe downlink relay transmission pattern as in [3]. Each RS receives and decodes the data transmitted from BS in the first subframe, and then forwards the data to the relevant MSs in the second subframe. For simplicity, we assume that the RS forwards the received data to the MS on the same subcarrier. It is further assumed that the channels are slowly-varying, and thus the channel state information of all links can be estimated and fed back to the BS.

Let $p_{sk}^n$ and $p_{km}^n$ denote the transmission powers of BS to $k$th RS and $k$th RS to $m$th MS spent on subcarrier $n$, respectively. $h_{sk}^n$ and $h_{km}^n$ represent the small-scale fading coefficients between BS and $k$th RS, $k$th RS and $m$th MS on subcarrier $n$. The path losses between BS and $k$th RS, $k$th RS and $m$th MS are $l_{sk}$ and $l_{km}$, respectively. Based on the Shannon formula, the average rate between BS and $k$th RS on subcarrier $n$ in two subframes is given by

$$r_{sk}^n = \frac{W}{2N}\log_2\left(1 + \frac{p_{sk}^n l_{sk}|h_{sk}^n|^2}{\Gamma W N_0/N}\right) \tag{1}$$

where $\Gamma$ is the signal to noise ratio (SNR) gap related to a target bit error rate (BER) [12], $\Gamma = -\ln(5\text{BER})/1.5$. $N_0$ denotes the power spectral density of the noise. Similarly, the average rate between $k$th RS and $m$th MS is

$$r_{km}^n = \frac{W}{2N}\log_2\left(1 + \frac{p_{km}^n l_{km}|h_{km}^n|^2}{\Gamma W N_0/N}\right). \tag{2}$$

The achievable rate of $m$th MS on subcarrier $n$ forwarded by $k$th RS is the minimum rate of $r_{sk}^n$ and $r_{km}^n$ [13]

$$r_{\mathrm{BS},km}^n = \min\{r_{sk}^n, r_{km}^n\}. \tag{3}$$

By the assumption that the RS forwards the received data to the MS on the same subcarrier, it is easy to get $r_{\mathrm{BS},km}^n = r_{sk}^n = r_{km}^n$ according to [6], thus further obtain that

$$p_{sk}^n = \frac{l_{km}|h_{km}^n|^2}{l_{sk}|h_{sk}^n|^2}p_{km}^n. \tag{4}$$

### B. Problem Formulation

We define $\rho_{km}^n \in \{0,1\}$ as the subcarrier assignment variable, where $\rho_{km}^n = 1$ indicates the assignment of subcarrier $n$ to $k$th RS and $m$th MS pair, and $\rho_{km}^n = 0$, otherwise. The instantaneous data rate of $m$th MS can be written as

$$R_m = \sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{km}^n r_{\mathrm{BS},km}^n. \tag{5}$$

A PF problem can be formulated as maximizing the sum of logarithmic user data rate [7], so that the optimization problem can be formulated as follows.

$$\begin{aligned}
&\max_{\mathbf{p_{sk}},\mathbf{p_{km}},\boldsymbol{\rho}} \sum_{m=1}^{M}\ln R_m \\
=\ &\max_{\mathbf{p_{sk}},\mathbf{p_{km}},\boldsymbol{\rho}} \sum_{m=1}^{M}\ln\left(\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{km}^n r_{\mathrm{BS},km}^n\right) \\
\text{s.t.}\quad &\text{(a)}\quad \rho_{km}^n \in \{0,1\}, \forall k,m,n; \\
&\text{(b)}\quad \sum_{m=1}^{M}\sum_{k=1}^{K}\rho_{km}^n \le 1, \forall n; \\
&\text{(c)}\quad p_{km}^n \ge 0, \forall k,m,n; \\
&\text{(d)}\quad p_{sk}^n \ge 0, \forall k,n; \\
&\text{(e)}\quad \sum_{m=1}^{M}\sum_{n=1}^{N}p_{km}^n \le P_k^{\max}, \forall k; \\
&\text{(f)}\quad \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}p_{sk}^n \le P_{\mathrm{BS}}^{\max}
\end{aligned} \tag{6}$$

where $P_{\mathrm{BS}}^{\max}$ and $P_k^{\max}$ stand for the total transmission power of BS and $k$th RS, respectively. Here, constraint (b) guarantees that each subcarrier is allocated to at most one RS-MS pair. The similar constraints are widely adopted in existing works, e.g., [3]–[6]. Note that the problem of intra-cell frequency reuse among RSs is not considered here. (e) and (f) correspond to the power constraints of BS and RSs, respectively.

However, problem (6) is a mixed binary integer programming problem, which is nondeterministic polynomial time (NP)-complete. An exhaustive search of the problem space is needed to find the optimal solution, but it is infeasible in practice. In the following section, we will obtain a near-optimal solution to (6) using the Lagrangian dual decomposition method.

## III. RESOURCE ALLOCATION ALGORITHM WITH PROPORTIONAL FAIRNESS

### A. Dual Decomposition Method

To make the problem (6) tractable, we relax $\rho_{km}^n$ to be a real number within the interval $[0,1]$. The continuous relaxation permits time sharing of each subcarrier [1]. It has been proved that the duality gap due to the relaxation becomes zero when the number of subcarriers goes to infinity [14]. Furthermore, it has been shown that with any practical number of subcarriers, the duality gap is virtually zero [15]. Since the duality gap of problem (6) is zero, the solution of its dual problem is equal to that of the primal one.

For the formulation of the dual problem, we first need the Lagrangian function of the primal problem (6). Upon rearranging the terms, the Lagrangian function can be written as follows.

$$\begin{aligned} &L(\mathbf{p_{sk}}, \mathbf{p_{km}}, \boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}) \\ =& \sum_{m=1}^{M} \ln\left(\sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{km}^n r_{\mathrm{BS},km}^n\right) \\ &+\sum_{n=1}^{N}\lambda_n\left(1-\sum_{m=1}^{M}\sum_{k=1}^{K}\rho_{km}^n\right) \\ &+\sum_{k=1}^{K}\mu_k\left(P_k^{\max}-\sum_{m=1}^{M}\sum_{n=1}^{N}p_{km}^n\right) \\ &+\eta\left(P_{\mathrm{BS}}^{\max}-\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}p_{sk}^n\right) \\ =& \sum_{m=1}^{M}\left[\ln\left(\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{km}^n r_{\mathrm{BS,km}}^n\right)-\sum_{n=1}^{N}\lambda_n\sum_{k=1}^{K}\rho_{km}^n\right. \\ &\left.-\sum_{k=1}^{K}\mu_k\sum_{n=1}^{N}p_{km}^n-\eta\sum_{k=1}^{K}\sum_{n=1}^{N}p_{sk}^n\right] \\ &+\sum_{n=1}^{N}\lambda_n+\sum_{k=1}^{K}\mu_k P_k^{\max}+\eta P_{\mathrm{BS}}^{\max} \end{aligned} \quad (7)$$

where $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_n]$, $\boldsymbol{\mu} = [\mu_1, \cdots, \mu_k]$, and $\boldsymbol{\eta} = [\eta]$ are Lagrangian multiplier vectors introduced by the constraints of (6b), (6e), and (6f), respectively. In detail, $\boldsymbol{\lambda}$ is the Lagrangian multiplier vector corresponding to the subcarrier assignment constraint, and $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are the vectors associated with the power constraints. Then, the Lagrangian dual function is given by

$$\begin{aligned} G(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}) = &\max_{\mathbf{p_{sk}}, \mathbf{p_{km}}, \boldsymbol{\rho}} \; L(\mathbf{p_{sk}}, \mathbf{p_{km}}, \boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}) \\ \text{s.t.} \quad & 0 \le \rho_{km}^n \le 1, \forall k, m, n, \\ & \text{(6c), (6d)} \end{aligned} \quad (8)$$

and the dual problem is

$$\begin{aligned} &\min_{\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}} \; G(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}) \\ &\text{s.t.} \;\; \boldsymbol{\lambda} \succeq 0, \boldsymbol{\mu} \succeq 0, \boldsymbol{\eta} \succeq 0. \end{aligned} \quad (9)$$

Using the decomposition method [16], problem (9) can be decomposed into lower level and higher level. At the lower level, we have $M$ subproblems, one for each MS $m$, in which (9) decouples

$$\begin{aligned} \max_{\mathbf{p_{sk}}, \mathbf{p_{km}}, \boldsymbol{\rho}} L_m = &\ln\left(\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{km}^n r_{\mathrm{BS},km}^n\right) \\ &-\sum_{n=1}^{N}\lambda_n\sum_{k=1}^{K}\rho_{km}^n-\sum_{k=1}^{K}\mu_k\sum_{n=1}^{N}p_{km}^n \\ &-\eta\sum_{k=1}^{K}\sum_{n=1}^{N}p_{sk}^n \\ \text{s.t.} \quad & 0 \le \rho_{km}^n \le 1, \forall k, m, n, \\ & \text{(6c), (6d).} \end{aligned} \quad (10)$$

At the higher level, we have the master dual problem in charge of updating the dual variables $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and $\boldsymbol{\eta}$ by solving

$$\begin{aligned} \min_{\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}} \quad & \sum_{m=1}^{M} G_m(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta}) + \sum_{n=1}^{N}\lambda_n \\ & + \sum_{k=1}^{K}\mu_k P_k^{\max} + \eta P_{\mathrm{BS}}^{\max} \\ \text{s.t.} \quad & \boldsymbol{\lambda} \succeq 0, \boldsymbol{\mu} \succeq 0, \boldsymbol{\eta} \succeq 0, \end{aligned} \quad (11)$$

where $G_m(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\eta})$ is obtained as the maximum value of $L_m$ in the subproblems for given $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and $\boldsymbol{\eta}$.

Consequently, we can obtain the solution of (9) by solving the subproblems and the master problem iteratively. In each iteration, the master problem provides the Lagrangian multiplier vectors for the subproblems, and in the reverse direction the resource allocation results are reported to the master problem.

### *B. Solution of the Dual Problem*

As described in subsection III.A, the duality gap of the primal problem (6) is zero, therefore, the solution of (6) is equal to that of its dual problem (9). Meanwhile, the dual problem (9) can be decomposed into several subproblems (10) and a master problem (11), thus the solution of the problem (6) can be obtained by solving (10) and (11).

#### B.1 Solution of the Subproblems

As we have obtained that $r_{\mathrm{BS,km}}^n = r_{sk}^n = r_{km}^n$ in subsection II.A, we will use $r_{km}^n$ to replace $r_{\mathrm{BS,km}}^n$ for simplicity. After substituting (4) into (10), and differentiating $L_m$ with respect to $\rho_{km}^n$ and $p_{km}^n$, respectively, we have

$$\begin{aligned} \frac{\partial L_m}{\partial \rho_{km}^n} &= \frac{r_{km}^n}{\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{km}^n r_{km}^n} - \lambda_n \\ &= \frac{r_{km}^n}{R_m} - \lambda_n, \end{aligned} \quad (12)$$

$$\frac{\partial L_m}{\partial p_{km}^n} = \frac{W\rho_{km}^n g_{km}^n}{2NR_m(1+p_{km}^n g_{km}^n)\ln 2} - \Phi_{km}^n \quad (13)$$

where $g_{km}^n = l_{km}|h_{km}^n|^2/(\Gamma W N_0/N)$ and $\Phi_{km}^n = \mu_k + \eta l_{km}|h_{km}^n|^2/(l_{sk}|h_{sk}^n|^2)$.

The subcarrier assignment variable $\rho_{km}^n$ is initialized to 1. Using Karush-Kuhn-Tucker (KKT) conditions [17], we can deduce the optimal power allocation from (13) as follows.

$$p_{km}^n = \left[\frac{W/2N}{\Phi_{km}^n R_m \ln 2} - \frac{1}{g_{km}^n}\right]^+ \quad (14)$$

where $[x]^+ = \max\{x, 0\}$.

In order to maximize the objective function $L_m$, we have

$$\rho_{km}^n \begin{cases} = 0, & \text{if } \frac{r_{km}^n}{R_m} < \lambda_n \\ \in (0,1), & \text{if } \frac{r_{km}^n}{R_m} = \lambda_n \\ = 1, & \text{if } \frac{r_{km}^n}{R_m} > \lambda_n \end{cases} \tag{15}$$

from (12).

The continuous relaxation yields a fractional-valued solution, and we should round $\rho_{km}^n$ to 0 or 1 to get an integer-valued solution. Since the constraint (6b) must be satisfied, (15) is equivalent to allocating subcarrier $n$ to the RS-MS pair with the maximum $r_{km}^n/R_m$. It follows that

$$(k^*, m^*) = \arg\max_{k,m} \frac{r_{km}^n}{R_m}, \tag{16}$$

$$\rho_{k^*m^*}^n = 1, \rho_{km}^n = 0, \quad \forall k \neq k^*, m \neq m^*. \tag{17}$$

The power allocation results and the subcarrier assignment results can be obtained by (14) and (16)–(17), where the Lagrangian multiplier vectors $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are provided by the master problem. And then, the resource allocation results are reported to the master problem.

B.2 Solution of the Master Problem

To solve the master problem (11), we use the subgradient method to update $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ [18].

$$\mu_k(i+1) = \left[\mu_k(i) - \alpha_1(i)(P_k^{\max} - \sum_{m=1}^{M}\sum_{n=1}^{N} p_{km}^n)\right]^+, \forall k, \tag{18}$$

$$\eta(i+1) = \left[\eta(i) - \alpha_2(i)(P_{\text{BS}}^{\max} - \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} p_{sk}^n)\right]^+ \tag{19}$$

where $i$ is the power iteration index, $\alpha_1(i)$ and $\alpha_2(i)$ are sequences of step sizes. Convergence to the optimal solution is guaranteed when the step sizes are chosen properly [17], [18]. The updated $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are supplied to the subproblems for the next iteration. The iteration process is repeated until the transmission powers converge.

Note that the Lagrangian multiplier $\boldsymbol{\lambda}$ introduced by the constraint (6b) needn't be updated by the subgradient method here. As described in subsection III.B.1, the value of $\rho_{km}^n$ can be obtained by (16) and (17), which are independent of $\boldsymbol{\lambda}$. Therefore, in the solution of the master problem in the BS, each $\lambda_n$ can be set to a certain value between the maximum $r_{km}^n/R_m$ and the secondary maximum one to denote that no subcarrier is shared.

### C. Proposed Resource Allocation Algorithm

In the solution of the subproblems, we can see from (14) and (16) that both the power allocation result $p_{km}^n$ and the subcarrier assignment result $\rho_{km}^n$ are related to $R_m$, while $R_m$ is also a function of $p_{km}^n$ and $\rho_{km}^n$. Therefore, $p_{km}^n$ and $\rho_{km}^n$ should be obtained by an iterative method. In this paper, we adopt the method of updating $R_m$ iteratively in each slot, which is defined as the resource allocation period. In each slot, $R_m$ is initialized to some nonnegative value, and then is updated iteratively until it converges. During the $l$th iteration, $R_m(l+1)$ can be calculated as

$$R_m(l+1) = R_m(l) - \beta_m(l)\left(R_m(l) - \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{km}^n(l) r_{km}^n(l)\right) \tag{20}$$

where $l$ is the rate iteration index, and $\beta_m(l) \in (0,1)$ is the sequence of step sizes of $m$th MS. The iteration process is guaranteed to converge to the optimal solution by choosing appropriate step sizes [17].

According to the results obtained in subsections III.B and III.C, the proposed joint subcarrier and power allocation algorithm with PF can be summarized in Table 1.

Table 1. Proposed resource allocation algorithm.

| | |
|---|---|
| 1) | Set power iteration index $i = 0$, initialize $\mu_k(0), \forall k$ and $\eta(0)$ with some nonnegative values.<br>Update the channel fading coefficients. |
| 2) | Set rate iteration index $l = 0$, and initialize $R_m(0), \forall m$ with some nonnegative values. |
| 3) | For subcarrier $n = 1$ to $N$<br>- a) Calculate $p_{km}^n, \forall k, m$, using (14).<br>- b) Obtain $r_{km}^n$, via (2).<br>- c) Subcarrier $n$ is allocated to $(k^*, m^*)$, according to (16) and (17). |
| 4) | For MS $m = 1$ to $M$<br>- Update $R_m$ according to (20). |
| 5) | Set $l = l + 1$.<br>Repeat step 3) and 4) until $R_m$ converges. |
| 6) | Update $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ by (18) and (19), set $i = i + 1$. |
| 7) | Go to step 2) until the transmission powers converge. |

## IV. SIMULATION RESULTS AND ANALYSIS

### A. Simulation Parameters

In this section, we will evaluate the system performance by simulations. Here, an OFDMA based relay cellular network with a single cell is considered. The outer and the inner boundaries have radii of 1 km and 0.6 km, respectively. The RSs are equally distributed at the inner boundary, and the MSs are uniformly distributed between the inner and the outer boundaries. The simulation parameters are listed in Table 2 in detail, and the simulation process is carried out according to Table 1. We run simulations for one hundred slots to achieve an average performance. The small-scale fading coefficients of all the channels are updated at the beginning of each slot.

### B. Results and Analysis

In our simulations, a heuristic algorithm, "NearRS + GreedySA + EquPA," is compared with our proposed algorithm. Here, NearRS means that the nearest RS of a MS is selected to forward this MS's data, GreedySA represents the greedy subcarrier assignment, and EquPA denotes the powers of BS and RSs are equally allocated to their transmitted subcarriers. In the

Table 2. Simulation parameters.

| Parameter | Value |
|---|---|
| Cell radius | 1 km |
| Inner cell radius | 0.6 km |
| Channel bandwidth $W$ | 1 MHz |
| Number of subcarriers $N$ | 32 |
| Number of RSs $K$ | 3 |
| Number of MSs $M$ | 2, 4, 6, 8, 10, 12 |
| $P_{\text{BS}}^{\max}$ | 22 dBm |
| $P_k^{\max}$ | 22 dBm |
| Noise power spectral density | −174 dBm/Hz |
| Propagation model (path-loss, in dB) | BS-RS: $128.1 + 28.8\log_{10}(R)$ RS-MS: $128.1 + 37.6\log_{10}(R)$ ($R$ is the distance in km) |
| Small-scale fading model | Rayleigh fading model |
| Target BER | $10^{-3}$ |

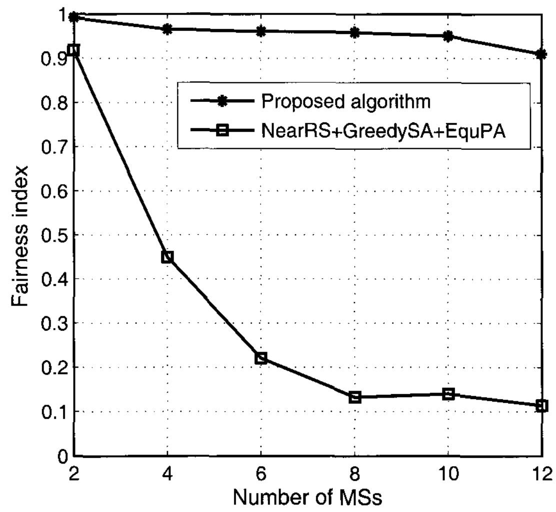



Fig. 3. Fairness index versus number of MSs.

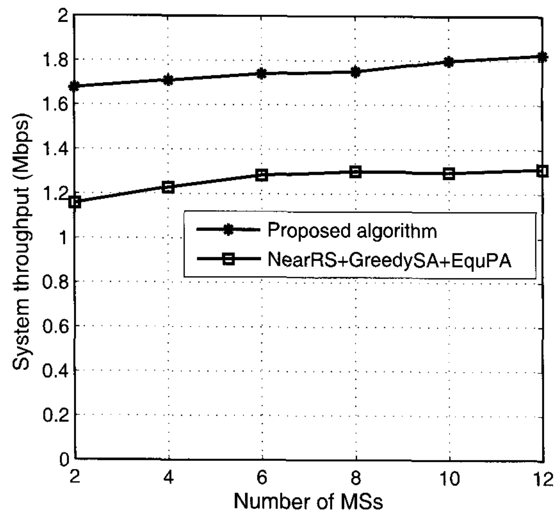



Fig. 2. System throughput versus number of MSs.

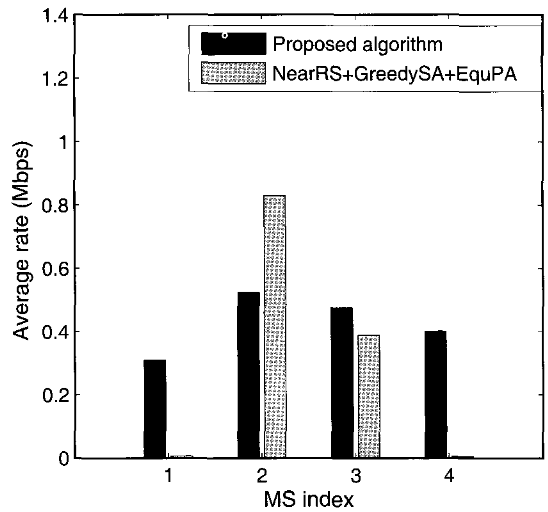



Fig. 4. The average rate of each MS (4 MSs).

following, we will show the performance of the proposed algorithm from two aspects: System throughput and user fairness.

Here, system throughput is defined as the sum rate of all the MSs. The performance of system throughput with respect to the number of MSs is shown in Fig. 2. Obviously, our proposed algorithm outperforms "NearRS+GreedySA+EquPA" algorithm for all number of MSs. It is for the reason that suitable RS-MS pair selection and adaptive power allocation strategy performs better than that of "NearRS" and "EquPA".

To evaluate the fairness, we first define fairness index (FI) as [7]

$$\text{Fairness index} = \frac{\left(\sum_{m=1}^{M} \bar{S}_m\right)^2}{M \sum_{m=1}^{M} (\bar{S}_m)^2} \tag{21}$$

where $\bar{S}_m$ is the average rate of *m*th MS over one hundred slots. The FI is a real number within the interval $[1/M,1]$. If the average rate of each MS is equal, FI is 1. It can be observed in Fig. 3 that the FI of our proposed algorithm is higher than that of "NearRS + GreedySA + EquPA" algorithm for all cases. Meanwhile, our FI decreases slowly with the increase of the number of MSs. In detail, all the values are above 0.9. However, the FI of "NearRS + GreedySA + EquPA" algorithm decreases rapidly. In other words, our proposed algorithm provides much better fairness among MSs, especially when the number of MSs is large. It is because that for greedy subcarrier assignment, as the number of MSs increases, there are no resources allocated to some users located at the edge of the cell due to their poor channel conditions.

Figs. 4 and 5 illustrate the average rate of each MS when there are 4 or 8 MSs in the system. We can see that the average rates vary significantly among MSs for "NearRS + GreedySA + EquPA" algorithm. The reason is that some MSs may occupy too many resources, so that the rates of other MSs become very low, especially for large number of MSs. In Fig. 5, the average rates of 3th and 8th MSs are zero, which means they obtain no resources. Contrastively, the average rates of MSs are much closer for our proposed algorithm. Note that the proposed al-

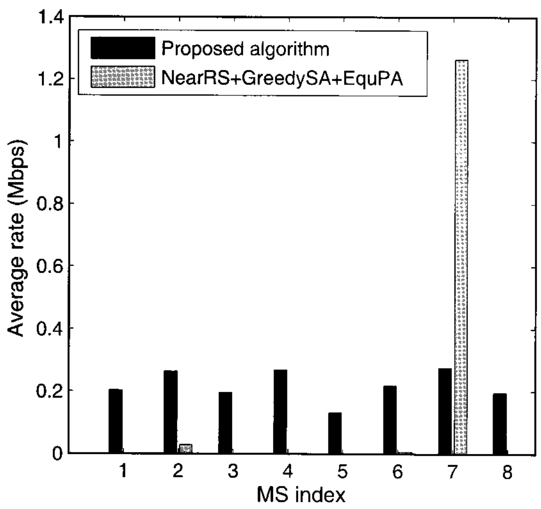



Fig. 5. The average rate of each MS (8 MSs).

gorithm not only achieves better fairness, but also has a good system throughput performance as shown in Fig 2.

## V. CONCLUSION

In this paper, we consider the PF based resource allocation in downlink OFDMA-based relay networks. A joint optimization problem for relay selection, subcarrier assignment and power allocation problem is formulated. Since the problem is NP-complete and cannot be solved directly, we make continuous relaxation and solve it by Lagrangian dual decomposition method. Using KKT conditions, we obtain a near-optimal solution, and a resource allocation algorithm with proportional fairness is proposed. Simulation results show that compared with "NearRS + GreedySA + EquPA" algorithm, our proposed algorithm performs better in terms of both system throughput and user fairness.

## REFERENCES


[1] C. Y. Wong, R. S. Cheng, K. B. Lataief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 1, pp. 1747–1758, Oct. 1999.

[2] Y. B. Lin, T. H. Chiu, and Y. T. Su, "Optimal and near-optimal resource allocation algorithms for OFDMA networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4066–4077, Aug. 2009.

[3] L. You, M. Song, J. Song, Q. Miao, and Y. Zhang, "Adaptive resource allocation in OFDMA relay-aided cooperative cellular networks," in *Proc. IEEE VTC*, May 2008, pp. 1925–1929.

[4] D. Zhang, Y. Wang, and J. Lu, "Resource allocation in OFDMA based cooperative relay networks," in *Proc. IEEE GLOBECOM*, Dec. 2008, pp. 1–5.

[5] T. C. Y. Ng and W. Yu, "Joint optimization of relay strategies and resource allocations in cooperative cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 328–339, Feb. 2007.

[6] Z. Tang and G. Wei, "Resource allocation with fairness consideration in OFDMA-based relay networks," in *Proc. IEEE WCNC*, Apr. 2009, pp. 1–5.

[7] T. D. Nguyen and Y. Han, "A proportional fairness algorithm with QoS provision in downlink OFDMA systems," *IEEE Commun. Lett.*, vol. 10, no. 11, pp. 760–762, Nov. 2006.

[8] L. Zhang, C. Jin, and W. Zhou, "Decomposition proportional fairness algorithm for multiuser OFDM systems," in *Proc. IEEE ICC*, May 2008, pp. 21–25.

[9] Y. J. Zhang and S. C. Liew, "Proportional fairness in multi-channel multi-rate wireless networks-part II: The case of time-varying channels with application to OFDM system", *IEEE Trans. Wireless Commun.*, vol. 7, no. 9, pp. 3457–3467, Sept. 2008.

[10] N. Ruangchaijatupon and Y. Ji, "Integrated approach to proportional-fair resource allocation for multiclass services in an OFDMA system," in *Proc. IEEE GLOBECOM*, Nov. 2009, pp. 1–6.

[11] B. Fan, W. Wang, Y. Lin, L. Huang, and K. Zheng, "Subcarrier allocation for OFDMA relay networks with proportional fair constraint," in *Proc. IEEE ICC*, June 2009, pp. 1–5.

[12] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 2, pp. 171–178, Feb. 2003.

[13] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[14] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, July 2006.

[15] K. Seong, M. Mohseni, and J. M. Cioffi, "Optimal resource allocation for OFDMA downlink systems," in *Proc. IEEE ISIT*, July 2006, pp. 1394–1398.

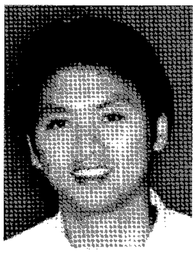
[16] D. P. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1439–1451, Aug. 2006.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[18] S. Boyd, L. Xiao, and A. Mutapcic, *Subgradient Methods*. Stanford, CA: Stanford Univ. Press, Autumn 2003.



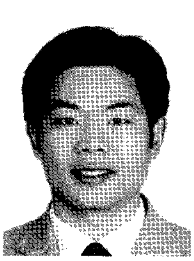
**Chang Liu** was born in 1983. He received his B.S. degree in Electrical Engineering (EE) from University of Science and Technology of China (USTC), Hefei, China, in 2006. Since 2008, he has been working toward the Ph.D. degree in EE at USTC. His research interests include radio resource management, cooperative communication, and network traffic modeling.


**Xiaowei Qin** was born in JiangSu, China, in 1979. He received the B.S. and Ph.D. degrees from the Department of Electrical Engineering and Information Science, University of Science and Technology of China (USTC), Hefei, China, in 2000 and 2008, respectively. Since 2002, he has been a member of staff in Wireless Information Network Laboratory at USTC. His research interests include optimization theory, service modeling in future heterogeneous networks, and self-organized networks.



**Sihai Zhang** received the B.S. degree in Computer Science (CS) from Ocean University of China, in 1996, and the M.S. and Ph.D. degrees in CS from University of Science and Technology of China (USTC), Hefei, China, in 2002 and 2006, respectively. He is currently a Lecturer in the Department of Electronic Engineering and Information Science, USTC. His research interests include evolutionary computation, wireless communication network, and social network.


**Wuyang Zhou** received his B.S. and M.S. degrees from Xidian University, Xi'an, China, in 1993 and 1996, respectively, and Ph.D. degree in Electrical Engineering (EE) from University of Science and Technology of China (USTC), Hefei, China, in 2000. He is currently a Professor in the Department of Electronic Engineering and Information Science, USTC. His research interests include error control coding, radio resource allocation, and wireless networking.