

# Utkin 정리의 단일입력 불확실 적분 선형 시스템에 대한 증명

논 문
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## A Poof of Utkin's Theorem for the SI Uncertain Integral linear Case

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**Abstract** - In this note, a proof of Utkin's theorem is presented for the SI(Single Input) uncertain integral linear case. The invariance theorem with respect to the two transformation methods so called the two diagonalization methods are proved clearly and comparatively for SI uncertain integral linear systems. With respect to the sliding surface transformation, the equation of the sliding mode, the sliding surface is invariant. Both the applied control inputs have the same gains. By means of the two transformation methods the same results can be obtained. Through an illustrative example and simulation study, the usefulness of the main results is verified.

**Key Words** : Variable structure system, Sliding mode control, Proof of Utkin's Theorem, Diagonalization methods, Uncertain integral systems

### 1. Introduction

The sliding mode control(SMC) can provide the effective means to the problem of controlling uncertain dynamical systems under parameter variations and external disturbances[1][2][3]. One of its essential advantages is the robustness of the controlled system to matched parameter uncertainties and matched external disturbances in the sliding mode on the predetermined sliding surface,  $s=0$ [4]. The proper design of the sliding surface can determine the almost output dynamics and its performances[5]. Many design algorithms including the linear(optimal control[6][9], geometric approach[7], pole assignment[8], eigenstructure assignment[9], differential geometric approach[10], integral augmentation[5][13][21], output feedback[15], Lyapunov approach[16][17], Ackermann's formula[18], dynamic sliding surface[19]), and nonlinear[14][20] techniques are reported. However, in these sliding surface design methods so far except [5], the existence condition of the sliding mode on the predetermined sliding surface is not proved.

To take the advantages of the sliding mode on the predetermined sliding surface, the existence condition of

the sliding mode,  $s \cdot \dot{s} < 0$  for the linear SI case and  $s_i \cdot \dot{s}_i < 0, i = 1, 2, \dots, m$  for the linear MI(Multi Input) case is satisfied. Therefore the existence condition of the sliding mode must be proved. For the linear SI case, that proof was reported in [22]. For an uncertain integral augmentation SI case for improving the output performance[5][13][18][21], that proof is hardly presented. For the linear MI case, a few design method was studied, those are hierarchical control methodology[1][3], diagonalization methods[1][3], simplex algorithm[10], Lyapunov approach[16][17][21], and so on. Until now in MIMO VSSs, it is difficult to prove the precise existence condition of the sliding mode on the predetermined sliding surface theoretically, but in [16][17][21], only the results of the derivative of the Lyapunov function is negative, i.e.  $\dot{V} < 0$  is obtained when  $V=1/2s^T s$  is proved. Utkin presented the two methodologies to prove the existence condition of the sliding mode on the sliding surface[1]. It is so called the invariance theorem, i.e the equation of the sliding mode is invariant with respect to the two nonlinear transformations. Those are the control input transformation and sliding surface transformation. so called the two diagonalization methods. The essential feature of these two methods is conversion of a multi-input design problem into m single-input design problems[2]. Those were reviewed in [2]. DeCarlo, Zak, and Matthews tried to prove Utkin's invariance theorem. But, the proof is not clear. In [15], Zak and Hui used the

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control input transformation without uncertainties and disturbance and suggested the sufficient condition for the existence and reachability of the sliding mode as in [1] and [2]. But, they did not explicitly prove for the control input transformation under uncertainties and disturbance. In [17], Su, Drakunov, and Ozguner mentioned the sliding surface transformation, which would diagonalize the control coefficient matrix to the dynamics for  $s$ . But they did not prove the existence condition of the sliding mode on the predetermined sliding surface. However, in [24], the existence condition of the sliding mode on the predetermined sliding surface. In [25], for the linear MI case, the Utkin's theorem is proved.

Until now, for SI uncertain integral systems, a rigorous proof of Utkin's theorem is not reported.

In this paper, a proof of Utkin's theorem is presented for the SI uncertain integral linear case. The invariance theorem with respect to the two transformation methods so called the two diagonalization methods are proved clearly and comparatively for SI uncertain integral linear systems. A design example and simulation study shows usefulness of the main results.

## 2. Main Results of Proof Utkin's Theorem for SI Uncertain Linear Integral System

The invariant theorem of Utkin is as follows[1]:

**Theorem 1:** The equation of the sliding mode is invariant with respect to the two nonlinear transformations, i.e. control input transformation and sliding surface transformation:

$$\begin{aligned} s^*(x) &= H_s(x, t) \cdot s(x) \\ u^*(x) &= H_u(x, t) \cdot u(x) \end{aligned} \quad (1)$$

for  $\det H_s \neq 0$  and  $\det H_u \neq 0$ . For the uncertain linear SI(single input) system:

$$\dot{x} = (A_0 + \Delta A)x + (B_0 + \Delta B)u + \Delta D(t) \quad (2)$$

where  $x \in R^n$  is the state,  $u \in R^1$  is the control input,  $A_0 \in R^{n \times n}$  is the nominal system matrix,  $B_0 \in R^{n \times 1}$  is the nominal input matrix,  $\Delta A$  and  $\Delta B$  are the system matrix uncertainty and input matrix uncertainty, those are bounded, and  $\Delta D(t)$  is bounded external disturbance, respectively.

The integral state is augmented as follows:

$$x_0(t) = \int_0^t x(\tau) d\tau + x_0(0) \quad (3)$$

Then, the integral sliding surface becomes

$$s = C \cdot x + C_0 x_0 = [C_0 \quad C] \bar{x}, \quad \bar{x} = [x_0^T \quad x^T]^T \quad (4)$$

The VSS control input are as follows:

$$u_1 = -K \cdot x - \Delta K \cdot x - G \cdot \text{sign}(s) \quad (5)$$

where  $K$  is a constant gain,  $\Delta K$  is a state dependent switching gain,  $G$  is a switching gain.

1) control input transformation[1][2][15][23]

$$\begin{aligned} u^* &= -H_u \cdot [Kx + \Delta Kx + G\text{sign}(s)], \quad H_u = (CB_0)^{-1} \\ &= -(CB_0)^{-1} \cdot [Kx + \Delta Kx + G\text{sign}(s)] \end{aligned} \quad (6)$$

Then, the real dynamics of  $s$ , i.e. the time derivative of  $s$  is as follows:

$$\begin{aligned} \dot{s} &= \dot{C}x + C_0 \dot{x}_0 \\ &= C(A_0 + \Delta A)x + C(B_0 + \Delta B)u^* + C\Delta D(t) + C_0 \dot{x}_0 \\ &\quad \Delta I = C\Delta B(CB_0)^{-1} \\ &= C(A_0 + \Delta A)x + (I + \Delta I)(-Kx - \Delta Kx - G\text{sign}(s)) \\ &\quad + C\Delta D(t) + C_0 \dot{x}_0 \\ &= CA_0 x + C_0 \dot{x}_0 - Kx + C\Delta Ax - C\Delta IKx - (I + \Delta I)\Delta Kx \\ &\quad + C\Delta D(t) - (I + \Delta I)G\text{sign}(s) \end{aligned} \quad (7)$$

By letting the constant gain

$$K = C_0 + CA_0 \quad (8)$$

then the real dynamics of  $s$  becomes

$$\dot{s} = [C\Delta A - C\Delta IK]x - (I + \Delta I)\Delta Kx + C\Delta D(t) - (I + \Delta I)G\text{sign}(s) \quad (9)$$

If one take the switching gain as design parameters

$$\Delta k_j = \begin{cases} \geq \frac{\max\{C\Delta A - C\Delta IK\}_j}{\min\{I + \Delta I\}} & \text{sign}(s x_j) > 0 \\ \leq \frac{\min\{C\Delta A - C\Delta IK\}_j}{\min\{I + \Delta I\}} & \text{sign}(s x_j) < 0 \end{cases} \quad j = 1, 2, \dots, n \quad (10)$$

$$G = \begin{cases} \geq \frac{\max\{C\Delta D(t)\}}{\min\{I + \Delta I\}} & \text{sign}(s) > 0 \\ \leq \frac{\min\{C\Delta D(t)\}}{\min\{I + \Delta I\}} & \text{sign}(s) < 0 \end{cases} \quad (11)$$

then one can obtain the following equation

$$s \cdot \dot{s} < 0 \quad (12)$$

The existence condition of the sliding mode is proved. The equation of the sliding mode, i.e. the sliding surface is invariant to the control input transformation.

2) sliding surface transformation[1][2][17][24]

$$s^* = (CB_0)^{-1} \cdot s, \quad H_s(x, t) = (CB_0)^{-1} \quad (13)$$

The transformation matrix is selected as  $H_s(x, t) = (CB_0)^{-1}$ . In [2], the proof is not sufficient.

Now, the VSS control input are taken as follows:

$$u_2 = -K \cdot x - \Delta K \cdot x - G \cdot \text{sign}(s^*) \quad (14)$$

The real dynamics of the sliding surface, i.e. the time derivative of  $s^*$  becomes

$$\begin{aligned} \dot{s}^* &= (CB_0)^{-1} \dot{s} = (CB_0)^{-1} \dot{C}x + (CB_0)^{-1} C_0 \dot{x}_0 \\ &= (CB_0)^{-1} C(A_0 + \Delta A)x + (CB_0)^{-1} C(B_0 + \Delta B)u_2 \\ &\quad + (CB_0)^{-1} C\Delta D(t) + (CB_0)^{-1} C_0 \dot{x}_0 \\ &= (CB_0)^{-1} C(A_0 + \Delta A)x + (I + \Delta I)u_2 \\ &\quad + (CB_0)^{-1} C\Delta D(t) + (CB_0)^{-1} C_0 \dot{x}_0, \quad \Delta I = (CB_0)^{-1} C\Delta B \\ &= (CB_0)^{-1} C(A_0 + \Delta A)x + (I + \Delta I)(-Kx - \Delta Kx - G\text{sign}(s^*)) \\ &\quad + (CB_0)^{-1} C\Delta D(t) + (CB_0)^{-1} C_0 \dot{x}_0 \\ &= (CB_0)^{-1} CA_0 x + (CB_0)^{-1} C_0 \dot{x}_0 - Kx + (CB_0)^{-1} C\Delta Ax - \Delta I Kx \\ &\quad - (I + \Delta I)\Delta Kx + (CB_0)^{-1} C\Delta D(t) - (I + \Delta I)G\text{sign}(s^*) \end{aligned} \quad (15)$$

By letting gain

$$K = (CB)^{-1}CA_0 + (CB)^{-1}C_0 \quad (16)$$

then the real dynamics of  $s^*$  becomes

$$\dot{s}^* = [(CB_0)^{-1}C\Delta A - \Delta I K]x - (I + \Delta I)\Delta Kx + (CB_0)^{-1}C\Delta D(t) - (I + \Delta I)G\text{sign}(s^*) \quad (17)$$

In [17], without uncertainty and disturbance, it is mentioned that the sliding surface transformation would diagonalize the control coefficient matrix to the dynamics for  $s$  and the  $\dot{V}(x) < 0$  is proved when  $V(x) = x^T P x > 0$ .

If one take the switching gains as follows:

$$\Delta k_j = \begin{cases} \geq \frac{\max\{(CB_0)^{-1}C\Delta A - \Delta I(CB_0)^{-1}CA_0\}_j}{\min\{I + \Delta I\}} & \text{sign}(s^*x_j) > 0 \\ \leq \frac{\min\{(CB_0)^{-1}C\Delta A - \Delta I(CB_0)^{-1}CA_0\}_j}{\min\{I + \Delta I\}} & \text{sign}(s^*x_j) < 0 \end{cases} \quad j=1,2,\dots,n \quad (18)$$

$$G = \begin{cases} \geq \frac{\max\{(CB_0)^{-1}C\Delta D(t)\}}{\min\{I + \Delta I\}} & \text{sign}(s^*) > 0 \\ \leq \frac{\min\{(CB_0)^{-1}C\Delta D(t)\}}{\min\{I + \Delta I\}} & \text{sign}(s^*) < 0 \end{cases} \quad (19)$$

then

$$s^* \cdot \dot{s}^* < 0 \quad (20)$$

If the sliding mode equation  $s^* = 0$ , then  $s = 0$  since  $CB_0 \neq 0$ . The inverse augment also holds, therefore the both sliding surfaces are equal i.e.,  $s = s^* = 0$ , which completes the proof of Theorem 1.

The sliding mode equation i.e. the sliding surface  $s = 0$  is the same as that of  $s^* = 0$ . To compare the control inputs,  $u_1$  and  $u_2$ , the form is the same but the gains of  $u_1$  are multiplied by  $(CB_0)$ . To compare the control input,  $u^*$  and  $u_2$ , the form and the gain is the same. The both methods equivalently diagonalize the system, so those are called the diagonalization methods.

### 3. Illustrative Example

Consider a third order uncertain system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 \pm 0.3 & 2 \pm 0.6 & 3 \pm 0.9 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \pm 0.1 \\ 3 \pm 0.2 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 5.0 \sin(10t) \end{bmatrix} \quad (21)$$

where the nominal parameter  $A_0$  and  $B_0$  and matched uncertainties  $\Delta A$ ,  $\Delta B$  are

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \Delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \pm 0.3 & \pm 0.6 & \pm 0.9 \end{bmatrix} \\ \Delta B = \begin{bmatrix} 0 \\ \pm 0.1 \\ \pm 0.2 \end{bmatrix}, \Delta D(t) = \begin{bmatrix} 0 \\ 0 \\ 5.0 \sin(10t) \end{bmatrix} \quad (22)$$

The integral state is augmented as follows:

$$x_0(t) = \int_0^t x(\tau) d\tau + x_0(0)$$

The coefficient of the sliding surface is determined as

$$C = [0.2 \ 0.2 \ 1] \quad \text{and} \quad C_0 = [3 \ 1 \ 10] \quad (23)$$

1) control input transformation

$$H_u = (CB_0)^{-1} = ([0.2 \ 0.2 \ 1] \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix})^{-1} = 3.4^{-1} \quad (24)$$

$$u^* = -3.4^{-1}[Kx + \Delta Kx + G\text{sign}(s)]$$

Then, the real dynamics of  $s$ , i.e. the time derivative of  $s$  is as follows:

$$\begin{aligned} \dot{s} &= \dot{C}x + C_0 \dot{x}_0 \\ &= CA_0x + C_0x - Kx + C\Delta Ax - C\Delta IKx \\ &\quad - (I + \Delta I)\Delta Kx + C\Delta D(t) - (I + \Delta I)G\text{sign}(s) \end{aligned} \quad (25)$$

where

$$\begin{aligned} \Delta I &= C\Delta B(CB_0)^{-1} = [0.2 \ 0.2 \ 1] \begin{bmatrix} 0 \\ \pm 0.1 \\ \pm 0.2 \end{bmatrix} 3.4^{-1} \\ &= \pm 0.22/3.4 = \pm 0.0647 \end{aligned} \quad (26)$$

By letting the constant gain

$$\begin{aligned} K &= CA_0 + C_0 = [0.2 \ 0.2 \ 1] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} + [3 \ 1 \ 10] \\ &= [4 \ 3.2 \ 13.2] \end{aligned} \quad (27)$$

then the real dynamics of  $s$  becomes

$$\dot{s} = [C\Delta A - C\Delta IK]x - (I + \Delta I)\Delta Kx + C\Delta D(t) - (I + \Delta I)G\text{sign}(s) \quad (28)$$

If one take the switching gain as design parameters

$$\begin{aligned} \Delta k_1 &= \begin{cases} 1.18 & \text{if } sx_1 > 0 \\ -1.18 & \text{if } sx_1 < 0 \end{cases} & \Delta k_2 &= \begin{cases} 1.304 & \text{if } sx_2 > 0 \\ -1.304 & \text{if } sx_2 < 0 \end{cases} \\ \Delta k_3 &= \begin{cases} 3.804 & \text{if } sx_3 > 0 \\ -3.804 & \text{if } sx_3 < 0 \end{cases} & G_3 &= \begin{cases} 5.35 & \text{if } s > 0 \\ -5.35 & \text{if } s < 0 \end{cases} \end{aligned} \quad (29)$$

then one can obtain the following equation

$$s \cdot \dot{s} < 0 \quad (30)$$

The existence condition of the sliding mode is proved. The equation of the sliding mode, i.e. the sliding surface is invariant to the control input transformation.

The simulation is carried out under 1[msec] sampling time and with  $x(0) = [5 \ 0 \ 0]^T$  initial state.

Fig. 1 shows the three output responses,  $X_1$ ,  $X_2$  and  $X_3$  by  $u^*$  with  $s$ .

The controlled system slides from the beginning as shown in the output responses  $X_2$  and  $X_3$  in Fig. 1.

2) sliding surface transformation

$$s^* = (CB_0)^{-1} \cdot s, \quad H_s(x, t) = (CB_0)^{-1} = 3.4^{-1} \quad (31)$$

Now, the VSS control input are taken as follows:

$$u_2 = -K \cdot x - \Delta K \cdot x - G \cdot \text{sign}(s^*) \quad (32)$$

The real dynamics of the sliding surface, i.e. the time derivative of  $s^*$  becomes

$$\begin{aligned} \dot{s}^* &= (CB_0)^{-1}\dot{s} = (CB_0)^{-1}C\dot{x} \\ &= (CB_0)^{-1}CA_0x - Kx + (CB_0)^{-1}C\Delta Ax - \Delta Kx \\ &\quad - (I + \Delta I)\Delta Kx + (CB_0)^{-1}C\Delta D(t) - (I + \Delta I)G\text{sign}(s^*) \end{aligned} \quad (33)$$

By letting gain

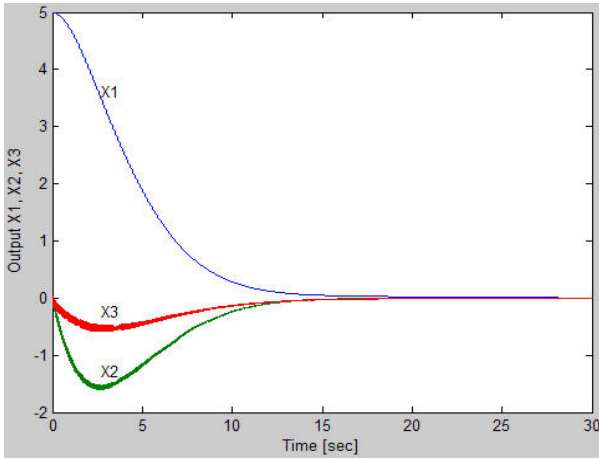


Fig. 1 Output responses,  $X_1$ ,  $X_2$ , and  $X_3$  by input diagonalization

$$\begin{aligned} K &= (CB_0)^{-1}CA_0 + (CB_0)^{-1}C_0 \\ &= 3.4^{-1}[1 \ 2.2 \ 3.2] + 3.4^{-1}[3 \ 1 \ 10] \\ &= [1.1765 \ 0.9412 \ 3.8824] \end{aligned} \quad (34)$$

then the real dynamics of  $s^*$  becomes

$$\dot{s}^* = [(CB_0)^{-1}C\Delta A - \Delta I K]x - (I + \Delta I)\Delta Kx + (CB_0)^{-1}C\Delta D(t) - (I + \Delta I)G\text{sign}(s^*) \quad (35)$$

$$\begin{aligned} \Delta I &= (CB_0)^{-1}C\Delta B \\ &= 3.4^{-1}[0.2 \ 0.2 \ 1] \begin{bmatrix} 0 \\ \pm 0.1 \\ \pm 0.2 \end{bmatrix} \\ &= \pm 2.2/3.4 = \pm 0.0647 \end{aligned} \quad (36)$$

If one take the switching gains as follows:

$$\begin{aligned} \Delta k_1 &= \begin{cases} 0.115 & \text{if } s^*x_1 > 0 \\ -0.115 & \text{if } s^*x_1 < 0 \end{cases} & \Delta k_2 &= \begin{cases} 0.234 & \text{if } s^*x_2 > 0 \\ -0.234 & \text{if } s^*x_2 < 0 \end{cases} \\ \Delta k_3 &= \begin{cases} 0.349 & \text{if } s^*x_3 > 0 \\ -0.349 & \text{if } s^*x_3 < 0 \end{cases} & G_3 &= \begin{cases} 1.573 & \text{if } s > 0 \\ -1.573 & \text{if } s < 0 \end{cases} \end{aligned} \quad (37)$$

then

$$s^* \cdot \dot{s}^* < 0 \quad (38)$$

if  $s^*=0$ , then  $\dot{s}^*=0$ . The inverse augment also holds. The switching gains in (36) can be obtained also from (28) by multiplying  $(CB_0)^{-1} = 3.4^{-1}$ .

The simulation is carried out under 1[msec] sampling time and with  $x(0) = [5 \ 0 \ 0]^T$  initial condition. Fig. 2 shows the three output responses,  $X_1$ ,  $X_2$  and  $X_3$ . Those are almost identical to Fig. 1 because the sliding surface  $s=0=s^*$  is equal and the continuous gains and discontinuous gains of the both controls,  $u^*$  and  $u_2$  are equal.

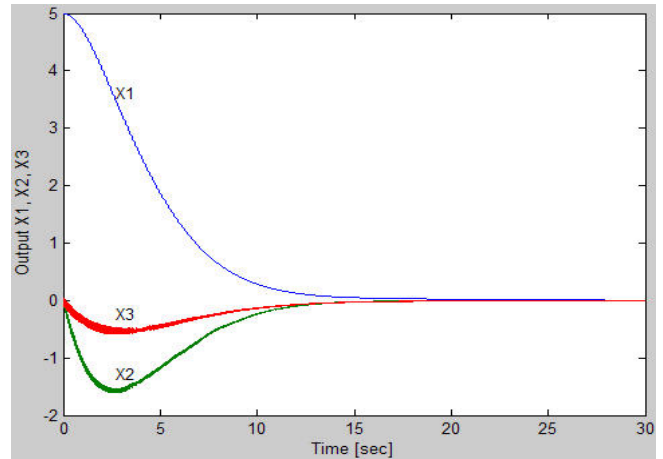


Fig. 2 Output responses,  $X_1$ ,  $X_2$ , and  $X_3$  by sliding surface diagonalization

#### 4. Conclusions

In this note, the theorem of Utkin's is proved rigorously for SI uncertain integral linear systems. The invariance theorem of the two diagonal methods, i.e., the control input transformation and sliding surface transformation is proved clearly and comparatively. Therefore, the equation of the sliding mode, i.e., the sliding surface is invariant with respect to the two diagonalization methods. These two methods diagonalize the input system of the real sliding dynamics of the sliding surface  $s$  or  $s^*$  so that the existence condition of the sliding mode on the predetermined sliding surface is easily proved. During the proof of Utkin's theorem, the design rules of both the control inputs are proposed. Through an illustrative example and simulation study, the effectiveness of the proposed main results is verified.

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