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A Note On Fuzzy r-*M* Precontinuity And Fuzzy r-Minimal Compactness On Fuzzy r-Minimal Spaces

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요 약

fuzzy r-minimal 공간에서 fuzzy r-M^* preopen 함수의 개념을 소개하며 fuzzy r-M pre-연속 함수, fuzzy r-M^* preopen 함수와 여러 종류의 fuzzy r-minimal 컴팩트성과의 관계에 대하여 조사한다.

Abstract

In this paper, we introduce and study the concept of fuzzy r-M^*preopen mappings between fuzzy r-minimal spaces. We also investigate the relationships among fuzzy r-M precontinuous mappings, fuzzy r-M^*-preopen mappings and several types of fuzzy r-minimal compactness.

Key Words : fuzzy r-minimal preopen, fuzzy r-M continuous, fuzzy r-M precontinuous, fuzzy r-minimal precompact, fuzzy r-M^*-preopen mappings

1. Intorduction

The concept of fuzzy set was introduced by Zadeh [7]. Chang [1] defined fuzzy topological spaces using fuzzy sets. In [2], Chattopadhyay, Hazra and Samanta introduced the smooth topological space which is a generalization of a fuzzy topological space. In [5], we introduced the concept of fuzzy r-minimal space which is an extension of the smooth topological space. The concepts of fuzzy r-open sets and fuzzy r-M continuous mappings are also introduced and studied. In [3], We introduced the concepts of fuzzy r-minimal preopen sets and fuzzy r-M precontinuous mappings, which are generalizations of fuzzy r-minimal open sets and fuzzy r-M continuous mappings, respectively. Yoo et al. introduced the concepts of fuzzy r-minimal compactness, almost fuzzy r-minimal compactness and nearly fuzzy r-minimal compactness on fuzzy r-minimal spaces in [6]. In this paper, we introduce and study the concept of fuzzy r-M^*-preopen mapping between fuzzy r-minimal spaces. We also investigate the relationships among fuzzy r-M precontinuous mappings, fuzzy r-M^*-preopen mappings and several types of fuzzy r-minimal compactness. In particular, in Theorem 3.11, we will show that: If a mapping $f:(X,M)\to(Y,N)$ is fuzzy r-M precontinuous and fuzzy r-M^*-preopen on two r-FMS's, and If A is a nearly fuzzy r-minimal precompact set, then $f(A)$ is a nearly fuzzy r-minimal compact set.

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2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a *fuzzy set* [6] of X. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on X with value 0 and 1, respectively. For any $A\in I^X$, A^c denotes the complement $\tilde{1}-A$. All other notations are standard notations of fuzzy set theory.

A *fuzzy point* x_α in X is a fuzzy set x_α is defined as follows

$$x_\alpha(y)=\begin{cases}\alpha, & \text{if } y=x,\\ 0, & \text{if } y\neq x.\end{cases}$$

A fuzzy point x_α is said to belong to a fuzzy set A in X, denoted by $x_\alpha\in A$, if $\alpha\le A(x)$ for $x\in X$.

A fuzzy set A in X is the union of all fuzzy points which belong to A.

Let $f:X\to Y$ be a mapping and $A\in I^X$ and $B\in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y)=\begin{cases}sup_{z\in f^{-1}(y)}A(z), & \text{if} f^{-1}(y)\neq\varnothing,\\ 0, & \text{otherwise},\end{cases}$$

for $y \in Y$ and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x)),\ x \in X$.

A *smooth topology* [2, 4] on X is a map $T: I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(A_1 \cap A_2) \geq T(A_1) \wedge T(A_2)$ for $A_1,\ A_2 \in I^X$.

(3) $T(\cup A_i) \geq \wedge T(A_i)$ for $A_i \in I^X$.

The pair (X, T) is alled a *smooth topological space.*

Let X be a nonempty set and $r \in (0,1] = I_0$. A fuzzy family $M: I^X \to I$ on X is said to have a *fuzzy r-minimal structure* [5] if the family

$$M_r = \{A \in I^X : M(A) \geq r\}$$

contains $\tilde{0}$ and $\tilde{1}$.

Then the (X,M) is called a *fuzzy r-minimal space* (simply r-*FMS*) [5]. Every member of M_r is called a *fuzzy r-minimal open set.* A fuzzy set A is called a *fuzzy r-minimal closed set* if the complement of A (simply, A^c) is a fuzzy r-minimal open set.

Let (X,M) be an r-FMS and $r \in (0,1] = I_0$. The fuzzy r-minimal closure of A, denoted by $\mathrm{mC}(A,r)$, is defined as

$$\mathrm{mC}(A,r) = \cap\{B \in I^X:\ B^c \in M_r \text{ and } A \subseteq B\}[4].$$

The fuzzy r-minimal interior of A, denoted by $\mathrm{mI}(A, r)$, is defined as

$$\mathrm{mI}(A,r) = \cup\{B \in I^X:\ B \in M_r \text{ and } B \subseteq A\}[4].$$

Theorem 2.1 ([5]). Let (X,M) be an r-*FMS* and $A,B \in I^X$. Then the following properties hold:

(1) $\mathrm{mI}(A,r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $\mathrm{mI}(A,r) = A$.

(2) $A \subseteq \mathrm{mC}(A,r)$ and if A is a fuzzy r-minimal closed set, then $\mathrm{mC}(A,r) = A$.

(3) If $A \subseteq B$, then $\mathrm{mI}(A,r) \subseteq \mathrm{mI}(B,r)$ and $\mathrm{mC}(A,r) \subseteq \mathrm{mC}(B,r)$.

(4) $\mathrm{mI}(A \cap B,r) \subseteq \mathrm{mI}(A,r) \cap \mathrm{mI}(B,r)$ and $\mathrm{mC}(A,r) \cup \mathrm{mC}(B,r) \subseteq \mathrm{mC}(A \cup B,r)$.

(5) $\mathrm{mI}(\mathrm{mI}(A,r),r) = \mathrm{mI}(A,r)$ and $\mathrm{mC}(\mathrm{mC}(A,r),r) = \mathrm{mC}(A,r)$.

(6) $\tilde{1} - \mathrm{mC}(A,r) = \mathrm{mI}(\tilde{1} - A,r)$ and $\tilde{1} - \mathrm{mI}(A,r) = \mathrm{mC}(\tilde{1} - A, r)$.

Let (X,M) be an r-*FMS* and $A \in I^X$. Then a fuzzy set A is called a *fuzzy r-minimal preopen set* [3] in X if $A \subseteq \mathrm{mI}(\mathrm{mC}(A, r),r)$.

A fuzzy set A is called a *fuzzy r-minimal spre-closed* set if the complement of A is fuzzy r-minimal preopen. We showed that any union of fuzzy r-minimal preopen sets is fuzzy r-minimal preopen [3].

For $A \in I^X$, $\mathrm{mpC}(A,r)$, $\mathrm{mpI}(A,r)$, respectively, are defined as the following:

$$\mathrm{mpC}(A,r) = \cap\{F \in I^X:\ A \subseteq F,\ F \text{ is fuzzy } r\text{-minimal preclosed}\}.$$

$$\mathrm{mpI}(A,r) = \cup\{B \in I^X:\ B \subseteq A,\ B \text{ is fuzzy } r\text{-minimal precopen}\}.$$

Theorem 2.2 ([3]). Let (X,M) be an r-*FMS* and A, $F \in I^X$. Then

(1) $\mathrm{mpI}(A,r) \subseteq A \subseteq \mathrm{mpC}(A,r)$.

(2) If $A \subseteq B$, then $\mathrm{mpI}(A,r) \subseteq \mathrm{mpI}(B,r)$ and $\mathrm{mpC}(A, r) \subseteq \mathrm{mpC}(B,r)$.

(3) A is fuzzy r-minimal preopen iff $\mathrm{mpI}(A,r) = A$.

(4) F is fuzzy r-minimal preclosed iff $\mathrm{mpC}(F,r) = F$.

(5) $\mathrm{mpI}(\mathrm{mpI}(A,r),r) = \mathrm{mpI}(A,r)$ and $\mathrm{mpC}(\mathrm{mpC}(A,r), r) = \mathrm{mpC}(A,r)$.

(6) $\mathrm{mpC}(\tilde{1} - A,r) = \tilde{1} - \mathrm{mpI}(A,r)$ and $\mathrm{mpI}(\tilde{1} - A,r) = \tilde{1} - \mathrm{mpC}(A,r)$.

3. Main Results

We recall the concepts of several types of fuzzy r-minimal compactness introduced in [6]. Let (X,M) be an r-FMS and $\Lambda = \{A_i \in I^X: i \in J\}$. Λ is called a *fuzzy r-minimal cover* if $\cup\{A_i: i \in J\} = \tilde{1}$. It is a *fuzzy r-minimal open cover* if each A_i is a fuzzy r-minimal open set. A subcover of a fuzzy r-minimal cover Λ is a subfamily of it which also is a fuzzy r-minimal cover. A fuzzy set A in X is said to be *fuzzy r-minimal compact* if every fuzzy r-minimal open cover $\Lambda = \{A_i \in M_r: i \in J\}$ of A has a finite subcover. A fuzzy set A in X is said to be *almost fuzzy r-minimal compact* if for every fuzzy r-minimal open cover $\Lambda = \{A_i \in I^X: i \in J\}$ of A, there exists $J_0 = \{j_1,\ j_2,\ \cdots,\ j_n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} \mathrm{mC}(A_i,r)$. A fuzzy set A in X is said to be *nearly fuzzy r-minimal compact* if for every fuzzy r-minimal open cover $\Lambda = \{A_i: i \in J\}$ of A, there exists $J_0 = \{j_1,\ j_2,\ \cdots,\ j_n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} \mathrm{mI}(\mathrm{mC}(A_i,r), r)$.

Definition 3.1. Let (X,M) be an r-FMS and $\Lambda = \{A_i \in I^X: i \in J\}$ a fuzzy r-minimal cover. It is a *fuzzy r-minimal preopen cover* if each A_i is a fuzzy r-minimal preopen set.

Definition 3.2. Let (X,M) be an r-FMS. A fuzzy set A in X is said to be

(1) *fuzzy r-minimal precompact* if every fuzzy r-minimal preopen cover $\Lambda = \{A_i \in M_r: i \in J\}$ of A has a finite subcover;

(2) *almost fuzzy r-minimal precompact* if for every fuzzy r-minimal preopen cover $\Lambda = \{A_i \in I^X: i \in J\}$ of A, there exists $J_0 = \{j_1,\ j_2,\ \cdots,\ j_n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} \mathrm{mpC}(A_i,r)$;

(3) *nearly fuzzy r-minimal precompact* if for every fuzzy r-minimal preopen cover $\Lambda = \{A_i \in I^X: i \in J\}$ of A, there exists $J_0 = \{j_1,\ j_2,\ \cdots,\ j_n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} \mathrm{mpI}(\mathrm{mpC}(A_i,r),r)$.

Let (X,M) and (Y,N) be r-FMS's. Then a mapping $f:(X,M)\to(Y,N)$ is said to be *fuzzy r-M precontinuous* [3] if for each point x_α and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal preopen set U containing x_α such that $f(U)\subseteq V$.

Theorem 3.3 ([3]). Let $f:(X,M)\to(Y,N)$ be a mapping on fuzzy r-minimal spaces (X,M) and (Y,N). Then the following are equivalent:

(1) f is fuzzy r-M precontinuous.

(2) $f^{-1}(V)$ is a fuzzy r-minimal precopen set for each fuzzy r-minimal open set V in Y.

(3) $f^{-1}(B)$ is a fuzzy r-minimal precclosed set for each fuzzy r-minimal closed set B in Y.

(4) $f(\mathrm{mpC}(A,r))\subseteq \mathrm{mC}(f(A),r)$ for $A\in I^X$.

(5) $\mathrm{mpC}(f^{-1}(B),r)\subseteq f^{-1}(\mathrm{mC}(B,r))$ for $B\in I^Y$.

(6) $f^{-1}(\mathrm{mI}(B,r))\subseteq \mathrm{mpI}(f^{-1}(B),r)$ for $B\in I^Y$.

Theorem 3.4. Let $f:(X,M)\to(Y,N)$ be a fuzzy r-M precontinuous mapping on two r-FMS's. If A is a fuzzy r-minimal precompact set, then $f(A)$ is fuzzy r-minimal compact.

Proof. Let $\{B_i\in I^Y:i\in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then since f is a fuzzy r-M precontinuous mapping, $\{f^{-1}(B_i):i\in J\}$ is a fuzzy r-minimal preopen cover of A in X. Since A is fuzzy r-minimal precompact, there exists a finite subset $J_0=\{j_1, j_2, \cdots, j_n\}\subseteq J$ such that $A\subseteq \cup_{i\in J_0} f^{-1}(B_i)$. It implies $f(A)\subseteq \cup_{i\in J_0} B_i$ for the finite subset J_0 of J, and hence $f(A)$ is fuzzy r-minimal compact.

Theorem 3.5. Let $f:(X,M)\to(Y,N)$ be a fuzzy r-M precontinuous mapping on two r-FMS's. If A is an almost fuzzy r-minimal precompact set, then $f(A)$ is almost fuzzy r-minimal compact.

Proof. Let $\{B_i\in I^Y:i\in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i):i\in J\}$ is a fuzzy r-minimal preopen cover of A in X. By almost fuzzy r-minimal precompact, there exists a finite subset $J_0=\{j_1, j_2, \cdots, j_n\}\subseteq J$ such that $A\subseteq \cup_{i\in J_0} \mathrm{mpC}(f^{-1}(B_i),r)$.

From Theorem 3.3 (5), it follows

$$\begin{aligned}\cup_{i\in J_0} \mathrm{mpC}(f^{-1}(B_i,\ r)) &\subseteq \cup_{i\in J_0} f^{-1}(\mathrm{mC}(B_i,\ r))\\ &= f^{-1}(\cup_{i\in J_0}\mathrm{mC}(B_i,\ r)).\end{aligned}$$

So $f(A)\subseteq \cup_{i\in J_0}\mathrm{mC}(B_i,r)$ and $f(A)$ is almost fuzzy r-minimal compact.

Definition 3.6. Let $f:(X,M)\to(Y,N)$ be a fuzzy mapping on two r-FMS's. Then f is said to be *fuzzy r-M^*-preopen* if for each fuzzy r-minimal preopen set U in X, $f(U)$ is fuzzy r-minimal open.

Theorem 3.7. Let $f:(X,M)\to(Y,N)$ be a fuzzy mapping on two r-FMS's.

(1) f is fuzzy r-M^*-preopen.

(2) $f(\mathrm{mpI(A},r))\subseteq \mathrm{mI}(f(\mathrm{A}),r)$ for $A\in I^X$.

(3) $\mathrm{mpI}f^{-1}(\mathrm{B}),r)\subseteq f^{-1}(\mathrm{mI(B},r)$ for $B\in I^Y$.

Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1)$\Rightarrow$ (2) For $A\in I^X$,

$$\begin{aligned}f(\mathrm{mpI}(A),r) &= f(\cup\{B\in I^X: B\subseteq A,\ B \text{ is fuzzy } r\text{-preopen}\})\\ &= \cup\{f(B)\in I^Y: f(B)\subseteq f(A),\ f(B) \text{ is fuzzy } r\text{-minimal open}\}\\ &\subseteq \cup\{U\in I^Y: U\subseteq f(A),\ U \text{ is fuzzy } r\text{-minimal open}\}\\ &= \mathrm{mI}(f(A),r)\end{aligned}$$

Hence $f(\mathrm{mpI(A)},r)\subseteq \mathrm{mI}(f(A),r)$.

(2) $\Rightarrow$ (3) For $B\in I^Y$, from (3) it follows that $f(\mathrm{mpI}(f^{-1}(\mathrm{B}),r))\subseteq \mathrm{mI}(f(f^{-1}(\mathrm{B})),r)\subseteq \mathrm{mI(B},r)$.

This implies (3).

Similarly, we get the implication (3) $\Rightarrow$ (2).

Example 3.8. Let $X=I$, and let N be the set of all natural numbers. For $n\in N$, consider each fuzzy set

$$A_n(x)=\frac{n}{n+1}x,\ x\in I.$$

And

$$A(x)=x,\ x\in I.$$

Define $$M(\sigma)=\begin{cases}\frac{n}{n+1}, & \text{if } \sigma=A_n,\\ 1, & \text{if } \sigma=\tilde{0},\tilde{1},\\ 0, & otherwise.\end{cases}$$

$$N(\beta)=\begin{cases}\frac{1}{3}, & \text{if } \beta=A,\\ \frac{2}{3}, & otherwise.\end{cases}$$

Consider the identity mapping $f:(X,M)\to(X,N)$. Then f satisfies the condition (2) of the above Theorem 3.7. For the fuzzy $\frac{1}{2}$-minimal preopen set A, $f(A)=A$ is not fuzzy $\frac{1}{2}$-minimal open in (X,N). Therefore, f is not fuzzy $\frac{1}{2}$-M^*-preopen.

Let X be a nonempty set and $M:I^X\to I$ a fuzzy family on X. The fuzzy family M is said to have the property (U) [5] if for $A_i\in M$ $(i\in J)$,

$$M(\cup A_i)\geq \wedge M(A_i).$$

Theorem 3.9 ([5]). Let (X,M) be an r-FMS with the property (U). Then

(1) $\mathrm{mI}(A,r)=A$ if and only if $A\in M_r$ for $A\in I^X$.

(2) $\mathrm{mC}(A,r)=A$ if and only if $\tilde{1}-A\in M_r$ for $A\in I^X$.

Corollary 3.10. Let $f:(X,M)\to(Y,N)$ be a fuzzy mapping on two r-FMS's. If N has property (U), then the following are equivalent:

(1) f is fuzzy r-M^*-preopen.

(2) $f(\mathrm{mpI(A},r))\subseteq \mathrm{mI}(f(A),r)$ for $A\in I^X$.

(3) $\mathrm{mpI}(f^{-1}(B),r)\subseteq f^{-1}(\mathrm{mI}(B,r))$ for $B\in I^Y$.

Theorem 3.11. Let a mapping $f:(X,M)\rightarrow(Y,N)$ be fuzzy r-M precontinuous and fuzzy r-M^*-preopen on two r-FMS's. If A is a nearly fuzzy r-minimal pre-compact set, then $f(A)$ is a nearly fuzzy r-minimal compact set.

Proof. Let $\{B_i\in I^Y : i\in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i\in J\}$ is a fuzzy r-minimal preopen cover of A in X. By nearly fuzzy r-minimal precompactness, there exists $J_0=\{j_1, j_2, \cdots, j_n\}\subseteq J$ such that $A\subseteq\cup_{i\in J_0}\text{mpI}(\text{mpC}(f^{-1}(B_i),r),r)$.

From Theorem 3.3 (5) and Theorem 3.7 (2), it follows

$$
\begin{aligned}
f(\text{A}) &\subseteq \cup_{i\in J_0} f(\text{mpI}(\text{mpC}(f^{-1}(B_i),r),r))\\
&\subseteq \cup_{i\in J_0} \text{mI}(f(\text{mpC}(f^{-1}(B_i),r)),r)\\
&\subseteq \cup_{i\in J_0} \text{mI}(f(f^{-1}(\text{mC}(B_i,r))),r)\\
&\subseteq \cup_{i\in J_0} \text{mI}(\text{mC}(B_i,r),r).
\end{aligned}
$$

Hence $f(A)$ is a nearly fuzzy r-minimal compact set.

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