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A Generalized Ratio-cum-Product Estimator of Finite Population Mean in Stratified Random Sampling

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Abstract

This paper suggests a ratio-cum product estimator of a finite population mean using information on the coefficient of variation and the fcoefficient of kurtosis of auxiliary variate in stratified random sampling. Bias and MSE expressions of the suggested estimator are derived up to the first degree of approximation. The suggested estimator has been compared with the combined ratio estimator and several other estimators considered by Kadilar and Cingi (2003). In addition, an empirical study is also provided in support of theoretical findings.

Keywords: Finite population mean, coefficient of variation, correlation coefficient, stratified random sampling, bias, mean squared error.

1. Introduction

The problem of the estimation of population parameters like mean, variance, and ratio of two population means are common in agriculture, economics, medicine, and population studies. The use of auxiliary information has been applied for improving the efficiencies of the estimators of population parameter(s) irrespective of sampling design. Ratio, product and regression methods of estimation are good examples in this context. Cochran (1940) used auxiliary information at the estimation stage and proposed a ratio estimator for the population mean. A ratio estimator is preferred when the correlation coefficient between the study variate and the auxiliary variate is positive. Robson (1957) defined a product estimator that was revisited by Murthy (1964). The product estimator is used when the correlation between the study variate and the auxiliary is negative.

Sisodia and Dwivedi (1981), Pandey and Dubey (1988), Upadhyaya and Singh (1999), Singh *et al.* (2004) and Singh and Tailor (2005) later used known values of various parameters of an auxiliary variate in simple random sampling.

A combined ratio and product are some basic estimators of the population mean that uses information on the auxiliary variate in stratified random sampling. Kadilar and Cingi (2003) defined, Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) and Singh *et al.* (2004) estimators in stratified random sampling.

Singh and Espejo (2003) suggested a ratio-cum-product estimator using scalar α. This estimator posses a nice property that for $\alpha = 1$, it becomes useful for a positive correlation while for $\alpha = 0$, it is quite effective in a negative correlation.

Singh and Tailor (2005) and Tailor and Sharma (2009) proposed ratio-cum-product estimator of a finite population mean in simple random sampling. Tailor (2009) defined a ratio-cum product estimator in stratified random sampling.

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Researchers are motivated to work in this direction and we propose a ratio-cum-product estimator using information on the coefficient of variation and the coefficient of the kurtosis of auxiliary variate in stratified random sampling.

Let $U = (U_1, U_2, \ldots, U_N)$ be a finite population of size N and it is divided into k homogeneous strata of size N_h $(h = 1, 2, \ldots, k)$. A sample of size n_h is drawn from each stratum using the simple random sampling without replacement.

Let y be the study variate taking values y_{hi} (i^{th} observation from h^{th} stratum) and x_{hi} be the auxiliary variate taking values x_{hi}.

Let $\bar{y}_{st} = \sum_{h=1}^{k} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{k} W_h \bar{x}_h$ be the unbiased estimators of the population mean $\bar{Y}$ and $\bar{X}$ of the study variate and auxiliary variate respectively, where

$$W_h = \frac{N_h}{N} \quad \text{: is the weight of } h^{th} \text{ stratum,}$$

$$\bar{y}_h = \frac{1}{n_h} \sum_{j=1}^{n_h} \bar{y}_{hj} \quad \text{: sample mean of the study variate } y \text{ for } h^{th} \text{ stratum,}$$

$$\bar{x}_h = \frac{1}{n_h} \sum_{j=1}^{n_h} \bar{x}_{hj} \quad \text{: sample mean of the auxiliary variate } x \text{ for } h^{th} \text{ stratum.}$$

The usual combined ratio and product estimators of population mean $\bar{Y}$ respectively are

$$\hat{\bar{Y}}_{RST} = \bar{y}_{st} \left(\frac{\bar{X}}{\bar{x}_{st}} \right), \tag{1.1}$$

$$\hat{\bar{Y}}_{PST} = \bar{y}_{st} \left(\frac{\bar{x}_{st}}{\bar{X}} \right). \tag{1.2}$$

The mean squared error(MSE) expressions of the combined ratio and product estimators up to the first degree of approximation are

$$\text{MSE}\left(\hat{\bar{Y}}_{RST}\right) = \sum_{h=1}^{k} W_h^2 \gamma_h \left(S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh} \right), \tag{1.3}$$

$$\text{MSE}\left(\hat{\bar{Y}}_{PST}\right) = \sum_{h=1}^{k} W_h^2 \gamma_h \left(S_{yh}^2 + R^2 S_{xh}^2 + 2RS_{yxh} \right), \tag{1.4}$$

where

$$R = \frac{\bar{Y}}{\bar{X}}, \qquad \gamma_h = \left(\frac{N_h - n_h}{N_h n_h} \right), \qquad S_{yh}^2 = \frac{1}{N_h - 1} \sum_{j=1}^{N_h} \left(y_{hj} - \bar{Y}_h \right)^2, \qquad S_{xh}^2 = \frac{1}{N_h - 1} \sum_{j=1}^{N_h} \left(x_{hj} - \bar{X}_h \right)^2$$

$$\text{and} \quad S_{yxh} = \frac{1}{N_h - 1} \sum_{j=1}^{N_h} \left(y_{hj} - \bar{Y}_h \right) \left(x_{hj} - \bar{X}_h \right).$$

Sisodia and Dwivedi (1981) suggested a ratio estimator of population mean $\bar{Y}$ using the coefficient of variation of auxiliary variate (C_x) as

$$\hat{\bar{Y}}_{SD} = \bar{y} \left[\frac{\bar{X} + C_x}{\bar{x} + C_x} \right], \tag{1.5}$$

Singh *et al.* (2004) proposed another ratio estimator for $\bar{Y}$, using the coefficient of kurtosis ($\beta_2(x)$) of the auxiliary variate x as

$$\hat{\bar{Y}}_{SE} = \bar{y}\left[\frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)}\right], \tag{1.6}$$

Upadhyaya and Singh (1999) used information on the coefficient of variation (C_x) and the coefficient of the kurtosis ($\beta_2(x)$) of the auxiliary variate x and suggested two estimators for $\bar{Y}$

$$\hat{\bar{Y}}_{US1} = \bar{y}\left[\frac{\bar{X}\beta_2(x) + C_x}{\bar{x}\beta_2(x) + C_x}\right] \tag{1.7}$$

and

$$\hat{\bar{Y}}_{US2} = \bar{y}\left[\frac{\bar{x}C_x + \beta_2(x)}{\bar{X}C_x + \beta_2(x)}\right], \tag{1.8}$$

Kadilar and Cingi (2003) defined $\hat{\bar{Y}}_{SD}$, $\hat{\bar{Y}}_{SE}$, $\hat{\bar{Y}}_{US1}$ and $\hat{\bar{Y}}_{US2}$ in stratified random sampling respectively as

$$\hat{\bar{Y}}_{SD}^{ST} = \bar{y}_{st}\left[\frac{W_h(\bar{X}_h + C_{xh})}{\sum_{h=1}^{k} W_h(\bar{x}_h + C_{xh})}\right], \tag{1.9}$$

$$\hat{\bar{Y}}_{SE}^{ST} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{k} W_h(\bar{X}_h + \beta_{2h}(x))}{\sum_{h=1}^{k} W_h(\bar{x}_h + \beta_{2h}(x))}\right], \tag{1.10}$$

$$\hat{\bar{Y}}_{US1}^{ST} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{k} W_h(\bar{X}_h\beta_{2h}(x) + C_{xh})}{\sum_{h=1}^{k} W_h(\bar{x}_h\beta_{2h}(x) + C_{xh})}\right], \tag{1.11}$$

$$\hat{\bar{Y}}_{US2}^{ST} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{k} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x))}{\sum_{h=1}^{k} W_h(\bar{x}_h C_{xh} + \beta_{2h}(x))}\right]. \tag{1.12}$$

To the first degree of approximation mean squared errors of $\hat{\bar{Y}}_{SD}^{ST}$, $\hat{\bar{Y}}_{SE}^{ST}$, $\hat{\bar{Y}}_{US1}^{ST}$ and $\hat{\bar{Y}}_{US2}^{ST}$ respectively are

$$\text{MSE}\left(\hat{\bar{Y}}_{SD}^{ST}\right) = \sum_{h=1}^{k} W_h^2\gamma_h\left(S_{yh}^2 + R_{SD}^2 S_{xh}^2 - 2R_{SD}S_{yxh}\right), \tag{1.13}$$

$$\text{MSE}\left(\hat{\bar{Y}}_{SE}^{ST}\right) = \sum_{h=1}^{k} W_h^2\gamma_h\left(S_{yh}^2 + R_{SK}^2 S_{xh}^2 - 2R_{SK}S_{yxh}\right), \tag{1.14}$$

$$\text{MSE}\left(\hat{\bar{Y}}_{US1}^{ST}\right) = \sum_{h=1}^{k} W_h^2\gamma_h\left(S_{yh}^2 + R_{US1}^2\beta_{2h}^2(x) S_{xh}^2 - 2R_{US1}\beta_{2h}(x)S_{yxh}\right), \tag{1.15}$$

$$\text{MSE}\left(\hat{\bar{Y}}_{US2}^{ST}\right) = \sum_{h=1}^{k} W_h^2\gamma_h\left(S_{yh}^2 + R_{US2}^2 C_{xh}^2 S_{xh}^2 - 2R_{US2}C_{xh}S_{yxh}\right), \tag{1.16}$$

where

$$R_{SD} = \bar{Y}\Big/\sum_{h=1}^{k} W_h\left(\bar{X}_h + C_{xh}\right), \qquad R_{SE} = \bar{Y}\Big/\sum_{h=1}^{k} W_h\left(\bar{X}_h + \beta_{2h}(x)\right),$$

$$R_{US1} = \bar{Y}\Big/\sum_{h=1}^{k} W_h\left(\bar{X}_h\beta_{2h}(x) + C_{xh}\right), \qquad R_{US2} = \bar{Y}\Big/\sum_{h=1}^{k} W_h\left(\bar{X}_hC_{xh} + \beta_{2h}(x)\right).$$

2. Suggested Ratio Estimator

Assuming that the population coefficient of the variation and the coefficient of kurtosis are known for all the stratum, the suggested ratio-cum product estimator is

$$\hat{\hat{Y}}_{bk} = \bar{y}_{st}\left[\alpha\left\{\frac{\sum_{h=1}^{k} W_h\left(\bar{X}_hC_{xh} + \beta_{2h}(x)\right)}{\sum_{h=1}^{k} W_h\left(\bar{x}_hC_{xh} + \beta_{2h}(x)\right)}\right\} + (1-\alpha)\left\{\frac{\sum_{h=1}^{k} W_h\left(\bar{x}_hC_{xh} + \beta_{2h}(x)\right)}{\sum_{h=1}^{k} W_h\left(\bar{X}_hC_{xh} + \beta_{2h}(x)\right)}\right\}\right]. \tag{2.1}$$

To obtain the bias and mean squared error of $\hat{\hat{Y}}_{bk}$, let $\bar{y}_h = \bar{Y}(1 + e_{0h})$ and $\bar{x}_h = \bar{X}_h(1 + e_{1h})$ such that

$$E(e_{0h}) = E(e_{1h}) = 0, \quad E(e_{0h}^2) = \frac{1}{\bar{Y}_h^2}\gamma_hS_{yh}^2, \quad E(e_{1h}^2) = \frac{1}{\bar{X}_h^2}\gamma_hS_{xh}^2 \quad \text{and} \quad E(e_0e_1) = \frac{1}{\bar{Y}_h\bar{X}_h}\gamma_hS_{yxh},$$

where $\gamma_h = (1/n_h - 1/N_h)$.

Expressing (2.1) in terms of e_{0h} and e_{1h}, we get

$$\hat{\hat{Y}}_{bk} = \bar{Y}(1 + e_0)\left\{\alpha(1 + e_1)^{-1} + (1-\alpha)(1 + e_1)\right\}, \tag{2.2}$$

where $e_0 = 1/\bar{Y}\sum_{h=1}^{k} W_h\bar{Y}_he_{0h}$ and $e_1 = \sum_{h=1}^{k} W_ha_he_{1h}$ here $a_h = (\bar{X}_hC_{xh})/(\bar{X}_{US2})$.

We now assume that $|e_1| < 1$ so that we may expand $(1 + e_1)^{-1}$ as a series in powers of e_1. To the first degree of approximation, the bias and mean squared error of the proposed estimator $\hat{\hat{Y}}_{bk}$ are

$$\text{Bias}\left(\hat{\hat{Y}}_{bk}\right) = \frac{1}{\bar{X}_{US2}}\sum_{h=1}^{k} W_h^2\gamma_hC_{xh}\left[\alpha R_{US2}C_{xh}S_{xh}^2 + (1-2\alpha)S_{yxh}\right], \tag{2.3}$$

$$\text{MSE}\left(\hat{\hat{Y}}_{bk}\right) = \sum_{h=1}^{k} W_h^2\gamma_h\left[S_{yh}^2 + 2R_{US2}(1-2\alpha)C_{xh}S_{yxh} + R_{US2}^2(1-2\alpha)^2C_{xh}^2S_{xy}^2\right]. \tag{2.4}$$

Mean squared error of $\hat{\hat{Y}}_{bk}$ is minimized for $\alpha = 1/2[1 + \{\sum_{h=1}^{k} W_h^2\gamma_hC_{xh}S_{yxh}\}/\{R_{US1}\sum_{h=1}^{k} W_h^2\gamma_hC_{xh}^2 S_{xh}^2\}]$.

By the substitution of α in (2.1) we get the asymptotically optimum estimator (AOE) for $\bar{Y}$ as

$$\hat{\hat{Y}}_{bk}^{(opt)} = \bar{y}_{st}\left[\frac{1}{2}\left(1 + \frac{\sum_{h=1}^{k} W_h^2\gamma_hC_{xh}S_{yxh}}{R_{US2}\sum_{h=1}^{k} W_h^2\gamma_hC_{xh}^2S_{xh}^2}\right)\left(\frac{\sum_{h=1}^{k} W_h\left(\bar{X}_hC_{xh} + \beta_{2h}(x)\right)}{\sum_{h=1}^{k} W_h\left(\bar{x}_hC_{xh} + \beta_{2h}(x)\right)}\right)\right.$$
$$\left. + \left\{1 - \frac{1}{2}\left(1 + \frac{\sum_{h=1}^{k} W_h^2\gamma_hC_{xh}S_{yxh}}{R_{US2}\sum_{h=1}^{k} W_h^2\gamma_hC_{xh}^2S_{xh}^2}\right)\right\}\left(\frac{\sum_{h=1}^{k} W_h\left(\bar{x}_hC_{xh} + \beta_{2h}(x)\right)}{\sum_{h=1}^{k} W_h\left(\bar{X}_hC_{xh} + \beta_{2h}(x)\right)}\right)\right]. \tag{2.5}$$

Substituting the value of α in (2.4), minimum mean squared error of $\hat{\hat{Y}}_{bk}$ is

$$\text{MSE}\left(\hat{\hat{Y}}_{bk}^{opt}\right) = \sum_{h=1}^{k} W_h^2 \gamma_h S_{yh}^2 - \frac{\left(\sum_{h=1}^{k} W_h^2 \gamma_h C_{xh} S_{yxh}\right)^2}{\sum_{h=1}^{k} W_h^2 \gamma_h C_{xh}^2 S_{xh}^2}. \tag{2.6}$$

3. Efficiency Comparisons

The variance of the usual unbiased estimator of the mean $\bar{y}_{st}$ in stratified random sampling is

$$V(\bar{y}_{st}) = \sum_{h=1}^{k} W_h^2 \gamma_h S_{yh}^2. \tag{3.1}$$

From (1.3), (1.13), (1.14), (1.15), (1.16), (2.4) and (3.1)

(i) $\text{MSE}\left(\hat{\hat{Y}}_{bk}\right) < V(\bar{y}_{st})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2} < \alpha < \left(\frac{1}{2} + \frac{B_1}{A_1 R_{US2}}\right) \\ \text{or} & \left(\frac{1}{2} + \frac{B_1}{A_1 R_{US2}}\right) < \alpha < \frac{1}{2} \end{array}\right\} \tag{3.2}$$

(ii) $\text{MSE}\left(\hat{\hat{Y}}_{bk}\right) < \text{MSE}\left(\hat{\hat{Y}}_{R}^{ST}\right)$ if

$$\frac{\left(R_{US2}^2 A_1 + R_{US2} B_1\right) - \sqrt{\left(R_{US2}^2 A_1 + R_{US2} B_1\right)^2 - R_{US2} A_1 C_1^*}}{2R_{US2}^2 A_1} < \alpha < \frac{\left(R_{US2}^2 A_1 + R_{US2} B_1\right) + \sqrt{\left(R_{US2}^2 A_1 + R_{US2} B_1\right)^2 - R_{US2} A_1 C_1^*}}{2R_{US2}^2 A_1} \tag{3.3}$$

(iii) $\text{MSE}\left(\hat{\hat{Y}}_{bk}\right) < \text{MSE}\left(\hat{\hat{Y}}_{SD}^{ST}\right)$ if

$$\frac{\left(R_{US2}^2 A_1 + R_{US2} B_1\right) - \sqrt{\left(R_{US2}^2 A_1 + R_{US2} B_1\right)^2 - R_{US2} A_1 C_2^*}}{2R_{US2}^2 A_1} < \alpha < \frac{\left(R_{US2}^2 A_1 + R_{US2} B_1\right) + \sqrt{\left(R_{US2}^2 A_1 + R_{US2} B_1\right)^2 - R_{US2} A_1 C_2^*}}{2R_{US2}^2 A_1} \tag{3.4}$$

(iv) $\text{MSE}\left(\hat{\hat{Y}}_{bk}\right) < \text{MSE}\left(\hat{\hat{Y}}_{SE}^{ST}\right)$ if

$$\frac{\left(R_{US2}^2 A_1 + R_{US2} B_1\right) - \sqrt{\left(R_{US2}^2 A_1 + R_{US2} B_1\right)^2 - R_{US2} A_1 C_3^*}}{2R_{US2}^2 A_1} < \alpha < \frac{\left(R_{US2}^2 A_1 + R_{US2} B_1\right) + \sqrt{\left(R_{US2}^2 A_1 + R_{US2} B_1\right)^2 - R_{US2} A_1 C_3^*}}{2R_{US2}^2 A_1} \tag{3.5}$$

(v) $\mathrm{MSE}\left(\hat{\bar{Y}}_{bk}\right) < \mathrm{MSE}\left(\hat{\bar{Y}}_{US1}^{ST}\right)$ if

$$\frac{\left(R_{US2}^2A_1 + R_{US2}B_1\right) - \sqrt{\left(R_{US2}^2A_1 + R_{US2}B_1\right)^2 - R_{US2}A_1C_4^*}}{2R_{US2}^2A_1} < \alpha < \frac{\left(R_{US2}^2A_1 + R_{US2}B_1\right) + \sqrt{\left(R_{US2}^2A_1 + R_{US2}B_1\right)^2 - R_{US2}A_1C_4^*}}{2R_{US2}^2A_1} \tag{3.6}$$

(vi) $\mathrm{MSE}\left(\hat{\bar{Y}}_{bk}\right) < \mathrm{MSE}\left(\hat{\bar{Y}}_{US2}^{ST}\right)$ if

$$\frac{(R_{US2}A_1 + B_1) - \sqrt{(R_{US2}A_1 + B_1)^2 - R_{US2}A_1C_5^*}}{2R_{US2}A_1} < \alpha < \frac{(R_{US2}A_1 + B_1) + \sqrt{(R_{US2}A_1 + B_1)^2 - R_{US2}A_1C_5^*}}{2R_{US2}A_1} \tag{3.7}$$

where

$$\begin{aligned}
A_1 &= \sum_{h=1}^{k} W_h^2\gamma_h C_{xh}^2 S_{xh}^2, \qquad B_1 = \sum_{h=1}^{k} W_h^2\gamma_h C_{xh} S_{yxh},\\
A_2 &= \sum_{h=1}^{k} W_h^2\gamma_h S_{xh}^2, \qquad B_2 = \sum_{h=1}^{k} W_h^2\gamma_h S_{yxh},\\
C_1^* &= R_{US2}^2A_1 + 2R_{US2}B_1 - R^2A_2 + 2RB_2,\\
C_2^* &= R_{US2}^2A_1 + 2R_{US2}B_1 - R_{SD}^2A_2 + 2R_{SD}B_2,\\
C_3^* &= R_{US2}^2A_1 + 2R_{US2}B_1 - R_{SE}^2A_2 + 2R_{SE}B_2,\\
C_4^* &= R_{US2}^2A_1 + 2R_{US2}B_1 - R_{US1}^2A_2 + 2R_{US1}B_2, \qquad \text{and}\\
C_5^* &= R_{US2}^2A_1 + 2B_1 - R_{US2}A_2 + 2B_2.
\end{aligned}$$

(3.2), (3.3), (3.4), (3.5), (3.6) and (3.7) provides the regions of preference in which the suggested estimator is more efficient than the usual unbiased estimator, conventional ratio estimator, estimators $\hat{\bar{Y}}_{SD}^{ST}$, $\hat{\bar{Y}}_{SE}^{ST}$, $\hat{\bar{Y}}_{US1}^{ST}$ and $\hat{\bar{Y}}_{US2}^{ST}$ given by Kadilar and Cingi (2003). The range of α provides enough scope of choosing many estimators that are more efficient than the above considered estimators.

4. Empirical Study

To show the performance of the suggested estimator in comparison to other estimators, a natural population data set is being considered. The description of the population is given below.

Population [Source: Singh and Mangat (1996), p.220]

Y: Total number of milch cows in 1993

X: Total number of milch cows in 1990

$N = 4810$ & $n = 24$		
$n_1 = 7$	$n_2 = 12$	$n_3 = 5$
$N_1 = 1260$	$N_2 = 2400$	$N_3 = 1150$
$\bar{X}_1 = 15.29$	$\bar{X}_2 = 17.25$	$\bar{X}_3 = 17.8$
$\bar{Y}_1 = 17.43$	$\bar{Y}_2 = 20.42$	$\bar{Y}_3 = 20.6$
$\beta_{21}(x) = 1.85$	$\beta_{22}(x) = 2.312$	$\beta_{23}(x) = 1.59$
$C_{x_1} = 0.2991$	$C_{x_2} = 0.3186$	$C_{x_3} = 0.1838$
$S^2_{x_1} = 20.905$	$S^2_{x_2} = 30.205$	$S^2_{x_3} = 10.7$
$S^2_{y_1} = 17.619$	$S^2_{y_2} = 18.386$	$S^2_{y_3} = 13.3$
$S_{yx_1} = 14.690$	$S_{yx_2} = 5.25$	$S_{yx_3} = 5.9$

Table 1: Biases (%) of different estimators

Estimators	$\hat{\hat{Y}}^{ST}_{R}$	$\hat{\hat{Y}}^{ST}_{SD}$	$\hat{Y}^{ST}_{SE}$	$\hat{\hat{Y}}^{ST}_{US1}$	$\hat{\hat{Y}}^{ST}_{US2}$	$\hat{\hat{Y}}^{(opt)}_{bk}$ $\alpha = 0.71675$
Bias	2.63	2.53	1.99	2.77	1.99	1.11

Table 2: Percent relative efficiencies of $\hat{\hat{Y}}^{ST}_{R}$, $\hat{\hat{Y}}^{ST}_{SD}$, $\hat{Y}^{ST}_{SE}$, $\hat{\hat{Y}}^{ST}_{US1}$, $\hat{\hat{Y}}^{ST}_{US2}$ and $\hat{\hat{Y}}^{(opt)}_{bk}$ with respect to $\bar{y}_{st}$

Estimators	$\bar{y}_{st}$	$\hat{\hat{Y}}^{ST}_{R}$	$\hat{\hat{Y}}^{ST}_{SD}$	$\hat{Y}^{ST}_{SE}$	$\hat{\hat{Y}}^{ST}_{US1}$	$\hat{\hat{Y}}^{ST}_{US2}$	$\hat{\hat{Y}}^{(opt)}_{bk}$ $\alpha = 0.71675$
PREs	100.00	62.96	64.55	74.12	119.54	11.37	121.81

Table 3: Range of α in which $\hat{\hat{Y}}_{bk}$ is more efficient $\bar{y}_{st}$, $\hat{\hat{Y}}^{ST}_{R}$, $\hat{\hat{Y}}^{ST}_{SD}$, $\hat{Y}^{ST}_{SE}$, $\hat{\hat{Y}}^{ST}_{US1}$ and $\hat{\hat{Y}}^{ST}_{US2}$

Estimators	$\bar{y}_{st}$	$\hat{\hat{Y}}^{ST}_{R}$	$\hat{\hat{Y}}^{ST}_{SD}$	$\hat{Y}^{ST}_{SE}$	$\hat{\hat{Y}}^{ST}_{US1}$	$\hat{\hat{Y}}^{ST}_{US2}$
Range	(0.50, 0.9335)	(−0.116, 1.5498)	(0.0835, 1.350)	(0.0981, 1.335)	(0.1373, 1.296)	(−0.730, 2.163)

From Table 1, which reveals the bias in percent of different estimators, it is observed that the bias of the suggested estimator at optimum α is minimum.

Table 2 the shows that the largest gain in efficiency is due to the suggested optimum estimator $\hat{\hat{Y}}_{bk}$ over the unbiased estimator $\bar{y}_{st}$, combined ratio estimator $\hat{\hat{Y}}^{ST}_{R}$, and estimators $\hat{\hat{Y}}^{ST}_{SD}$, $\hat{Y}^{ST}_{SE}$, $\hat{\hat{Y}}^{ST}_{US1}$ and $\hat{\hat{Y}}^{ST}_{US2}$ given by Kadilar and Cingi (2003). This implies that the gain in efficiency due to the proposed class of estimators $\hat{\hat{Y}}_{bk}$ can be obtained even when the scalar α deviates from its exact optimum value $\alpha_{(opt)}$.

Table 3 exhibits the range of α in which the suggested estimator is more efficient than other estimators. We further note from Table 3, that there is enough scope in selecting the value of scalar α to obtain better estimators from the suggested class of estimators $\hat{\hat{Y}}_{bk}$.

Conclusion

Our empirical study shows that the proposed class of estimators provides estimators that are less biased and more efficient than other considered estimators. It gives the freedom to choose more efficient estimators, even if α deviates from its optimum value. Thus the suggested class of estimators is recommended for its use in practice.

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