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Robust Unit Root Tests for a Panel TAR Model

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Abstract

Robust unit root tests are developed for dynamic panels consisting of TAR processes. The test statistics are all based on diverse combinations of individual t-type tests for significance of TAR coefficients. Limiting null distributions are established. A Monte-Carlo experiment compares the proposed tests. The tests are applied to a panel data set of Canadian unemployment rates which show asymmetric features as well as having outliers.

Keywords: Asymmetry, instrumental variable estimation, robustness, TAR process, unemployment rate, unit root test.

1. Introduction

During the last decade, testing for panel unit roots have attracted signifficant attention. In the early years, various tests for cross-sectionally uncorrelated error models were developed by Choi (2001), Levin *et al.* (2002), Im *et al.* (2003), and others. Shortly after these papers, extensions to models with cross-sectionally correlated errors were made by Phillips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004), Shin and Kang (2006), and Choi and Chue (2007). Some results for correlated models were summarized by Pesaran (2007), and others. In the recent years, diverse studies were made by Herwartz and Siedenburg (2008) for wild bootstrapping, Gengenbach *et al.* (2010) for comparative study, and many others.

Robustness is an important issue for testing unit root. Various results are available by Lucas (1995) for a test based by M-estimator, Herce (1996) for a test constructed from a least absolute deviation(LAD) estimator, Shin and So (1999) for a test obtained by using a semiparametric adaptive M-estimator, So and Shin (2001) for a sign test, for Shin *et al.* (2009) for panel sign test.

Dynamic asymmetry is apparent in many economic and finance variables. Many authors argue that such variables have dynamics different from up-times, that is, the times at that these variables are increasing, to down-times. Examples of asymmetric variables are unemployment rates that show patterns depending on the business cycle: rapid deterioration for recession times and slow recovery for expansion times. See Koop and Potter (1999) and references therein. In addition, many finance variables such as stock prices and foreign exchange rates show wilder behaviors for big bad news than for big good news. See Tsay (2005) and references therein.

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Classes of nonstationary TAR(threshold autoregressive) models were generally utilized to represent such dynamic asymmetry. Koop and Potter (1999) considered diverse TAR models in their Bayesian analysis of asymmetric features of US unemployment rates. Enders and Granger (1998) and Caner and Hansen (2001) provided unit root tests for TAR models. More studies were made by Shin and Lee (2001) for unit root tests and tests for asymmetry, Hansen and Seo (2002) for testing cointegration, Shin and Lee (2008) for panel unit root tests, and others.

We are interested in robust unit root tests for panel TAR models. Robustness is achieved by adopting the strategy of Huber (1981) of discounting large errors to smaller ones. Following the instrumental variable approach of Shin and Kang (2006) for estimating unit root, diverse statistics are constructed. Standard asymptotic null distributions are established for the statistics. A Monte-Carlo experiment compares the proposed tests. The proposed tests are applied to a panel data set of yearly unemployment rates of Canadian provinces which show asymmetric features as well as outlying observations.

The remaining of the paper is organized as follows. In Section 2, robust tests are proposed and their limiting null distributions are established. In Section 3, a Monte-Carlo comparison of the proposed tests is made. In Section 4, a real data set is analyzed. In Section 5, a concluding remark is provided.

2. A TAR Model and Robust Tests

We are interested in a dynamic panel model given by

$$z_{it} = \sum_{k=1}^{2} \rho_{ki}(y_{i,t-1} - \mu_{i,t-1}) I_{kit} + u_{it}, \tag{2.1}$$

$t = 1, \ldots, T$, $i = 1, \ldots, n$, where $\{y_{it},\ t = 1, \ldots, T,\ i = 1, \ldots, n\}$ is the set of observations over n-panel units for time period $t = 1, \ldots, T$, $z_{it} = \Delta y_{it} = y_{it} - y_{i,t-1}$, $I_{1it} = I(z_{i,t-1} > 0)$ are the indicator variables for the events$(z_{i,t-1} > 0)$, $I_{2it} = 1 - I_{1it}$ are indicators for the other events$(z_{i,t-1} \leq 0)$, ρ_{ki} are unknown autoregressive(AR) parameters, μ_{it} are unknown mean functions, and u_{it} are regression errors. The errors u_{it} are allowed to be cross-sectionally dependent. Let $u_t = (u_{1t}, \ldots, u_{nt})'$. Let $\Sigma_u = \mathrm{var}(u_t)$. For each i, the component series y_{it} is a TAR process whose AR coefficient is ρ_{1i} for up-times, *i.e.*, times such that $y_{i,t-1} > y_{i,t-2}$ and is ρ_{2i} for down-times.

For the time being, we assume given values for μ_{it} and Σ_u for a simple description of the main idea. Later test statistics are constructed with estimated values. We construct robust tests for the null hypothesis of unit roots

$$H_0 : \rho_{1i} = \rho_{2i} = 0, \quad \text{for all} \ \ i = 1, \ldots, n,$$

against the alternative hypothesis $H_1 : \rho_{ki} \leq 0$ for all (k, i) and $\rho_{ki} < 0$ for at least one (k, i). Let $\rho_k = (\rho_{k1}, \ldots, \rho_{kn})'$, $k = 1, 2$. Let $z_t = (z_{1t}, \ldots, z_{nt})'$ and let

$$X_{kt} = \mathrm{diag}((y_{1,t-1} - \mu_{1,t-1}) I_{k1t}, \ldots, (y_{n,t-1} - \mu_{n,t-1}) I_{knt}), \quad k = 1, 2.$$

Then model (1) can be rewritten as

$$z_t = \rho_1 X_{1t} + \rho_2 X_{2t} + u_t.$$

Since X_{1t} and X_{2t} are orthogonal, the ordinary least squares estimator(OLSE) is

$$\bar{\rho}_k = \left(\sum_t X'_{kt} X_{kt}\right)^{-1} \left(\sum_t X'_{kt} z_t\right), \quad k = 1, 2.$$

Owing to the cross-sectional correlation of z_t, the distribution of the OLSE is not free from nuisance parameters arising from the cross-sectional correlation. In stationary cases, nuisance parameter dependency is removed if we use the Generalized Least Squares Estimator(GLSE) given by

$$\tilde{\rho}_k = \left(\sum_t X^{*'}_{kt} X^*_{kt}\right)^{-1} \left(\sum_t X^{*'}_{kt} z^*_t\right), \quad k = 1, 2,$$

where

$$X^*_{kt} = \Gamma_u X_{kt}, \qquad z^*_t = \Gamma_u z_t = \Gamma_u u_t$$

and Γ_u, Γ'_u are LU decomposition of Σ_u^{-1} such that $\Gamma_u \Gamma'_u = \Sigma_u^{-1}$.

However, unlike the stationary cases, as pointed out by Phillips and Sul (2003) and Shin and Kang (2006) for panels of AR processes, the null distributions of test statistics based on the GLSE depend on the nuisance parameter arising from cross-sectional correlation. Moreover, the tests based on the GLSE is not robust because the GLSE is sensitive to outlying errors u^*_{it} which are, under H_0, equal to z^*_{it}.

The object of robustness is achieved by discounting large errors to smaller ones through

$$z^{*H}_{it} = h_\ell(z^*_{it}),$$

where

$$h_\ell(x) = -1, \quad \text{if} \quad x \le -\ell; \qquad h_\ell(x) = \frac{x}{\ell}, \quad \text{if} \ \ |x| < \ell; \qquad h_\ell(x) = 1, \quad \text{if} \quad x \ge \ell$$

and $\ell \ge 0$ is a real number. The function h_ℓ was proposed by Huber (1981) and was widely used in robust estimation.

The object of nuisance-parameter-free asymptotics is achieved by adopting the instrumental variable (IV) approach of Shin and Kang (2006) with instrument

$$\begin{aligned} H_{kt} &= \operatorname{diag}\left(h_m\left(\frac{y_{1,t-1} - \mu_{1,t-1}}{\sigma_1}\right) I_{k1t}, \ldots, h_m\left(\frac{y_{n,t-1} - \mu_{n,t-1}}{\sigma_n}\right) I_{knt}\right) \\ &= \operatorname{diag}(h_{k1t}, \ldots, h_{knt}), \end{aligned}$$

where $m \ge 0$ is a real number,

$$h_{kit} = h_m\left(\frac{y_{i,t-1} - \mu_{i,t-1}}{\sigma_i}\right) I_{kit},$$

and $\sigma_i^2 = \text{var}(u_{it})$ is the (i, i) component of Σ_u.

In the instrumental variable, the purpose of using the discounting function $h_m()$ is for resolving nuisance parameter dependency and attaining normality rather than for robustness. Under H_0, as $T \to \infty$, $(y_{i,t-1} - \mu_{1,t-1})$ is of probabilistic order $t^{1/2}$ and $|(y_{i,t-1} - \mu_{1,t-1})| > m$ with high probability. Therefore, for large T,

$$\sum_t h_m \left(\frac{y_{i,t-1} - \mu_{i,t-1}}{\sigma_i} \right) I_{kit} z^*_{it} \cong \sum_t \text{sign}(y_{i,t-1} - \mu_{i,t-1}) I_{kit} z^*_{it}, \quad i = 1, \ldots, n, \;\; k = 1, 2, \quad (2.2)$$

which are asymptotically independent and standard normal if normalized by $T^{1/2}$. See Appendix for a formal justification.

The resulting robust IV-estimator is

$$\hat{\rho}_k = \left(\sum_t H_{kt} X^*_{kt} \right)^{-1} \left(\sum_t H_{kt} z^{*H}_t \right),$$

with estimated variance

$$V_{\hat{\rho}_k} = \left(\sum_t H_{kt} X^*_{kt} \right)^{-1} \left(\sum_t H_{kt} V_{z^{*H}} H_{kt} \right) \left(\sum_t X^{*'}_{kt} H_{kt} \right)^{-1},$$

where $z^{*H}_t = (z^{*H}_{1t}, \ldots, z^{*H}_{nt})'$ and $V_{z^{*H}}$ is an estimated variance of z^{*H}_t. Since elements of z^*_t are uncorrelated with zero mean and common variance, so are z^{*H}_t. Therefore, a natural estimator of the variance of z^{*H}_t is

$$V_{z^{*H}} = \hat{\sigma}^{*2} I_n, \quad \hat{\sigma}^{*2} = (nT)^{-1} \sum_{i=1}^{n} \sum_{t=1}^{T} \left(z^{*H}_{it} \right)^2.$$

Now, the Wald test for H_0 is

$$W = \sum_{k=1}^{2} \hat{\rho}'_k V^{-1}_{\hat{\rho}_k} \hat{\rho}_k = \sum_{k=1}^{2} \left(\sum_t H_{kt} z^{*H}_t \right)' \left(\sum_t H_{kt} V_{z^{*H}} H_{kt} \right)^{-1} \left(\sum_t H_{kt} z^{*H}_t \right).$$

Since H_{kt} is diagonal, so is $(\sum_t H_{kt} V_{z^{*H}} H_{kt})$. Hence the Wald test for H_0 becomes

$$W = \sum_{k=1}^{2} \sum_{i=1}^{n} (\hat{\tau}_{ki})^2,$$

where

$$\hat{\tau}_{ki} = \frac{\hat{\rho}_{ki}}{\text{se}(\hat{\rho}_{ki})} = (\hat{\sigma}^*)^{-1} \frac{\sum_t h_{kit} z^H_{it}}{(\sum_t h^2_{kit})^{\frac{1}{2}}},$$

a t-type test for significance of ρ_{ki}, where $\text{se}(\hat{\rho}_{ki})$ is the square root of the i^{th} diagonal element of $V_{\hat{\rho}_{ki}}$. Taking advantage of the one-sided nature of the testing problem, we obtain the following modifications of the Wald test

$$W^- = \sum_{k=1}^{2} \sum_{i=1}^{n} \{(\hat{\tau}_{ki})^-\}^2$$

and

$$\bar{W}^- = \sum_{k=1}^{2} \left\{ \left(\sum_{i=1}^{n} \hat{\tau}_{ki} \right)^- \right\}^2,$$

where $x^- = x$ if $x < 0$, $x^- = 0$ if $x \geq 0$. Instead of summing squares of $\hat{\tau}_{ki}$, following the averaging scheme of Im *et al.* (2003) and the p-value approach of Choi (2001), we obtain the following statistics

$$\bar{\tau} = (2n)^{-\frac{1}{2}} \sum_{k=1}^{2} \sum_{i=1}^{n} \hat{\tau}_{ki},$$

$$P = -2 \sum_{k=1}^{2} \sum_{i=1}^{n} \ln(p_{ki}),$$

where

$$p_{ki} = \Phi(\hat{\tau}_{ki}) \quad \text{is the } p\text{-value of } \hat{\tau}_{ki},$$

and Φ is the cumulative distribution of the standard normal distribution.

Since Σ_u and mean function μ_{it} are unknown, they should be replaced by their estimators $\hat{\Sigma}_u$ and $\hat{\mu}_{it}$. Then $\hat{\tau}_{ki} = \sum_t h_{kit} z_{it}^{*H} / (\sum_t h_{kit}^2)^{1/2}$, $i = 1, \ldots, n$, $k = 1, 2$ are constructed using $z_t^* = (z_{1t}^*, \ldots, z_{nt}^*)' = \hat{\Gamma}_u z_t$ and $h_{kit} = h_m((y_{i,t-1} - \hat{\mu}_{i,t-1})/\hat{\sigma}_i) I_{kit}$ where $\hat{\Gamma}_u \hat{\Gamma}_u' = \hat{\Sigma}_u^{-1}$ is the LU-decomposition of $\hat{\Sigma}_u^{-1}$ and $\hat{\sigma}_i^2$ is the (i,i)-element of $\hat{\Sigma}_u$. The test statistics $W, \bar{\tau}, P, W^-, \bar{W}^-$ are constructed using these $\hat{\tau}_{ki}$.

As a H_0-consistent estimator of Σ_u, we may use $\hat{\Sigma}_u = T^{-1} \sum_t z_t z_t'$. This estimator is recommended in case of large T relative to n because it is positive definite for $T > n$ and is simple to compute. When n is large, one can construct other estimators based on factor models as adopted by Phillips and Sul (2003), Moon and Perron (2004) and Bai and Ng (2004). More specifically, estimator $\hat{\Sigma}_u = \Sigma(\hat{\theta})$ is constructed using an estimator $\hat{\theta}$ of parameters for a factor model $u_{it} = \sum_{\ell=1}^{L} \delta_{i\ell} f_{\ell t} + e_{it}$ where $f_{\ell t}$ are independent zero mean errors having unit variance and e_{it} are independent zero-mean errors having variance σ_{ii}^e independent of $f_{\ell t}$.

The mean function $\mu_{i,t-1}$ is recursively estimated in order for $\sum_t \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) I_{kit} z_{it}^*$ of (2) with estimated mean function $\hat{\mu}_{i,t-1}$ to have a martingale structure, which is essential for asymptotic normality, see Shin and So (2001) and the Appendix. For models with simple mean $\mu_{it} = \mu_i$, we use $\hat{\mu}_{i,t-1} = \bar{y}_{i,t-1} = (t-1)^{-1} \sum_{s=1}^{t-1} y_{is}$, the sample average of $\{y_{i1}, \ldots, y_{i,t-1}\}$, to adjust $\mu_{i,t-1}$ in $y_{i,t-1} - \mu_{i,t-1}$. For models with time trend $\mu_{it} = \beta_{i0} + \beta_{i1} t$, we use $\hat{\mu}_{i,t-1} = \bar{\beta}_{i0,t-1} + \bar{\beta}_{i1,t-1}(t-1)$, where $(\bar{\beta}_{i0,t-1}, \bar{\beta}_{i1,t-1})$ are the coefficients of $(1, s)$ in the regression of y_{is} on $(1, s)$, $s = 1, \ldots, t-1$.

In the following theorem, limiting null distributions of the test statistics are established.

Theorem 2.1. *Consider model (1) with (a) $E[h_\ell(u_{it}^*)|\mathcal{F}_{t-1}] = 0, i = 1, \ldots, n$, (b) for each $i \neq j$, $E[h_\ell(u_{it}^*) h_\ell(u_{jt}^*)|\mathcal{F}_{t-1}] = 0$, (c) finite Σ_u, where $F_t = \sigma(y_{is},\ s = 1, \ldots, t,\ i = 1, \ldots, n)$ is the σ-algebra generated by $(y_{is},\ s = 1, \ldots, t,\ i = 1, \ldots, n)$. Assume that $\hat{\Sigma}_u$ is consistent as $T \to \infty$ and $\hat{\mu}_{i,t-1}$ is F_{t-1}-measurable. Then, under H_0, as $T \to \infty$, we have*

$$Pr[W \leq x] \to Pr[\chi_{2n}^2 \leq x],$$

$$Pr[W^- \leq x] \to \sum_{\ell=0}^{2n} \binom{2n}{\ell} 0.5^n Pr[\chi_\ell^2 \leq x],$$

$$Pr[\bar{W}^- \leq x] \to \sum_{\ell=0}^{2} \binom{2}{\ell} 0.5^2 Pr[\chi_\ell^2 \leq x],$$

$$Pr[\bar{\tau} \leq x] \to Pr[Z \leq x],$$

$$Pr[P \leq x] \to Pr[\chi_{4n}^2 \leq x],$$

where Z is a standard normal random variable, χ^2_ℓ is a chi-square random variable with ℓ degrees of freedom, and $\chi^2_0 = 0$.

Proof. See the Appendix. □

The null distributions of test statistics W and P are asymptotically chi-squared with degrees of freedom $2n$ and $4n$, respectively, for large T. The limiting null distributions of W^- and $\bar{W}^-$ are linear combinations of chi-square distributions which are called chi-bar square distributions. The statistic $\bar{\tau}$ is asymptotically standard normal under H_0. These distributions are free from nuisance parameters and are valid for both mean-adjusted tests and for trend-adjusted tests as well as for other adjustments for mean functions.

For serially independent u_{it}, conditions (a) and (b) become

$$E[h_\ell(u^*_{it})] = 0, \qquad E[h_\ell(u^*_{it})h_\ell(u^*_{jt})] = 0,$$

which are satisfied if $u_{it}, i = 1, \ldots, n$ are jointly symmetrically distributed with origin zero. Since h_ℓ is an odd function, the distribution of $h_\ell(u^*_{it})$ is symmetric about zero satisfying $E[h_\ell(u^*_{it})] = 0$, and the joint distribution of $(h_\ell(u^*_{1t}), \ldots, h_\ell(u^*_{nt}))'$ is symmetric about the origin, satisfying $E[h_\ell(u^*_{it})h_\ell(u^*_{jt})] = 0$.

The parameters ℓ is related with robustness and power of the test statistics. The tests are more robust for smaller ℓ. If error distributions are not heavy tailed, then the tests are more powerful for larger ℓ. The parameter m is related with the robustness of sizes of the tests. Sizes are more stable for smaller m. This issue will be addressed in a Monte-Carlo study in the following section.

3. A Monte Carlo Study

Finite sample size and power properties of the proposed tests are investigated for a model with one factor error,

$$z_{it} = \sum_{k=1}^{2} \rho_{ki}(y_{i,t-1} - \mu_i)I_{kit} + u_{it}, \quad u_{it} = \delta_i f_t + e_{it}.$$

We consider 4 errors for u_{it} and f_t:

$D_1:$ $N(0,1)$, the standard normal,

$D_2:$ $0.9N(0,1) + 0.1N(0,10)$, a normal mixture,

$D_3:$ $C(0,1)$, the standard Cauchy,

$D_4:$ $u_{it} = \epsilon_{it}\sqrt{1 + 0.9u^2_{i,t-1}}$ and $f_t = \epsilon_t\sqrt{1 + 0.9f^2_{t-1}}$, ARCH errors,

where $\epsilon_{it}, \epsilon_t$ are *i.i.d.* $N(0,1)$ errors and u_{it}, f_t are independent.

The factor loading coefficients δ_i are independently generated from $U(d_0, d_1)$ with $(d_0, d_1) = (0, 0.5), (1, 3)$, where $U(d_0, d_1)$ is the uniform distribution on (d_0, d_1). The corresponding cases are denoted by C_1, C_2, respectively, which correspond to weak and strong cross-sectional correlation, respectively: errors under C_1 have average cross-section correlation smaller than 0.1 and errors under C_2 have average cross-section correlation larger than 0.6.

Table 3.1. Sizes(%) of tests based on 10,000 replications. $\rho_{1i} = \rho_{2i} = 0$.

	n	T	$\ell=0, m=0$				$\ell=0, m=2$				$\ell=2, m=0$				$\ell=2, m=2$			
			$\bar{W}^-$	$\bar{\tau}$	P	W^-	$\bar{W}^-$	$\bar{\tau}$	P	W^-	$\bar{W}^-$	$\bar{\tau}$	P	W^-	$\bar{W}^-$	$\bar{\tau}$	P	W^-
									$u_{it}, f_t \sim N(0,1)$									
C_1	5	50	4.8	4.8	4.8	4.7	7.4	7.8	7.4	6.5	4.5	4.8	4.5	4.4	7.6	8.5	7.7	6.7
C_1	20	50	3.9	3.9	4.0	4.1	8.1	8.8	8.5	7.3	3.7	3.4	3.9	4.1	8.7	9.5	9.6	8.5
C_1	5	100	4.8	4.7	4.4	4.5	6.2	6.3	6.1	5.8	4.5	4.5	4.5	4.5	6.6	6.9	6.7	6.2
C_1	20	100	4.2	4.5	4.6	4.7	7.2	8.0	7.6	7.1	4.3	4.4	4.4	4.5	8.2	8.6	8.8	8.0
C_2	5	50	4.6	4.6	4.7	4.7	6.1	6.6	5.9	5.3	4.6	4.6	4.5	4.2	6.4	6.7	5.9	5.2
C_2	20	50	4.6	4.5	4.4	4.3	6.8	6.9	6.5	5.6	4.5	4.8	4.3	4.0	7.3	7.8	7.1	6.0
C_2	5	100	4.8	4.9	4.8	4.9	5.8	6.2	5.8	5.5	4.7	5.0	4.9	4.9	5.9	6.3	5.7	5.8
C_2	20	100	4.8	4.8	4.5	4.6	6.1	6.7	6.4	5.9	4.4	4.7	4.5	4.4	6.4	7.0	6.7	6.3
									$u_{it}, f_t \sim 0.9N(0,1) + 0.1N(0,10)$									
C_1	5	50	4.9	5.2	5.0	4.9	7.0	7.7	7.0	6.4	5.2	5.1	5.0	4.8	7.8	8.3	7.9	7.1
C_1	20	50	4.2	4.4	4.7	4.6	7.7	8.6	8.9	7.8	3.7	3.7	4.4	4.5	8.6	9.1	9.8	8.8
C_1	5	100	5.2	5.2	5.2	5.1	6.6	6.6	6.8	6.4	5.0	5.0	5.2	5.3	7.2	7.5	7.3	6.9
C_1	20	100	4.8	4.9	5.1	5.0	7.6	8.3	7.9	7.3	4.5	4.3	4.8	5.0	8.0	8.5	8.8	8.2
C_2	5	50	4.5	4.8	4.4	4.7	5.9	6.2	5.8	5.2	4.5	4.8	4.7	4.7	6.2	6.4	6.4	5.9
C_2	20	50	5.2	5.4	5.0	4.8	7.3	7.8	6.9	5.8	4.9	4.7	4.9	4.6	7.2	8.0	7.3	6.3
C_2	5	100	4.6	4.9	4.7	4.8	5.8	5.8	5.6	5.4	4.7	4.9	4.8	4.7	6.3	6.3	6.1	5.9
C_2	20	100	5.4	5.1	4.8	4.5	6.7	6.7	6.4	5.8	5.1	4.9	4.7	4.6	6.8	6.9	6.8	6.2
									$u_{it}, f_t \sim C(0,1)$									
C_1	5	50	4.8	4.5	4.5	4.6	5.3	5.4	4.8	4.5	4.5	4.6	5.3	5.8	5.4	5.9	6.6	6.9
C_1	20	50	4.9	4.6	4.6	4.6	6.4	6.6	6.3	5.3	4.6	4.2	5.1	6.0	6.5	6.6	6.7	7.5
C_1	5	100	4.7	4.7	4.7	4.6	5.2	5.4	5.2	4.8	5.0	4.6	5.6	6.3	5.4	5.9	6.5	7.1
C_1	20	100	5.4	5.0	5.3	5.3	6.9	6.7	6.7	6.3	5.0	4.6	5.9	7.2	6.7	7.3	8.2	9.3
C_2	5	50	5.0	5.0	5.0	4.9	5.8	5.9	5.6	5.3	5.0	5.2	5.5	5.9	6.1	6.2	6.8	7.2
C_2	20	50	7.6	7.1	6.6	5.9	9.3	8.6	7.8	6.4	5.9	5.9	6.1	5.9	7.9	7.8	7.6	7.6
C_2	5	100	5.6	5.3	4.8	4.9	5.9	5.8	5.6	5.2	5.1	5.0	5.5	6.2	6.0	5.5	6.3	7.4
C_2	20	100	9.9	8.6	8.2	6.9	11.6	10.5	9.4	7.8	7.6	7.2	7.3	7.7	9.7	9.1	9.3	9.1
									$u_{it} = \epsilon_{it}\sqrt{1+0.9u_{i,t-1}^2},\ f_t = \epsilon_t\sqrt{1+0.9f_{t-1}^2}, \epsilon_{it}, \epsilon_t \sim N(0,1)$									
C_1	5	50	4.7	4.7	4.6	4.7	7.4	7.7	7.0	6.3	4.6	4.4	4.4	4.3	9.6	9.7	11.0	10.8
C_1	20	50	4.4	4.2	4.3	4.4	8.7	9.7	9.5	8.2	4.0	3.8	3.8	3.8	11.0	11.4	15.3	15.1
C_1	5	100	5.1	5.3	5.3	5.3	6.6	6.9	6.9	6.5	5.3	5.2	5.3	5.3	8.4	8.8	9.4	9.1
C_1	20	100	4.6	4.5	4.4	4.6	8.0	8.4	8.2	7.2	4.6	4.3	4.2	4.3	10.2	10.4	12.6	12.8
C_2	5	50	5.3	5.1	4.8	4.5	7.1	7.0	6.2	5.4	5.2	5.2	4.9	4.7	8.4	8.5	8.6	7.8
C_2	20	50	5.7	5.1	5.3	5.1	7.8	7.8	7.0	6.1	5.1	4.6	4.7	4.4	8.2	8.5	9.0	8.0
C_2	5	100	5.4	5.2	5.1	5.0	6.8	6.7	6.4	6.0	5.4	5.3	5.0	5.1	7.5	7.8	7.8	7.3
C_2	20	100	6.7	6.2	5.8	5.5	8.7	8.2	7.6	6.9	6.3	5.6	5.2	5.0	8.9	8.4	8.7	8.1

Note: $u_{it} = \delta_i f_t + \epsilon_{it}, \delta_i \sim U(0, 0.5)$ for C_1, $\delta_i \sim U(1,3)$ for C_2

The autoregressive parameters are generated as: $\rho_{1i} = \rho_{2i} = 0$ for size study and $\rho_{1i} \sim U(-0.1, 0)$, $\rho_{2i} \sim U(-0.2, 0)$ for power study. The other parameters are set to: $n = 5, 20$; $T = 50, 100$.

In simulating data y_{it}, standard normal, standard Cauchy, and uniform errors are generated by IMSL (1989) FORTRAN subroutines RNNOA, RNCHY, RNUN, respectively. Initial values y_{it} and others for $t = 0$ are all set to zero.

We choose $\ell = 0, 2; m = 0, 2$. We use $\hat{\Sigma}_u = T^{-1} \sum_t z_t z_t'$ and use mean adjustments $\hat{\mu}_{i,t-1} = \bar{y}_{i,t-1}$. Rejected percentages of level 5% tests out of 10,000 replications are reported in Tables 3.1, 3.2.

Table 3.2. Powers (%) of tests based on 10,000 replications. $\rho_{1i} \sim U(-0.1, 0), \rho_{2i} \sim U(-0.2, 0)$.

	n	T	$\ell = 0, m = 0$				$\ell = 0, m = 2$				$\ell = 2, m = 0$				$\ell = 2, m = 2$			
			$\bar{W}^-$	$\bar{\tau}$	P	W^-	$\bar{W}^-$	$\bar{\tau}$	P	W^-	$\bar{W}^-$	$\bar{\tau}$	P	W^-	$\bar{W}^-$	$\bar{\tau}$	P	W^-
							$u_{it}, f_t \sim N(0,1)$											
C_1	5	50	31.1	33.8	29.6	22.3	49.5	53.6	46.4	35.5	43.5	47.7	39.8	30.0	67.9	73.0	64.9	50.5
C_1	20	50	69.0	72.6	63.0	47.0	91.1	93.3	87.8	72.7	86.9	89.6	81.0	62.7	98.9	99.3	97.9	90.4
C_1	5	100	63.7	67.6	62.1	51.6	82.1	84.2	81.2	71.3	82.9	85.4	81.8	70.9	95.1	96.6	95.3	90.3
C_1	20	100	98.6	99.1	98.1	93.1	100.	100.	99.9	99.3	100.	100.	99.9	99.3	100.	100.	100.	100.0
C_2	5	50	19.7	20.3	21.6	20.2	28.3	28.7	31.5	28.8	26.3	26.3	29.6	27.1	39.1	38.7	45.7	43.2
C_2	20	50	28.0	28.5	32.5	29.7	42.5	42.8	51.4	47.0	39.1	38.6	47.5	45.3	57.5	56.6	70.6	68.9
C_2	5	100	38.0	36.8	45.8	45.3	50.3	48.5	61.8	61.0	50.9	48.1	62.5	62.6	64.0	61.5	79.6	80.1
C_2	20	100	63.0	58.6	78.0	78.9	76.8	72.8	91.4	92.5	77.2	72.6	92.4	93.7	88.0	85.0	98.5	98.9
							$u_{it}, f_t \sim 0.9N(0,1) + 0.1N(0,10)$											
C_1	5	50	39.9	43.3	38.3	30.5	60.9	64.8	59.4	47.7	48.0	51.6	44.4	34.5	72.8	77.4	70.4	57.7
C_1	20	50	77.8	81.7	73.5	57.6	95.7	96.9	94.3	84.5	88.3	91.3	83.9	66.4	99.3	99.6	98.7	93.1
C_1	5	100	76.5	79.7	76.0	66.5	90.9	92.7	91.4	85.5	86.9	89.7	86.5	77.2	97.3	98.2	97.5	94.2
C_1	20	100	99.8	99.9	99.8	98.3	100.	100.	100.	100.	100.	100.	100.	99.7	100.	100.	100.	100.0
C_2	5	50	23.5	23.9	27.0	25.4	34.9	35.3	40.2	37.7	28.3	28.4	33.0	31.0	42.7	42.5	50.6	48.3
C_2	20	50	35.1	35.2	42.4	40.2	49.0	49.3	61.3	58.8	42.8	42.2	53.5	51.0	60.5	59.2	74.8	74.7
C_2	5	100	46.3	44.5	57.0	56.6	59.4	57.4	73.5	74.0	54.8	52.3	67.7	68.2	68.4	66.0	84.2	84.9
C_2	20	100	71.8	67.7	87.5	88.9	83.7	80.1	96.4	97.0	80.8	76.7	95.3	96.0	90.7	87.7	99.0	99.2
							$u_{it}, f_t \sim C(0,1)$											
C_1	5	50	74.6	76.8	82.0	79.1	90.3	91.0	95.4	94.5	55.0	58.2	55.6	46.9	84.1	86.0	87.5	81.6
C_1	20	50	91.6	92.6	96.1	94.8	97.8	97.8	99.3	99.4	82.3	84.2	85.4	75.3	96.6	96.6	98.7	97.9
C_1	5	100	91.9	92.1	97.2	97.3	97.1	96.7	99.3	99.4	89.1	90.6	91.3	87.7	97.2	96.9	99.0	98.7
C_1	20	100	99.2	98.8	99.9	100.	99.4	99.1	100.	100.	98.5	98.4	99.4	99.2	99.2	99.0	99.9	99.9
C_2	5	50	53.0	53.4	67.3	68.4	68.0	68.3	84.1	85.5	38.8	39.1	46.9	44.8	59.7	58.6	74.5	73.0
C_2	20	50	62.2	61.2	81.7	84.6	75.7	74.5	92.6	95.4	51.8	50.2	67.7	66.2	71.3	68.7	90.0	91.3
C_2	5	100	76.4	75.0	90.5	92.2	85.3	84.0	96.4	97.1	67.3	66.1	80.3	79.8	82.1	79.3	94.8	95.5
C_2	20	100	87.7	84.6	98.3	99.2	91.5	88.6	99.1	99.5	82.4	79.5	95.0	95.9	89.7	86.6	98.7	99.3
							$u_{it} = \epsilon_{it}\sqrt{1 + 0.9u_{i,t-1}^2},\ f_t = \epsilon_t\sqrt{1 + 0.9f_{t-1}^2}, \epsilon_{it}, \epsilon_t \sim N(0,1)$											
C_1	5	50	33.7	36.5	32.4	26.2	54.6	59.0	53.4	42.8	41.2	44.3	37.5	27.8	69.7	71.6	70.3	61.0
C_1	20	50	67.7	70.8	63.5	49.2	91.5	93.4	89.2	77.1	82.5	85.0	77.0	58.5	98.1	98.5	98.3	94.3
C_1	5	100	68.1	71.0	67.4	58.5	87.1	89.0	86.9	79.3	79.3	81.9	78.4	67.8	94.1	94.8	95.0	91.1
C_1	20	100	98.7	99.1	98.6	94.8	99.9	99.9	99.9	99.7	99.8	99.9	99.8	98.8	100.	99.9	100.	100.0
C_2	5	50	22.4	22.8	24.9	23.0	33.8	33.6	37.9	35.7	27.4	27.2	30.2	28.5	43.9	42.8	52.8	51.1
C_2	20	50	33.4	33.7	39.9	37.6	49.7	49.1	59.2	56.8	42.4	41.8	51.0	47.7	62.1	60.6	77.0	77.2
C_2	5	100	44.1	42.9	52.6	51.4	59.3	56.1	70.8	70.1	53.2	50.9	63.4	62.1	69.6	66.0	83.0	83.1
C_2	20	100	67.3	63.7	83.0	83.9	80.0	76.4	94.1	95.4	77.8	74.2	92.4	93.1	88.8	85.8	98.5	98.9

Note: $u_{it} = \delta_i f_t + \epsilon_{it}, \delta_i \sim U(0, 0.5)$ for C_1, $\delta_i \sim U(1, 3)$ for C_2

The size results are reported in Table 3.1. We see that all the four tests $\bar{W}^-$, $\bar{\tau}$, P, W^- have similar size performances. The table show us stable null behaviors of the proposed tests whose empirical size values are reasonably close to the nominal level 5% except for a few cases of $\ell = m = 2$. The tests corresponding to $m = 0$ have size values very close to 5% under all cases considered here. The tests corresponding to $m = 2$ are slightly oversized, especially under ARCH error.

The strong cross-sectional dependence corresponding to C_2 does not prevent the proposed tests from having good size values. Size performances are similar for all (n, T) considered here except for a few cases.

Under the well-behaved standard normal error D_1 and moderately outlying errors of the normal mixture error D_2, size values of all the tests seem good. Even under the outlying Cauchy errors D_3 in which covariance matrix Σ_u does not exist, rotation by the sample covariance matrix $\hat{\Sigma}_u$ is successful in resolving cross-section dependence of C_2, producing stable size values of the tests except for ($n = 20$, $T = 100$, $m = 2$). Under ARCH error of D_4, sizes are all good except for $(\ell, m) = (2, 2)$.

Power performances of the tests are reported in Table 3.2. We see that powers are sensitive to ℓ, m. Relative power performances vary according to error distributions depending on whether error variances are finite as in D_1, D_2, D_4 or infinite as in D_3.

For the normal error D_1, as expected, tests with $(\ell, m) = (2, 2)$ seem to have the highest powers. The tests with $(\ell, m) = (2, 2)$ still seem to hold a power advantage over tests with other (ℓ, m) for the moderately outlying normal mixture error D_2 and ARCH errors D_4. For errors D_1, D_2 and D_4, tests with $(\ell, m) = (0, 2)$ seem to have better powers than the tests with $(\ell, m) = (0, 0), (2, 0)$. However, we should note that the high power values of tests with $(\ell, m) = (0, 2), (2, 2)$ due in part to the oversizes as seen in Table 3.1. If the oversizes are adjusted, the tests with $(\ell, m) = (0, 2), (2, 2)$ would lose much of their power advantages over the tests with $(\ell, m) = (2, 0)$. We may say that tests with $(\ell, m) = (2, 0)$ have powers comparable to those with $(\ell, m) = (0, 2), (2, 2)$. For the outlying Cauchy error D_3, tests with $(\ell, m) = (0, 2)$ have highest power values.

Relative performances of the four tests vary according to cross-section dependence structure. For the weakly dependent case of C_1, the test $\bar{\tau}$ seem to have the highest powers. For the strongly dependent case of C_2, the two tests P, W^- have higher powers than the other tests.

Combining the size results and power results we conclude the following: for the finite variance errors, tests with $(\ell, m) = (2, 0)$ are best in that they have best size performance and have powers not worse than tests with other (ℓ, m) considered here; for the infinite variance error, tests with $(\ell, m) = (0, 2)$ are best; if cross-section dependence is not strong, the test $\bar{\tau}$ performs better than the other three tests. If cross-section dependence is strong, the tests P, W^- perform better than the other two tests.

4. An Example

The proposed tests are illustrated by analyzing a panel data set of 10 Canadian provinces. Annual unemployment rates(%) for the period of 1976–2009 are depicted in Figure 4.1. The data set is obtained from Statistics Canada. We have $n = 10$, $T = 34$.

Many people argued that data generating processes of unemployment rates are nonstationary. The classical Dickey-Fuller tests are not usually rejected. Some people such as Caner and Hansen (2001) claimed that unemployment rates are better modelled by stationarity than nonstationarity if we allow asymmetry via TAR models. This issue will be investigated in panel context for the Canadian unemployment rates.

In Figure 4.2, the differences of the state unemployment rates are displayed. The figure reveals apparent asymmetric features that positive peaks are sharper than negative peaks. This is related with the asymmetry of the business cycle where recessions are usually sharper than expansions. Moreover, some sharp increases indicate that the distributions are heavier tailed than normal distributions. For examples, in the year 1982, three provinces(BC, MB, QC) have unemployment increases greater than the corresponding (sample mean + 3 × sample standard deviation). The

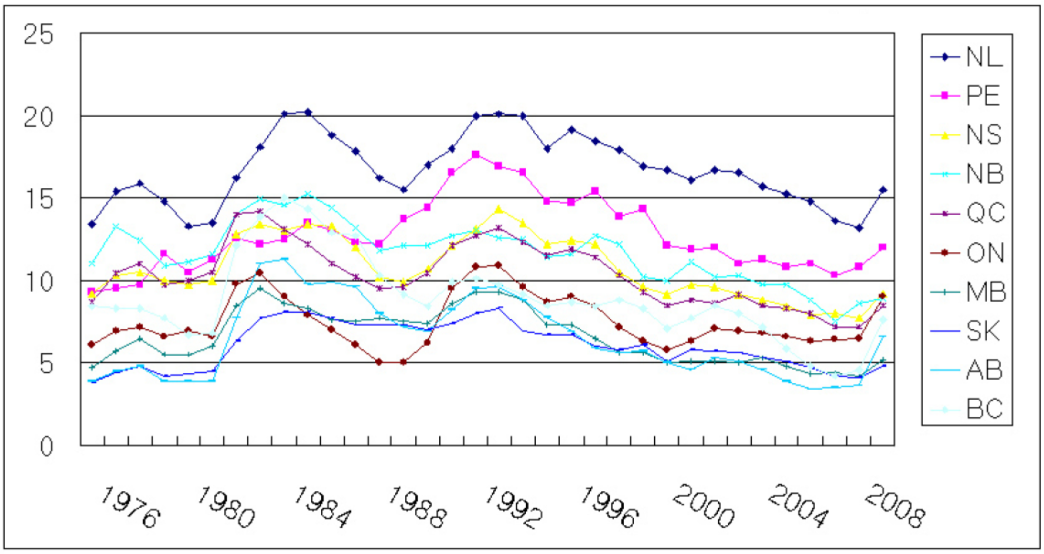

Figure 4.1. Canadian regional unemployment rates(%)

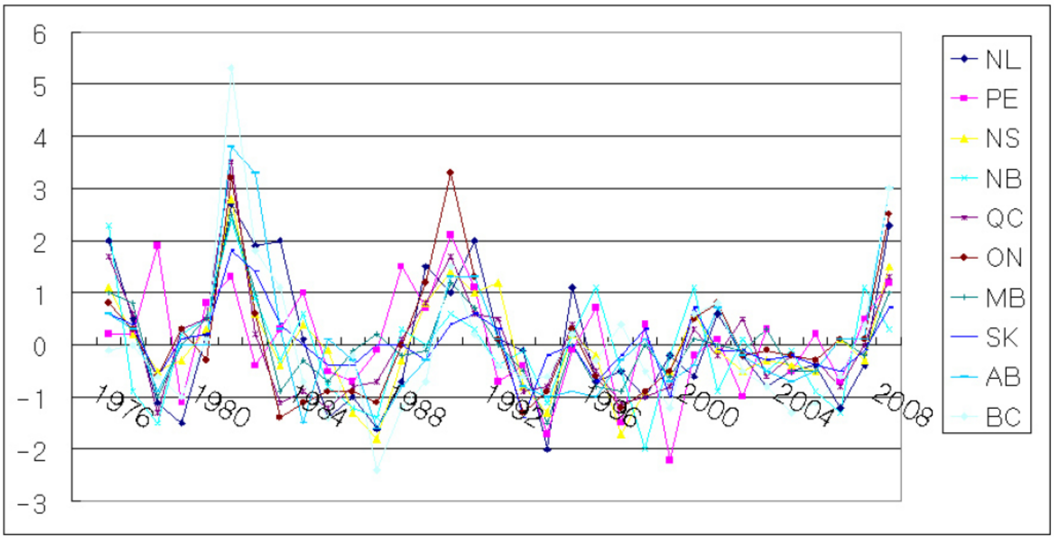

Figure 4.2. Canadian regional unemployment rate changes(%)

average of sample correlation coefficients of all the pairs of the differenced series is 0.626. This average value is considerably large, indicating a strong cross-sectional correlation. These observations motivate us to use the proposed robust tests taking account asymmetry and cross-sectional correlation.

Test statistics as well as their p-values are provided in Table 4.1 for values of $\ell, m = 0, 2$. In computing the test statistics, simple means are adjusted and $\hat{\Sigma}_u = T^{-1}\sum_t z_t z_t'$ are used. For all ℓ, m considered here, all statistics have p-values greater than 0.1 indicating presence of unit roots. Even if asymmetry is allowed, our analysis is in favor of nonstationarity of the data generating

Table 4.1. Panel unit root tests for yearly Canadian provincial unemployment rates.

ℓ	m	$\bar{W}^-$	p-value	$\bar{\tau}$	p-value	P	p-value	W^-	p-value
0	0	.00	.75	1.81	.96	27.77	.93	4.46	.89
0	2	.50	.44	.80	.79	32.94	.78	6.32	.75
2	0	.00	.75	1.46	.93	31.15	.84	7.26	.67
2	2	.33	.49	.36	.64	35.66	.67	6.78	.71

process of the unemployment rates rather than stationarity.

5. Conclusion

Robust unit root tests are developed for panel data sets having asymmetric and outlying aspects. Asymmetry is addressed by employing a TAR model and robustness is attained by adopting the discounting principle of Huber (1981) for robust M-estimation. The test statistics have standard limiting null distributions which are simple functions of standard normal distributions. A Monte-Carlo experiment is conducted to investigate sizes and powers of the proposed tests in diverse situations of error distributions and tuning parameters ℓ, m. A Canadian regional unemployment rate data set is analyzed by the proposed test statistics, revealing evidence for unit roots.

Appendix : Proof of Theorem 1.

Under H_0, for each i, y_{it} is a random walk and is of probabilistic order $t^{1/2}$. Therefore, according to Lemma 1(b) of Shin and Lee (2001), $\hat{h}_{kit} \cong \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) I_{kit}$ in that

$$\sum_t \hat{h}_{kit}^2 = \sum_t \{\text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) I_{kit}\}^2 + o_p(T) = \sum_t I_{kit} + o_p(T)$$

and that, together with consistency of $\hat{\Sigma}_u$,

$$\sum_t \hat{h}_{kit} \hat{z}_{it}^* = \sum_t \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) I_{kit} u_{it}^* + o_p\left(T^{\frac{1}{2}}\right).$$

Therefore, by the law of large numbers,

$$T^{-1} \sum_t \hat{h}_{kit}^2 = T^{-1} \sum_t I_{kit} + o_p(T) \xrightarrow{p} q_{ki},$$

where $q_{ki} = E(I_{kit}) = P(z_{it} > 0)$. We show that $T^{-1/2} \sum_t \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) u_{it}^* \xrightarrow{d} q_{ki} E_{ki}$, where E_{ki}, $i = 1, \ldots, n$, $k = 1$, 2 are independent standard normal random variables. Then $\hat{\tau}_{ki} = (\sum_t h_{kit}^2)^{-1/2} \sum_t \hat{h}_{kit} \hat{z}_{it}^* \xrightarrow{d} E_{ki}$. Therefore, we get

$$W = \sum_k \sum_i \hat{\tau}_{ki}^2 = \sum_k \sum_i E_{ki}^2 + o_p(1) \xrightarrow{d} \chi_{2n}^2$$

and other limiting results. It remains to show

$$T^{-\frac{1}{2}} \sum_t \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) u_{it}^* \xrightarrow{d} q_{ki} E_{ki}.$$

Let λ_{ki} be real numbers. We have

$$T^{-\frac{1}{2}} \sum_k \sum_i \sum_t \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) I_{kit} u_{it}^* = T^{-\frac{1}{2}} \sum_t x_t + o_p(1),$$

where $x_t = \sum_k \sum_i \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) I_{kit} u_{it}^*$. Since $\text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) I_{kit}$, $i = 1, \ldots, n$, $k = 1, 2$ are F_{t-1} measurable, $E(x_t | \, F_{t-1}) = 0$ and hence x_t is a F_t-martingale difference. Since I_{1it}, I_{2it} are orthogonal and $\{u_{it}^*, i = 1, \ldots, n\}$ is a set of uncorrelated random variable with unit variances, $E(x_t^2 | \, F_{t-1}) = \sum_k \sum_i \lambda_{ki}^2 q_{ki}$. Noting $T^{-1} \sum_t E(x_t^2 | \, F_{t-1}) \to \sum_k \sum_i \lambda_{ki}^2 q_{ki}$ and applying a version of martingale central limit theorem, we get $T^{-1/2} \sum_t x_t \xrightarrow{d} N(0, \sum_k \sum_i \lambda_{ki}^2 q_{ki})$. Therefore, we get $T^{-1/2} \sum_t \text{sign}(y_{i,t-1} - \hat{\mu}_{i,t-1}) u_{it}^* \xrightarrow{d} q_{ki} E_{ki}$ from the Cramer-Wold device.

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