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Uncertainty, View, and Hedging: Optimal Choice of Instrument and Strike for Value Maximization*

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ABSTRACT

This paper analytically studies how to choose hedging instrument for firms with steady operating cash flows from value maximization perspective. I derive a formula to determine option's optimal strike that makes hedged cash flow have the best monetary payoff given a hedger's view on the underlying asset. I find that not only the expected mean but also the expected standard deviation of the underlying asset in relation to the forward price and the implied volatility play a crucial role in making optimal hedging decision. Higher moments play a certain part in hedging decision but to a lesser degree.

Keywords: Optimal Strike, Option, Value Maximization

1. Introduction

Firms face many different kinds of financial uncertainties resulting from interest rates, foreign exchange, and commodities and it is generally advised to hedge, if not all, its exposures for the firm to be able to concentrate on its core business operations. Financial derivatives have been invented to serve the needs to hedge the risks resulting from the uncertainties and nowadays a variety of the derivatives is simply enormous. When it comes to the question of what is the best hedging strategy among them, however, it is not as easy and straightforward to answer as it first seemed. Market consensus is that forward contract is a best hedging tool as it can remove the future uncertainty in terms of price. In fact, forward contract or its equivalent forms

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are so widely used (e.g., BIS, 2007; Jesswein *et al*., 1995; Adam, 2002) that even people tend to think that anything other than forward contract such as option are not for hedging but just for speculation.

While a forward contract is often viewed as a market-neutral instrument (see, Cho, 2010, for the hazard and perils of forward contracts), it is in fact based on an implicit view on future market movements. Suppose that a hedger is in a long position in an underlying asset, the spot price of the underlying asset is 100, and the forward price is identically 100. If the hedger *believes* that the spot price in the future would be 110, he should not sell the forward contract at 100 *given* his view. Put differently, if the hedger decides to sell the forward at 100, his view could be that the price in the future is unlikely to rise significantly. In other words, the decision to enter a forward contract or not is implicitly but surely dependent upon the view about the price in the future. To form a probabilistic view on future price movement is inevitable when facing uncertainty. In other words, even a decision of no hedging implicitly takes a view (Bodnar and Gebhardt, 1999; Brown, 2001).

A standard explanation for the decision of adopting forward contract in such a situation is that even if the hedger might think that the price in the future would rise, he could wish to eliminate the uncertainty in exchange for the might-be gain of 10, i.e., the 10 is a risk premium. However, such an explanation is hard to be applicable for the hedgers who have the opposite side, i.e., those who wish to buy the underlying asset. To them, forward contract not only enables to eliminate the uncertainty but also makes a gain of 10 simultaneously.

Forward contract does have virtues such as: i) it can completely eliminate uncertainty in future price, ii) it does not incur upfront cost, if might do so later, and iii) so-called hedge accounting is applicable almost for sure. Hedge accounting is a special accounting treatment for derivatives trade, whereby the mark-to-market variation of underlying asset and of derivatives instrument to hedge the exposure is not taken to the profit and loss account of income statement but directly reserved at shareholder's equity on balance sheet. There is no doubt about the first point and particularly so if the size of the exposure is big and the exposure is not repetitive and concentrated. A good example for that would be long-term bond issued in a different currency rather than in the firm's accounting currency. The second and the third ones can, however, be controversial in that what should matter ultimately is its cost adjusted benefit on realized basis and that the rules regarding hedge accounting are actually redundant once we move to mark-to-market based accounting scheme for financial derivatives. As a matter of fact, the existence of the hedge accounting has been one of the main reasons why forward contract is so popular, for the dichotomy between hedge accounting and no hedge accounting is critical for the finance managers of firms. In other words, the agents of firms have little incentive to search for better hedging strategy than simply executing forward contracts, because, if they do something else other than forward contracts and the trade goes against the firm, then they usually have to defend their decision and likely to be internally penalized. In addition, they can claim the notion that forward contract is a market-neutral instrument as an excuse when the forward contract makes a significant loss.

By nature, the probabilistic views under uncertainty are individually different thus subjective. Recall that without differing views it is not possible for a simple transaction of buying and selling to occur in the market. The other aspect is that to explain price changes in the past and predict price movement in the future many researchers have attempted to find a formula or rule as if it were the Holy Grail. In foreign exchange market, for instance, more than several competing theories have been suggested. Traditional ones based on macroeconomic theories are purchasing power parity (Ross, Westerfield, and Jaffe, 2002), which states that exchange rate should align with the basket of goods provided there is no tariff and shipping cost, and uncovered interest rate parity, which asserts that the exchange rate in the future is determined by the interest rate in both currencies involved on mathematical expectation basis (Billingsley, 2006). More recent theories have a different theoretical background such as behavioral economics (Grauwe and Grimaldi, 2006), which focuses on the interaction of a number of agents with certain rules and their perceptions that may not necessarily be rational in a traditional sense, or microstructure finance (Lyons, 2001), which cares about market mechanisms such as order flow and bid-offer spread. However, in spite of all the efforts to rationalize the price change it does not appear that single theory can prove to be right all the time (Sarno and Taylor, 2002).

Depending on one's view on future price movement and what hedging should mean, there may be a better hedging instrument: options. The purchaser of an option can exercise it only when it is beneficial to do so and is not obliged to make a loss when the price in the future is disadvantageous. Surely the uncertainty with respect to the price in the future is not completely eliminated. However, it is exactly the uncertainty that makes the option-based hedging better off in terms of expected payoff than forward-based hedging. Then it is a bit puzzling why purchased option is less used by corporate than forward contract despite the benefits of option hedging. The less popularity of option is often ascribed to the upfront cost incurred, i.e., firms, more precisely, the agents of the firms, do not want to pay any upfront cost for hedging, which is myopic given that they usually do not care about *ex post* cost of forward contract. A lack of any guideline as to how to determine an optimal strike of option could be another reason why option is not as popular as forward.

Academic circle seem to have paid lesser attention to the problem of choosing a right hedging strategy. In fact, the term "hedging" can mean very different notions depending on one's perspective. Usual approach on hedging is concerned about instantaneous neutralization with respect to infinitesimal price and market parameters changes (Black and Scholes, 1973). Another one is hedging effectiveness of a futures contract from the mean-variance optimization perspective to derive an optimal hedge ratio (Heaney and Poitras, 1991). These approaches are mainly useful for financial institutions as a derivatives market maker as they do so-called dynamic hedging, but not much relevant for corporate. The objective of hedging for corporate could be to make sure a certain economic outcome when the expiry is reached, and temporal price fluctuation might be of less relevance once the hedge is in place. More fundamentally, as far as corporate risk management is concerned, risk-neutral, arbitrage based arguments may not be so pertinent, i.e., all the market prices, whether that is forward price or implied volatility, are exogenous and the firms are to best utilize available financial instruments for hedging as a price-taker.

Stulz (1984) is one of the pioneers tackling optimal hedging problem for non-financial firms. He studies the issues related to whether firms should mechanically hedge their all exposures and shows that sometimes it is optimal for firm to take a larger or smaller position in forward contracts than the notional of the underlying exposure under the assumption that managers of the firms maximize their expected lifetime utility and that their income from the firm is an increasing function of the changes in the value of the firm. In this paper, forward contract is considered the only available hedging instrument. Later Stulz (1996) focuses on bankruptcy costs as the primary concern for risk management and argues that out-of-money (OTM) option could be a useful hedging vehicle in that respect.

Giddy and Dufey (1995) extensively analyzes various hedging strategies made of vanilla options and summarily argues that when the quantity of an underlying exposure is known forward contract is the best solution and when the quantity is unknown vanilla option is advisable, which has been cited as 'Giddy Rule' (e.g., Steil, 1993). It appears that the term, hedging, means to him only a complete elimination of uncertainty regardless of whether or not the uncertainty is beneficial to the firm.

Beneda (2004) looks at the optimal selection problem of hedging instruments from the perspective of usual portfolio theory. Relying on Monte-Carlo simulation technique to obtain efficient frontier of the portfolio comprising underlying exposure and hedging instrument, she shows that forward contract is the worst in terms of the cost among the various hedging strategies and also worse than the naked position, i.e. no hedging instrument used.

Albuquerque (2007) studies an optimal hedging problem under the assumptions that there is a bankruptcy cost or convex tax schedule, or loss-aversion. He claims that forward contract is better in implementing hedging than options against downside risk based on the assumption that transaction cost is all the same for forward contract and options.

The main findings of this paper are as follows. First, view on future price movement, in particular the expectations on the mean and the standard deviation of the view in relation to forward price and implied volatility, plays a crucial role in determining which hedging instrument is better off between forward contract and option. Let me call the first one, which is the relationship between the mean and the forward price, mean effect, and the second one, which is the one between the standard deviation and the implied volatility, volatility effect. Second, the mean effect requires that a hedger who is in a long position in an underlying asset choose option rather than forward contract when his expected mean is greater than forward price. Similarly, by symmetry a hedger who is in a short position in an underlying asset should choose option, not forward contract, when his expected mean is smaller than forward price. Third, the volatility effect determines that hedgers, whether they are long hedgers or short hedgers, should buy option rather than enter forward contract when the expected standard deviation is greater than the relevant implied volatility. The volatility effect has a priority over the mean effect in that option is better off than forward for the short hedgers who need to sell the underlying asset even when the expected mean is no greater than the forward price, if the expected standard deviation is meaningfully greater than the implied volatility. Lastly, the optimal strike of option tends to be close to the forward price, i.e., at-the-money-forward, when the expected standard deviation is greater than the implied volatility, and one or two standard deviation away from the forward price, i.e., a little out-of-money (OTM), when the expected standard deviation is smaller than the implied volatility. Note that in the latter case the upfront premium of the option can be small.

The structure of this paper is as follows. Section 2 describes the models to compare the forward hedging and option hedging analytically and derives the optimal strike of option. It is assumed that firm is all equity firm and the equity holder wishes to maximize the expected payoff from an asset received in the future. In contrast to a common notion that the goal of hedging is to eliminate uncertainty completely, the objective of sound hedging policy for corporate is assumed i) to be prepared for a worst case scenario (thus no hedging is disregarded as it can be a decision but not be a hedging) and ii) to maximize its payoff in a statistical sense ex ante. A key premise for the above to hold is that the future cash flows are not concentrated but scattered and repetitive so that the statistical expectation bears a practical meaning. Section 3 analyzes the parameters affecting the optimal hedging decision and examines under which conditions option-hedging is better off than forward-hedging and vice versa. Consideration for higher order moments of the view such as skewness and kurtosis is taken additionally. Lastly, Section 4 summarizes the findings in terms of optimal hedging decision and offers some concluding remarks.

2. Model

2.1 Repetitive Economic Flow

Consider a firm that has a repetitive, economically valuable flow such as products or foreign currency to sell in the future, of which the future price is not certain. In order to get around the problems related to the principal-agent model and bankruptcy cost, it is assumed that the firm's capital structure is all equity financed and shareholder's future value is to be maximized. The assumption of repetitive flow of products or foreign currency is not too hard to justify if we imagine a firm producing goods such as oil or manufacturers that export to overseas countries. This assumption is quite critical in that if there is just one or a few concentrated economic flows we may just want to eliminate any uncertainty regarding the future price via forward contract regardless of the view on the underlying price in the future, as it is simply too risky. To state this in a different manner, mathematical expectation on just one realization of a random variable can be obtained theoretically but practically almost meaningless.

Without loss of generality, the value of firm's normalized future economic flow is given by

$$V_u(T, S) = 1 \times S(T) \tag{1}$$

where 1 denotes a normalized unit of a future economic flow and S(T) is the price of the underlying asset at a future time T. Notice that I do not make any assumption on probability distribution of the underlying at this point and that there is no uncertainty regarding the number of units that the firm will receive in the future, which may not be applicable for asset managers and funds who invest in foreign assets.

2.2 Forward Contract

In terms of hedging instrument, two very basic derivatives are considered: forward contract and put option. Forward contract is a bilateral obligation whereby each counterpart is bound to exchange one asset for another at a predetermined price at a predetermined future time. Its payoff at maturity, T, is

$$V_f(T, S) = K_f - S(T) \tag{2}$$

where K_f is the predetermined fixed price, called forward rate or forward price depending on the kind of the underlying asset. The above equation is for forward sell contract as I consider hedging for incoming flow, i.e., hedgers who are in a long position in the underlying asset flow. It is a market convention that when both parties enter a forward contract, there is no upfront payment from either side, which is often quoted as the trade is done at zero-cost. One should not be confused that even though a trade is done at zero cost, i.e., no upfront payment, the forward price available to hedgers is *always* away from the reference spot price not only due to the difference between the mid spot price and the mid forward price but also due to bid-offer spread, market slippage, counterparty credit charge, and margin for the market maker, and so on.

The forward price is given by the following equation (e.g., Billingsely, 2006)

$$K_f = S(0)\, e^{(r-d)T} \tag{3}$$

where S(0) is the spot price of the underlying asset at initial time, r is a risk-free, continuously compounded interest rate of the domestic currency and d is continuously compounded yield of the underlying asset. For the foreign exchange rate, d is a continuously compounded interest rate of the foreign currency involved.

2.3 Put Option

Put option is a right, but not an obligation for the holder of it whereby the holder can choose to sell one asset at a predetermined price called strike, K_P, at a predetermined future time if that is beneficial to the holder. Put option involves an outright cost, which is usually paid upfront but can be deferred to a later date if needed. Given that the upfront premium is $p(K_P, S(0), T, r, \sigma)$ the payoff at time T is

$$V_p(T, S) = \max[0, K_p - S(T)] - p(K_p, S(0), T, r, \sigma) * e^{rT} \tag{4}$$

One can further refine Equation (4) by using a more appropriate interest rate such as the cost of capital for the firm or the credit included interest rate for calculating the future value of the upfront premium, which is ignored here for simplicity.

2.4 Expectation on Hedged Portfolio Payoff

Suppose that S(T) has a certain probability distribution, which is not necessarily the same as one under risk-neutral measure or the historical probability. Let the probability distribution be f(S). Then under the probability distribution, the expected value of the underlying asset at time T is

$$E[V_u] = \int_0^{\infty} V_u f(S)dS = \int_0^{\infty} Sf(S)dS = E[S] = \mu_s \quad (5)$$

where E[·] is the expectation under the probability distribution of f(S).

As can be seen from Equation (5), the expectation of unhedged underlying asset is the same as the expected value of the price of the underlying asset. In addition, the future value of the unhedged underlying asset will have a certain distribution that is derived from the assumed probability distribution of the asset price.

I postulate that the shareholders of the firm wish to maximize its expectation on future economic value by judicious choice of hedging strategies. Formally, the optimization problem to solve is

$$max\ E[V_u+V_h] \quad (6)$$

where V_h is the payoff of hedging instrument at time T, which could be either V_f or V_p.

If forward contract is used for hedging, the above optimization problem becomes trivial. To show this

$$max\ E[V_u+V_h] = max\ E[S+K_f-S] = K_f \quad (7)$$

In other words, the future value of the underlying asset is fixed as the forward price and there will be no uncertainty. At a glance, the fact that there is no uncertainty can be seen as a virtue but it is conceivable to think of a case that the lowest possible price in the future is higher than the forward rate for some reason. In that case, entering the forward contract will be always worse off than doing nothing. While this is not a usual situation in a real world, one should be reminded that forward contract is not a panacea and eliminating of uncertainty could be disadvantageous in some cases.

In case that put option is chosen as a hedging tool, Equation (6) becomes

$$max\ E[V_u+V_h] = max\ E[S+max(0, K_p-S)-p(K_p, S(0), T, r, \sigma) * e^{rT}] \quad (8)$$

While all the other variables in Equation (8) are given exogenously, i.e., underlying asset's current price, tenor, and domestic interest rate are not something that corporate can determine, the put option strike is the only control variable that hedgers need to solve for. Note that it is not certain whether or not there exists an optimal strike that maximizes the value of combined portfolio of the underlying asset and the put option at this point.

Given the assumed probability distribution of f(S) for the underlying asset, the expected value of the portfolio of the underlying asset and its put option is

$$E\left[V_u + V_p\right] = \int_0^{\infty}\left(S + \max(0, K_p - S) - p\cdot e^{rT}\right) f(S)dS$$

$$= \mu_S - p\cdot e^{rT} + \int_0^{K_p}\left(K_p - S\right) f(S)dS = \mu_S - p\cdot e^{rT} + K_p\int_0^{K_p} f(S)dS - \int_0^{K_p} Sf(S)dS \qquad (9)$$

2.5 Optimal Condition

In order to obtain an optimal strike, I take a partial derivative of Equation (9) with respect to put option strike, which results in

$$\frac{\partial E\left[V_u + V_p\right]}{\partial K_p} = -e^{rT}\frac{\partial p}{\partial K_p} + \int_0^{K_p} f(S)dS \qquad (10)$$

If there exists a certain K_P that makes Equation (10) equal to zero and that makes the second partial derivative of Equation (9) negative, the K_P is the optimal strike. The put premium, p, is given by (e.g., Hull, 2003)

$$p = K_p e^{-rT} N(-d_2) - S(0)e^{-dT} N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K_p}\right) + \left(r - d + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S(0)}{K_p}\right) + \left(r - d - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad (11)$$

where N(·) is a cumulative probability distribution function of normal distribution and σ is implied volatility of the underlying asset. Even though the implied volatility is assumed to be constant in Black-Scholes original formulation, it is observed to be a function of strike and tenor (Gatheral, 2006)

Although an approximate formula made of polynomials of the argument for N(·) is readily available (Abramowitz and Stegun, 1972), it is practically easier to implement numerical calculation for partial derivative of the put option premium with respect to put option strike. It should be mentioned that locally $\partial p/\partial K_P$ behaves like $\partial p/\partial S(0)$, which is well known as one of the option's Greeks, delta, i.e., qualitatively speaking, delta can be used as a good proxy in evaluating existence of optimal strike in Equation (10). Similarly, the right hand side term of Equation (10) can be obtained through numerical integration regardless of assumed probability distribution of the underlying asset.

With the equation to solve for optimal condition, the questions to ask are i) if there is an optimal strike for put option hedging strategy, ii) if so under what conditions, and most importantly iii) if the put option with the optimal strike can beat forward contract in terms of risk management, which will be addressed in detail in following sections.

In summarizing Section 2, one should be noted that the derivation above is generic enough to deal with any kinds of views without modification, whether that is conventional log-normal distribution or even exotic price distribution. Likewise, the above methodology can be straightforwardly extended to the cases where corporate needs to buy an asset such as commodities or foreign currency at a future date. Apparently in that case one should consider call option as a hedging tool.

3. Parametric Study

To illustrate how the derived formulas can be used to determine an optimal hedging policy, various parametric studies are conducted on a fictitious underlying asset, which can be thought of as a currency or a commodity. Suppose that a firm needs to sell one unit of the underlying asset in one year. And as a given variable, let the current price of the underlying asset one thousand dollar, the tenor one year, the interest rate for dollar one percent, the yield of the underlying asset one percent, and the implied volatility be ten percent. Note that the forward price is the same as the current spot price given that there is no yield differential between the underlying asset and the dollar.

As for the assumed probability distribution, normal distribution for the price is taken. This is not to say that the normal distribution is a best one to represent real price dynamics but it can be a good starting point as long as we are mindful that it is quite possible for actual distribution to differ. Once basic characteristics are studied, more realistic ones can be incorporated into the analysis to make a better decision on the hedging strategy. One might say that log-normal distribution is more appropriate than normal distribution. In practice, the numerical difference between the two distributions is not too discernable and the impact of truncation on negative price for normal distribution is simply negligible given its very small probability density for the region. In addition, normal distribution has a certain appeal for assets that are bounded and mean-reverting, particularly in situation where the hedging period is not too instant but reasonably long.

Once the basic input variables are set, there are nine, basic scenarios to study: three situations for the expected mean of the probability distribution, that is, the mean is larger than, equal to, or smaller than the forward price, times three situations for the expected standard deviation, i.e., larger than, equal to, or smaller than the implied volatility.

3.1 Identical Standard Deviation and Implied Volatility

3.1.1 When Mean is Equal to Forward Price

In the beginning, let the mean and the standard deviation be the same as the forward price and the implied volatility, respectively. Figure 1 shows the expected payoff for the hedged portfolio by forward contract and put option for different strikes. The first thing to note is that there exists a certain put option strike that makes the expected payoff the highest among all the feasible strikes. The strike is around 900, about one standard deviation from its mean to the left, and its premium at initial time is about 0.71% of the notional. The result suggests that if we were to choose put option for hedging, then to buy an out-of-money option by one standard deviation is actually optimal for this particular set of market parameters.

Compared to the payoff of the portfolio hedged by forward contract, it is certain that the optimal put option portfolio shows a slightly higher payoff by about 0.15%. The difference, however, could be viewed as quite small a number in practical sense and one may prefer to lock in the payoff completely by forward contract in this situation. However, that does not nullify the fact that there does exist an optimal strike of option that results in better expected payoff than by forward contract for the terms assumed.

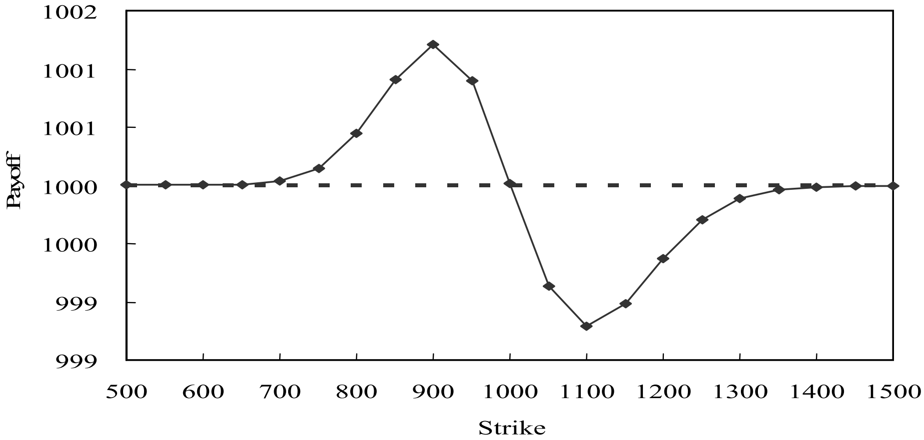


Figure 1. The expected payoff of the put option hedging portfolio and the forward hedging portfolio when the expected mean is the same as the forward price and the expected standard deviation is the same as the implied volatility. Y−axis represents the expected payoff and X−axis represents the strike of put option, not the underlying price in the future. Note that the expected payoff of forward hedging portfolio is represented by dashed line regardless of the strike. Expected mean is 1000, forward price 1000, expected standard deviation 10%, implied volatility 10% and time−to−maturity is one year. Solid line with solid dots represents put option hedging portfolio

3.1.2 When Mean is Greater than Forward Price

More interesting results come out when we change our views on the mean of the probability distribution. As displayed in Figure 2, when the expected mean is greater than the forward price by 50, i.e., 5% higher than the forward price, put option is always better off than forward contract. The put option portfolio has the maximum payoff of 1050, which is larger than that of the forward contract by 5%. The maximum payoff occurs when the put option strike is at 742, and it can be seen that even when the strike is at 800, the payoff is almost the same as when the strike is at the optimal strike. Theoretical Black-Scholes option premiums when the strike is 800 and 742 are 0.04% and 0.003% respectively, which is very small. Clearly the put option purchase strategy is a dominant one over forward contract in this situation.

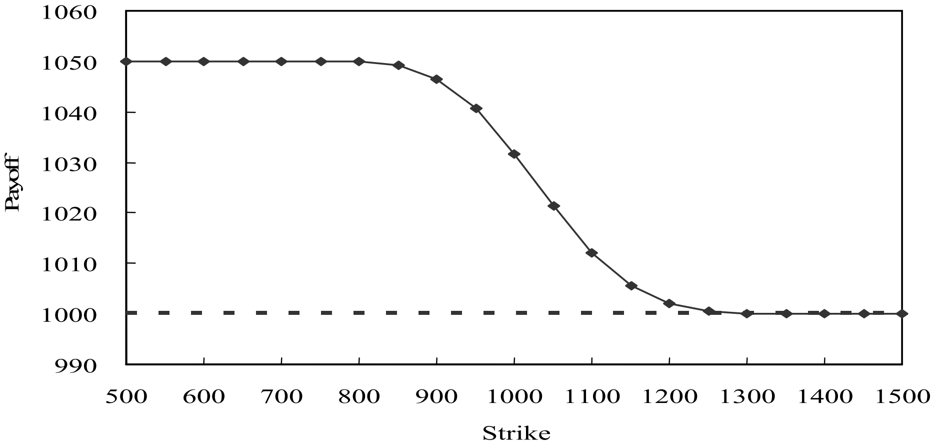


Figure 2. The expected payoff of the put option hedging portfolio and the forward hedging portfolio when the expected mean is greater than the forward price and the expected standard deviation is the same as the implied volatility. Solid line with solid dots represents put option hedging portfolio and dashed line does forward hedging portfolio. Expected mean is 1050, forward price 1000, expected standard deviation 10%, implied volatility 10%, and time-to-maturity is one year

3.1.3 When Mean is Smaller than Forward Price

At a glance, it may be presumed that put option purchase would pay off even more when the mean of the probability distribution is smaller than the forward price because then put option is more likely to end in the money at maturity, which is not the case as illustrated in Figure 3. When the predicted mean is smaller than the forward price, the payoff is always smaller than that of the forward contract. The reason is that the forward price is already in the money, i.e., you can sell the underlying asset at a price higher than your expectation by forward contract.

The transition from when the expected mean is almost equal to the forward price to when the mean is greater than the forward price by 1% is depicted in Figure 4. One can notice that even with just 1% higher expectation of the mean, buying a put option struck at about one to two standard deviation less than the mean beat the forward contract.

The reason that forward contract is worse off than put option in some situations lies in the fact that there are good uncertainty and bad uncertainty and forward contract does not distinguish good from bad. The payoff of the portfolio with put option would be left only with *positive* surprise, as long as the minimum payoff is above the

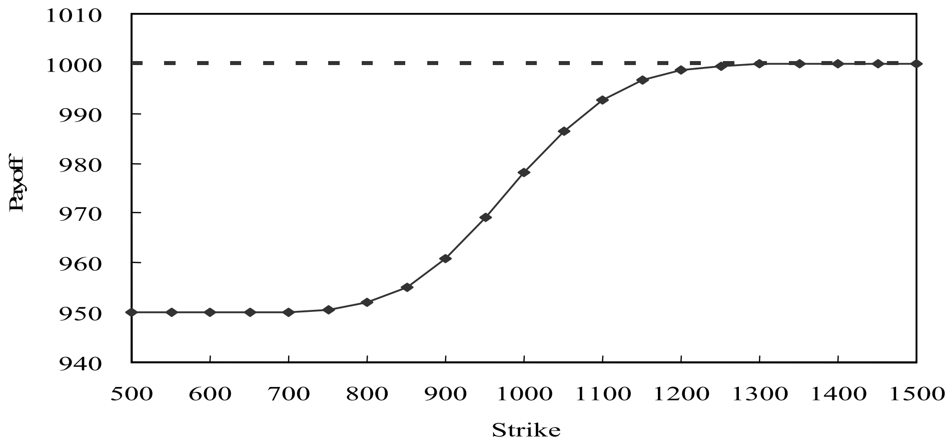


Figure 3. The expected payoff of the put option hedging portfolio and the forward hedging portfolio when the expected mean is smaller than the forward price and the ex－pected standard deviation is the same as the implied volatility. Solid line with solid dots represents put option hedging portfolio and dashed line does forward hedging portfolio. Expected mean is 950, forward price 1000, expected standard deviation 10%, implied volatility 10%, and time－to－maturity is one year

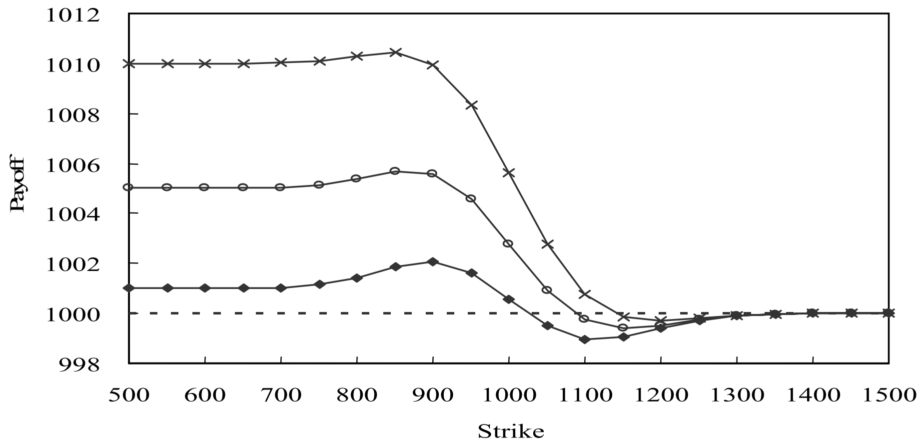


Figure 4. The expected payoff of the put option hedging portfolios and the forward hedging portfolio when the expected mean is greater than the forward price and the ex－pected standard deviation is the same as the implied volatility. This figure is to see the effect of gradual change of expected mean when the mean is close to the for－ward price. Solid line with solid dots represents put option hedging portfolio for the mean of 1001, solid line with hollow dots for the mean of 1005, solid line with cross for the mean of 1010 and dashed line does forward hedging portfolio. Forward price is 1000, expected standard deviation 10%, implied volatility 10%, and time－to－maturity is one year

level that the firm sees no operational problem with. Qualitatively speaking, it is similar to consider Sortino Ratio (Lhabitant, 2004) rather than Sharpe Ratio in evaluating hedge fund's performance.

3.2 Non-Equal Standard Deviation and Implied Volatility

3.2.1 When Mean is Equal to Forward Price

So far it has been assumed that the expected standard deviation of the probability distribution of the underlying asset is the same as the implied volatility. Figure 5 displays the expected payoff when the expected mean of the probability distribution is the same as the forward price and the standard deviation is 9.5%, 10%, 10.5%, respectively. Given that the expected payoff of the portfolio with put option is marginally better or almost the same as that of the portfolio with forward contract when the mean is the same as the forward price and the standard deviation is close to the implied volatility, we may affirm that forward contract is not too worse than put option in terms of the expected payoff of the hedged portfolio for the case at hand.

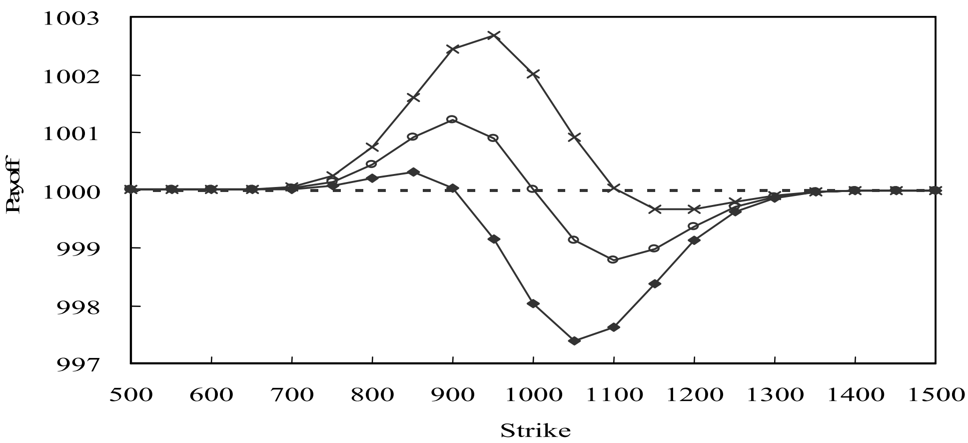
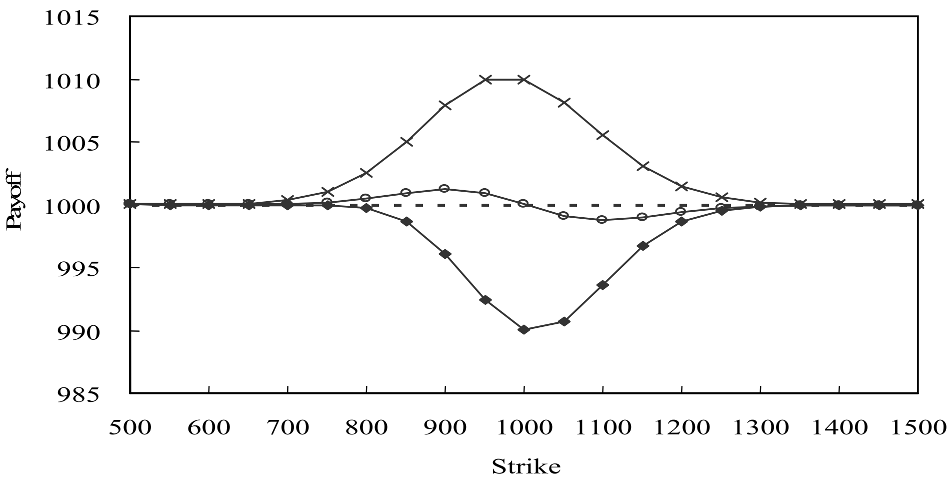


Figure 5. The expected payoff of the put option hedging portfolios and the forward hedging portfolio when the expected mean is the same as the forward price and the expected standard deviation varies near the implied volatility. This figure is to see the effect of micro−change of expected standard deviation near the implied volatility. Solid line with solid dots represents put option hedging portfolio for the standard deviation of 9.5%, solid line with hollow dots for the standard deviation of 10%, solid line with cross for the standard deviation of 10.5% and dashed line does forward hedging portfolio. Expected mean is 1000, forward price 1000, implied volatility 10%, and time−to−maturity is one year

However, when there is a meaningful difference between the implied volatility and the expected standard deviation of the underlying asset, different conclusions come out. Figure 6 shows the expected payoff when the standard distribution is 7.5%, 10%, 12.5% while the mean is the same as the forward price. Now it becomes clear that the expected payoff of the portfolio with put option when the strike is optimal is meaningfully higher than that of the portfolio with forward contract. If the standard deviation is higher than the implied volatility, i.e., the implied volatility is cheap based on your assessment on the future realized volatility, then hedging with put option does make sense as it is expected to have higher payoff. For the standard deviation of 12.5% while the implied volatility is 10%, the optimal strike for the put option is 976.4, which is reasonably close to the forward price. In this circumstance, the portfolio hedged with the put option of the optimal strike has about 1% more expected value.



Figuer 6. The expected payoff of the put option hedging portfolios and the forward hedging portfolio when the expected mean is the same as the forward price and the expected standard deviation varies substantially across the implied volatility. This figure is to see the effect of macro-change of expected standard deviation across the implied volatility. Solid line with solid dots represents put option hedging portfolio for the standard deviation of 7.5%, solid line with hollow dots for the standard deviation of 10%, solid line with cross for the standard deviation of 12.5% and dashed line does forward hedging portfolio. Expected mean is 1000, forward price 1000, implied volatility 10%, and time-to-maturity is one year

On the other hand, if the standard deviation is smaller than the implied volatility while the mean is the same as the forward price, hedging with put option is always worse off than hedging with forward contract as its expected payoff is smaller than or at best the same as the payoff from the forward contract regardless of the put option strike.

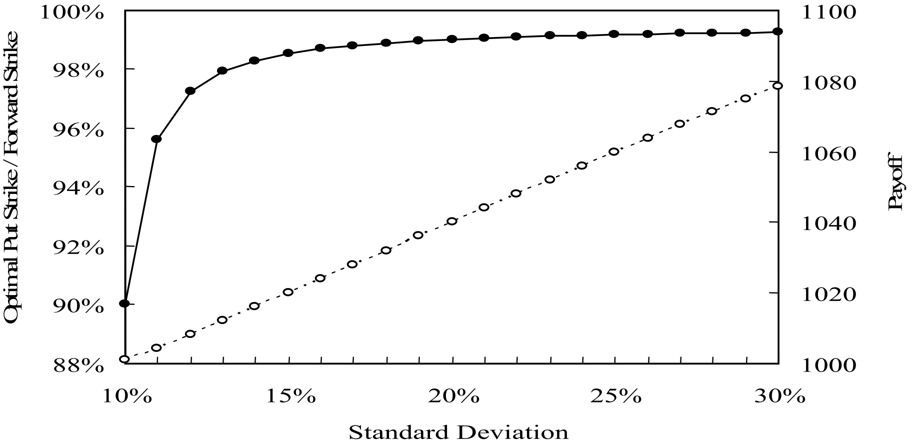


Figure 7. The relative ratio of optimal strike of put option over forward price and the incremental payoff versus the expected standard deviation. This figure is to see how close the optimal strike of put option can be with respect to the forward price depending on the expected standard deviation and also to see the incremental payoff of the put option hedging portfolio when the standard deviation increases. Solid line with solid dots represents optimal strike of put option divided by the forward price given each expected standard deviation and dotted line with hollow dots does the expected payoff of the portfolio. Expected mean is 1000, forward price 1000, implied volatility 10%, and time−to−maturity is one year

It is useful to know how sensitive the ratio between the optimal put strike and the forward price is to the relative strength of the standard deviation with respect to the implied volatility and how much the expected payoff can increase by 1% increase of the standard deviation. As exhibited in Figure 7, the optimal put strike quickly converges to 99% of the forward price and when the standard deviation is higher than the implied volatility by 4% and the optimal put strike is above 98% of the forward price. This is not trivial as in a situation where your prediction of the future standard deviation is not too marginally higher than the implied volatility, the actual payoff, not just the expectation, of the portfolio with the optimal put is just a few percent off at worst. In terms of the sensitivity of the expected payoff with respect to the expected standard deviation, it shows an almost linear relation and the expected payoff increases by 0.39% when the standard deviation increases by 1% on average.

3.2.2 When Mean is Smaller than Forward Price

Now, consider the effect of relative strength of the implied volatility with respect to the standard deviation when the mean is smaller than the forward price. Figure 8 displays the expected payoffs when the mean is smaller than the forward price for different standard deviations that are smaller than, equal to, or greater than the implied volatility. When the standard deviation is smaller than the implied volatility, the portfolio with forward contract has no smaller than expected payoff than the portfolio with put option regardless of the choice of the put strike, just like when the standard deviation is the same as the implied volatility. What is interesting is when the standard deviation is greater than the implied volatility. In this case, it is possible

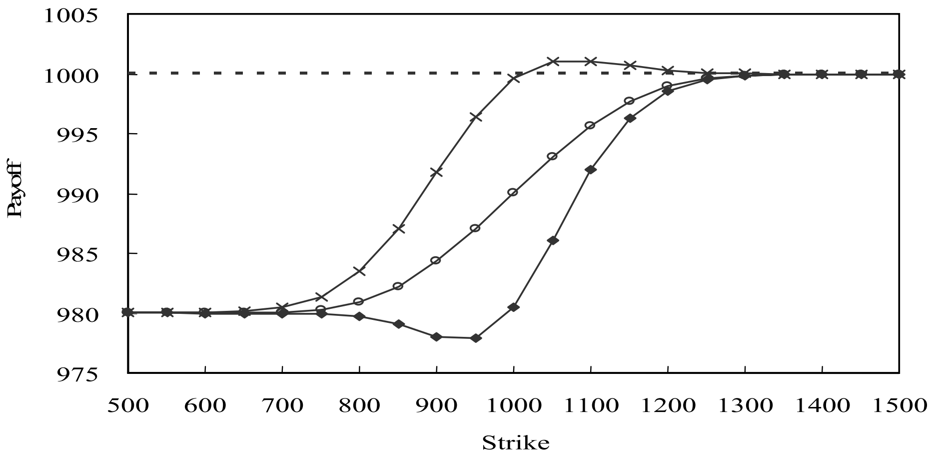


Figure 8. The expected payoff of the put option hedging portfolios and the forward hedging portfolio when the expected mean is smaller than the forward price and the expected standard deviation varies. This figure is to see the effect of change of expected standard deviation when the mean is smaller than the forward price. Solid line with solid dots represents put option hedging portfolio for the standard deviation of 7.5%, solid line with hollow dots for the standard deviation of 10%, solid line with cross for the standard deviation of 12.5% and dashed line does forward hedging portfolio. Expected mean is 980, forward price 1000, implied volatility 10%, and time-to-maturity is one year

for the expected payoff to be larger than using forward contract as hedging instrument, although the number may be too small to be seriously considered in practical sense depending on how close the mean is to the forward price and how different the standard deviation is from the implied volatility. It suffices to say that we cannot completely rule out the possibility that put option can be better hedging tool than forward contract even for the case that the expected mean is smaller than the forward price. Relative cheapness of the implied volatility versus the expected standard deviation can play such an important role in the decision of hedging strategies. In addition, it should be noted that the optimal put strike when the implied volatility is smaller than the standard deviation and the mean is smaller than the forward price is likely to be in-the-money, i.e., the strike is higher than the forward price, therefore the actual payoff at worst including the effect of the option premium is not too far from the payoff of the portfolio with forward contract.

3.2.3 When Mean is Greater than Forward Price

The relative strength of the standard deviation versus the implied volatility is a factor to be considered, too, when the mean is greater than the forward price. As demonstrated in Figure 9, the expected payoff of the portfolio with an optimal put option is higher than that of the portfolio with forward contract regardless of whether the standard deviation is smaller than, equal to, or larger than the implied volatility. It is interesting to note that even if the implied volatility is expensive compared to the expected standard deviation, put option is a better hedging tool than forward contract. The optimal strike when the standard deviation is smaller than or equal to the implied volatility is quite out-of-money, i.e., the strike is at least one or two standard deviation away from the forward price thus its premium can be very small in a pure Black-Scholes environment where there is no volatility skew, no bid-offer spread, and no margin. More interesting case is when the standard deviation is greater than the forward price. While the conclusion that put option is better than forward in that situation still holds, the optimal strike can be close to the forward price so that the difference between the optimal put strike and the forward price is no larger than one standard deviation. Besides, the expected payoff can be even larger than when the standard deviation is no greater than the implied volatility. In other words, buying a cheap volatility via option can pay off regardless of the view about the mean.

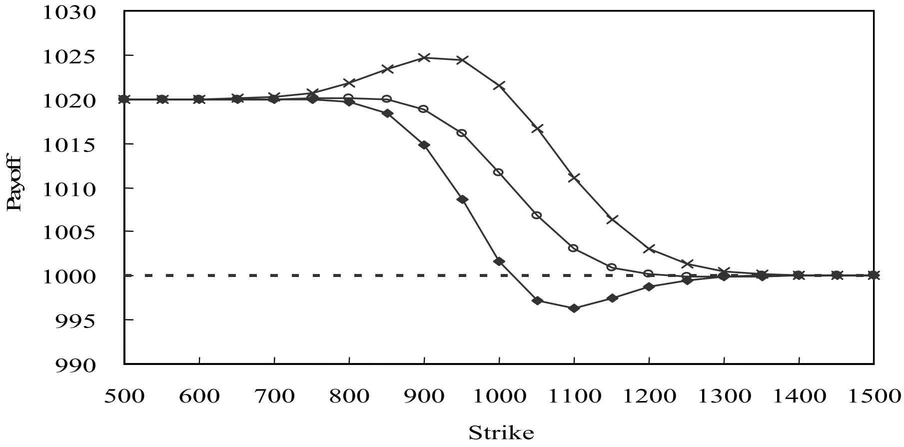


Figure 9. The expected payoff of the put option hedging portfolios and the forward hedging portfolio when the expected mean is greater than the forward price and the expected standard deviation varies. This figure is to see the effect of change of expected standard deviation when the mean is greater than the forward price. Solid line with solid dots represents put option hedging portfolio for the standard deviation of 7.5%, solid line with hollow dots for the standard deviation of 10%, solid line with cross for the standard deviation of 12.5% and dashed line does forward hedging portfolio. Expected mean is 1020, forward price 1000, implied volatility 10%, and time−to−maturity is one year

3.3 Non-equal Forward Price and Initial Spot Price

So far, we have looked at the cases that the forward price is the same as the current spot price, which is not always the case in reality. Depending on the yield of the underlying asset and the interest rate of the domestic currency, the forward price can differ from the current spot price significantly. Since all the analyses and discussions above were based on the relative magnitude of the forward price and the mean, we can just focus on whether the forward price, regardless of whether it is higher or smaller than the current spot price, is greater or smaller than the mean to determine the best hedging policy. For example, there is no fundamental difference in terms of hedging policy decision between when the forward price is 1100 and the mean is also 1100 while the current spot price is 1000, and when the forward price, the mean, and the current spot price is all 1000. What matters is not whether the forward price is greater or smaller than the current spot price, but whether it is greater or smaller than the mean.

In fact, this can raise an inherently unique issue for hedging decision. For instance, much has been debated on whether there is a risk premium in the forward price of the foreign exchange market (e.g., Froot and Thaler, 1990) but it is now well argued that the forward price has little predictive power of the future spot price and the best predictor is ironically the current spot price itself. This is referred to as forward rate bias in many articles (e.g., Aggarwal, Lucey, and Mohanty, 2006). The implication is that when there is a carry between two currency pairs, i.e., the interest rate differential is non-zero, your best prediction of the mean is the current spot and the forward price is either greater than or smaller than the predicted mean by nature. An example is the US dollar-Japanese yen exchange rate. Until most recently, the interest rate differential between US dollar and Japanese Yen was at least a few percent per annum and therefore the forward price of Japanese yen per the US dollar was chronically smaller than the prevailing spot rate. Given the difference, the US dollar-Japanese yen forward market has been dominated by Japanese importers who wish to lock in a favorable rate whereas Japanese exporters have been reluctant to use plain vanilla forward contract for their incoming US dollar cash flows. Some Japanese exporters might have let the exposure unhedged purposefully, albeit without a deeper comprehension of the problem.

3.4 Higher Moment Effects

Two completely different probability distributions can have exactly the same mean and standard deviation and certainly the expected payoffs will be different even if the averages and standard deviations are identical to each other. The question is to what extent the optimal hedging decision can be affected by a different higher moment of the distributions. While normal distribution can be a good starting point to evaluate statistical properties of real phenomena, actual distributions are likely to be different from an ideal normal distribution. The 'fat tail' is something that cannot be easily ignored. Apparently, the methodology of this paper has no difficulty in dealing with the fat tail events as there is no fundamental restriction in terms of perceived probability distribution.

In order to assess the impact of the higher moments of a distribution, it is necessary to inspect three different scenarios: i) symmetric probability distribution around the mean but has a higher kurtosis than the normal distribution, ii) a left-skewed one with the same mean as the normal distribution, and iii) a right-skewed one with the same mean as the normal distribution. For each assumed distribution, it is important to ensure that the normal distribution to compare has not only the same mean but also the same standard deviation as the assumed distribution to eliminate the effect of those lower moments of the distribution.

3.4.1 Kurtosis

A distribution in Figure 10 is an example of the symmetric probability distribution but has a higher kurtosis than the relevant normal distribution. Note that the reference normal distribution has the same mean and the same standard deviation as the distribution in case. Figure 11 and Figure 12 display the expected payoff of the portfolio hedged by put option when the standard deviation is greater or smaller than the implied volatility for a higher mean than the forward price. In overall, the payoff changes a little as expected, but it appears that the significance of a higher kurtosis is smaller than the mean or standard deviation in qualitative sense. When the standard deviation is smaller than the implied volatility, there is little impact on hedging decision as the optimal strike is almost identical at far out-of-money and the expected payoff at the optimal strike is essentially the same. However, in case that the standard deviation is greater than the implied volatility, the expected payoff at the optimal

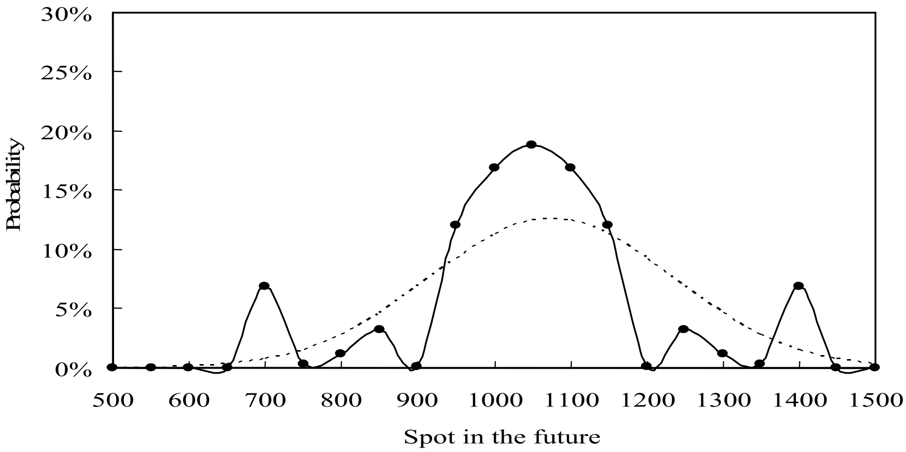


Figure 10. A symmetric probability distribution of a higher kurtosis than normal distribution. These distributions will be used in Figure 11 and Figure 12 to see the effect of kurtosis on the expected payoff of put option hedging portfolio. Solid line with solid dots represents the assumed, artificial distribution and dotted line does a normal distribution. The kurtosis is 3.806. Expected mean is 1050 and expected standard deviation is 15%

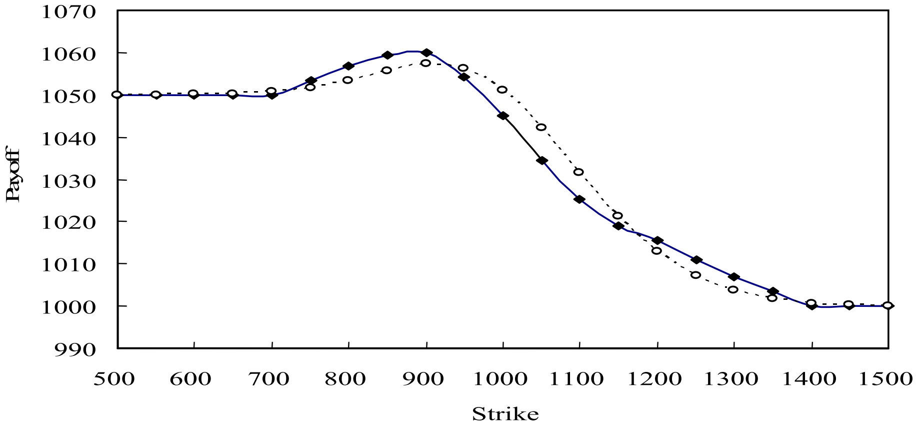


Figure 11. The expected payoff of the put option hedging portfolios under the distributions of Figure 10 when the expected standard deviation is greater than the implied volatility. This figure is to see the effect of kurtosis when the expected mean is greater than the forward price and the expected standard deviation is greater than the implied volatility. Solid line with solid dots represents put option hedging port-folio for the high kurtosis and dotted line with hollow dots does for the normal distribution. Expected mean is 1050, forward price 1000, expected standard de-viation 15%, implied volatility 10%, and time-to-maturity is one year

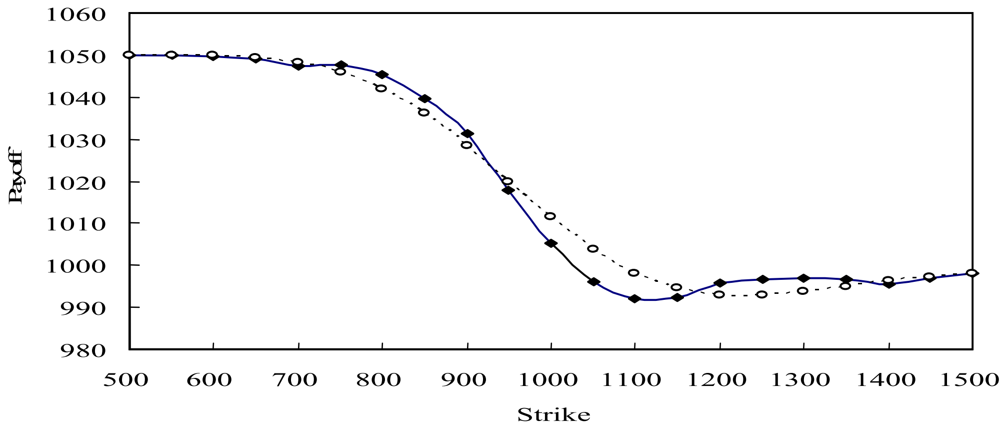


Figure 12. The expected payoff of the put option hedging portfolios under the distributions of Figure 10 when the expected standard deviation is smaller than the implied volatility. This figure is to see the effect of kurtosis when the expected mean is greater than the forward price and the expected standard deviation is greater than the implied volatility. Solid line with solid dots represents put option hedging port-folio for the high kurtosis and dotted line with hollow dots does for the normal distribution. Expected mean is 1050, forward price 1000, expected standard de-viation 15%, implied volatility 20%, and time-to-maturity is one year

strike can increase meaningfully while the optimal strike itself stays more or less the same, which can make option as a hedging tool more advantageous. This makes sense because, for the same implied volatility, i.e., price of option, the fatter the tail of the distribution the higher the expected payoff. The implication remains largely the same even when the mean is smaller than the forward price in that in this case forward contract tends to dominate option as hedging tool for most values of the standard deviation. However, it is possible that higher kurtosis overturns optimal hedging decision in favor of option in some situations as exemplified in Figure 13, which is based on a probability distribution that is similar to the one in Figure 10 but shifted to left so that the mean is smaller than the forward price.

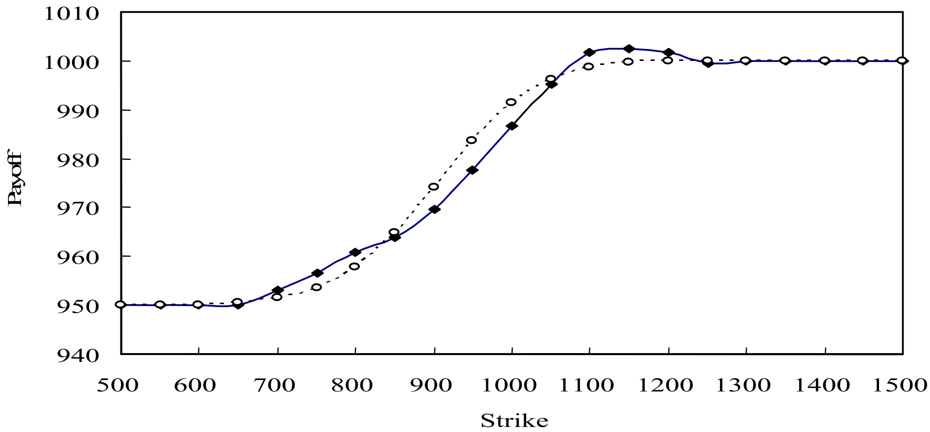


Figure 13. The expected payoff of the put option hedging portfolios under the distributions of Figure 10 when the expected mean is smaller than the forward price. This figure is to see the effect of kurtosis when the expected mean is smaller than the forward price and the expected standard deviation is greater than the implied volatility. Solid line with solid dots represents put option hedging portfolio for the high kurtosis and dotted line with hollow dots does for the normal distribution. The kurtosis is 3.886. Expected mean is 950, forward price 1000, expected standard deviation 13.85%, implied volatility 10%, and time-to-maturity is one year

3.4.2 Skewness

Skewness can also play a role in affecting optimal hedging decision. Intuitively the effect of non-zero skewness is similar to that of higher kurtosis in that optimal hedging decision can be affected to a certain extent but not as significantly as either the mean or the standard deviation. In order to verify the intuition, two discrete probability distributions are assumed as in Table 1. Note that the kurtosis of the distributions is 3.283, which is not exactly the same as that of normal distribution but reasonably close to it. Thus it is expected that the impact of kurtosis is certainly there but should not be too noticeable. As demonstrated in Figure 14 and Figure 15, when put option is involved and there is negative skewness, the expected payoff to the left is enlarged and conversely when there is positive skewness, the expected payoff to the left is suppressed.

Table 1. Two Probability Distributions that have Non−Zero Skewness

These distributions are artificially generated in order to see the effect of positive and negative skewness of the view for the price distribution in the future in relation to optimal hedging decision. By construction, the mean, the standard deviation, and the kurtosis of the two distributions are identical. These distributions are used in Figure 14 and Figure 15.

Spot Price in the future	Distribution A Probability	Distribution B Probability
500	0%	0%
550	0%	0%
600	0%	0%
650	0%	0%
700	0%	0%
750	5%	0%
800	5%	0%
850	5%	0%
900	5%	0%
950	5%	25%
1000	0%	50%
1050	0%	0%
1100	50%	0%
1150	25%	5%
1200	0%	5%
1250	0%	5%
1300	0%	5%
1350	0%	5%
1400	0%	0%
1450	0%	0%
1500	0%	0%
Mean	1050	1050
Standard Deviation	122.5	122.5
Equivalent Volatility	11.66%	11.66%
Skewness	-1.327	1.327
Kurtosis	3.283	3.283

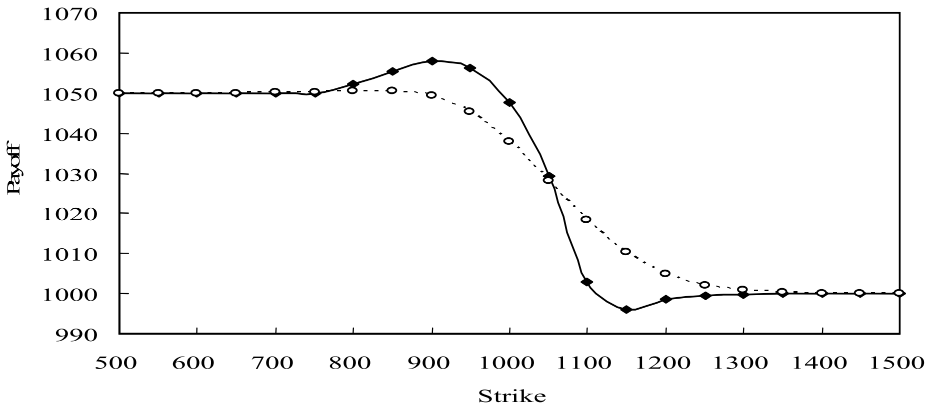


Figure 14. The expected payoff of the put option hedging portfolios under the Distribution A of Table 1 when the expected mean is greater than the forward price. This figure is to see the effect of negative skewness when the expected mean is greater than the forward price and the expected standard deviation is greater than the implied volatility. Solid line with solid dots represents put option hedging portfolio for the negative skewness and dotted line with hollow dots does for the normal distribution. The skewness is −1.327, and the kurtosis is 3.283, which is close to that of normal distribution. Expected mean is 1050, forward price 1000, expected standard deviation 11.66%, implied volatility 10%, and time−to−maturity is one year

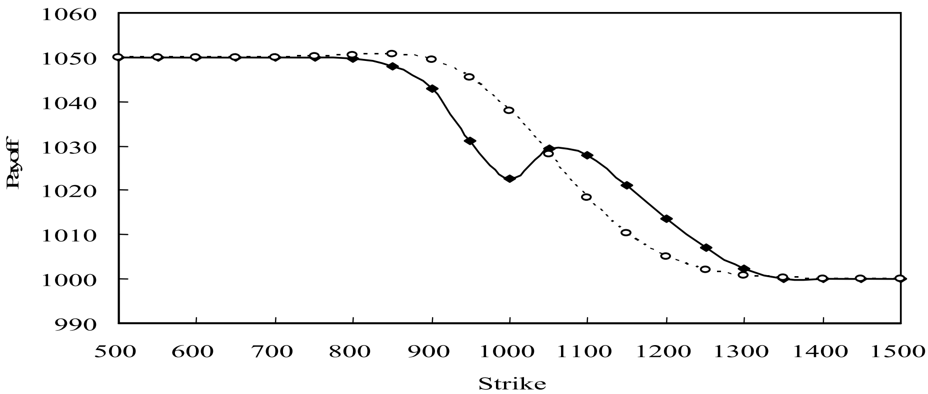


Figure 15. The expected payoff of the put option hedging portfolios under the Distribution B of Table 1 when the expected mean is greater than the forward price. This figure is to see the effect of positive skewness when the expected mean is greater than the forward price and the expected standard deviation is greater than the implied volatility. Solid line with solid dots represents put option hedging portfolio for the positive skewness and dotted line with hollow dots does for the normal distribution. The skewness is 1.327, and the kurtosis is 3.283, which is close to that of normal distribution. Expected mean is 1050, forward price 1000, expected standard deviation 11.66%, implied volatility 10%, and time−to−maturity is one year

In sum, the existence of higher moments should be considered in determining optimal hedging policy and option is more favorably affected than forward contract. Those higher moments, particularly the even numbered moments such as kurtosis, play an almost the same role in affecting hedging decision as standard deviation does but to a lesser degree. Odd numbered higher moments such as skewness can also act to increase the expected payoff of the option when the direction of skewness matches the direction of the option, e.g., negative skewness for put option and positive skewness for call option. These enhanced effects of the higher moments on to the expected payoff and the optimal strike may be somewhat nullified by the volatility smile in practice, though. In the end, hedgers ought to do their best in formulating realistic probability distributions and draw a conclusion on their hedging policy based on them.

Last but not least, one should be reminded that owning a put option even at a very remote strike for a small premium does have a virtue from an insurance-like risk management perspective. The option may have little impact on the expected payoff if the strike is too out-of-money for the option to have any expected value given the predicted probability distribution, but this can act as a last cushion in case that an unimagined situation occurs. Owning a cheap, out-of-money option for a small premium can be a prudent approach in itself.

4. Conclusion

When firms have to determine their hedging policy, they face two distinctive categories of parameters: one is given exogenously and the other endogenously. The ones that are given exogenously are actual market prices such as forward price and implied volatility, which the firms have no choice but to accept. The others that are to be formed endogenously are their view for the underlying asset price in the future. Given that firms can only determine their endogenous parameters, the latter may be where the finance managers of the firm should claim their competency in risk management, together with their hedging decision.

In order to draw qualitative conclusions on optimal hedging decision, extensive parametric study has been performed under the assumption that the underlying asset price follows a normal distribution. Given the assumption, ultimately two parameters become critical in making an optimal hedging decision in conjunction with their counterparts: forward price versus expected mean and implied volatility versus expected standard deviation. For the short hedgers who are in a long position in an underlying asset, put option is better off than forward contract when the expected mean is greater than the forward price, regardless of the relative strength of the expected standard deviation relative to the implied volatility.

Interestingly, the reverse of the above statement is not true, i.e., even if the expected mean is smaller than the forward price, it is not always the case that forward contract is better off than put option. What matters more in that situation is the relative cheapness of the implied volatility relative to the expected standard deviation. If the expected standard deviation is meaningfully greater than the implied volatility, then put option is better off than forward contract.

A case that is a little degenerate in practical sense is when the expected mean is exactly the same as the forward price. In that case, hedging with option outperforms hedging with forward contract when the expected standard deviation is greater than the implied volatility. If the standard deviation is smaller than or equal to the implied volatility, put option cannot beat forward contract in terms of expected payoff thus is definitely worse.

Symmetry can command the following rules for long hedgers who are in a short position of an underlying asset. When the expected mean is smaller than the forward price, call option is always better off than forward contract regardless of the relative cheapness of the expected standard deviation against the implied volatility. Although the expected mean is greater than the forward price, as long as the standard deviation is meaningfully greater than the implied volatility, call option is better off than forward contract.

In situations in which option is no worse than forward contract in terms of expected payoff, the formula derived in the paper can be used to find an optimal strike of the option. When the implied volatility is no smaller than the standard deviation, the optimal strike is away from the forward price by at least one to two standard deviations so the premium tends to be small. If the implied volatility is smaller than the standard deviation, the optimal strike can come close to the forward price within one standard deviation, almost the same as the forward price when the forward price is equal to the mean, and can be even greater than the forward price, i.e., the put option is in-the-money. The formula should be particularly useful for firms that want to use options for hedging.

Higher moments such as skewness and kurtosis play a certain role in affecting optimal hedging decision, although they are less critical than mean and standard deviation. The overall effect is that they tend to make the expected payoff of portfolio hedged by option more pronounced on top of the effect of standard deviation of the distribution. A part of the pronounced effect can be offset by volatility smile in practice, thus it needs to be examined as a whole together with all the main factors and market variables.

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