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Optimizing Zone-dependent Two-level Facility Location Problem

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Zone을 고려한 2단계 시설배치 계획 최적화

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This paper considers a problem of locating both distribution centers and retailers in a zone-dependent two-level distribution network where either a distribution center or a retailer should be located in each zone. Customer demands of each zone should be satisfied directly from either its own distribution center or its own retailer being supplied from a distribution center of another zone. The objective of the proposed problem is to minimize total cost being composed of distribution center/retailer setup costs and transportation costs. In the analysis, the problem is proved to be NP-hard, so that a branch-and-bound algorithm is derived for the problem. Numerical experiments show that the proposed branch-and-bound algorithm provides the optimal solution efficiently for some small problems.

Keyword: two-level, facility location, zone constraints, branch-and-bound

1. Introduction

The success or failure of business and public facilities depends in large part on their locations, which can lead to various economic benefits including increased profit, increased market share, reduced operating cost, and improved customer satisfaction for many business. Accordingly, the issue of facility location network has received an increasing research attention in recent years. Unlike the usual single-level location problems, the associated research efforts have been made to enhance the real world applicability. The two-level uncapacitated facility location problem can be stated as follows. A single product is transported from some distribution centers to some retailers and then to customers, or from some distribution centers to customers directly. The objective is to determine suitable locations from a set of potential distribution centers and retailers, such that the total cost is minimal. This cost consists of distribution center/retailer setup costs and transportation costs. <Figure 1>

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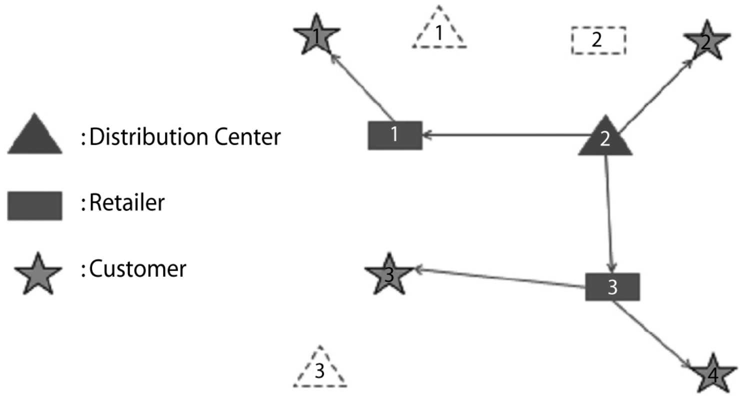

Figure 1. A general two-level facility location problem

represents theassociated general two-level facility location problem.

Recently, the issue of improving service quality and customer satisfaction have become as important as maximizing total profit or minimizing total cost. Accordingly, this paper considers the issue of not only minimizing total costs, but also satisfying fixed service quality. In case of just transporting products, the issue of locating distribution centers and retailers effects total cost only. In case of applying to post offices or service centers, the location issue affects not only total cost but also customer satisfaction. Satisfying a fixed customer service level in terms of total transportation cost and facility location is important. The issue of satisfying the fixed customer service level can be expressed subject to zone constraints such that either a distribution center or a retailer should be located in each zone. Accordingly, the objective of the associated zone-dependent two-level facility location problem can be defined as to minimize total cost subject to such zone constraints. <Figure 2> depicts a zone-dependent two-level facility location problem.

The zone-constraints mean that customer demands of each zone should be satisfied directly from either its own distribution center or its retailer being supplied from a distribution center of another zone. For example(<Figure 2>), if there are no zone-constraints, it will be optimal to satisfy the demands of customer 3 by retailer 3, while due to zone-constraints, it will be optimal to satisfy the demands of customer 3 demands by distribution center 3 but it will be infeasible to satisfy customer 3 by retailer 3. In case of foreign trades between different countries(especially for cigarettes or alcohol), it may be allowed to locate production plants or retailers trade declares in foreign countries, while it may not be allowed to sell products to foreign customers directly. For the case, it could be better to apply not a general two-level facility location problem but the zone-dependent two-level facility location problem.

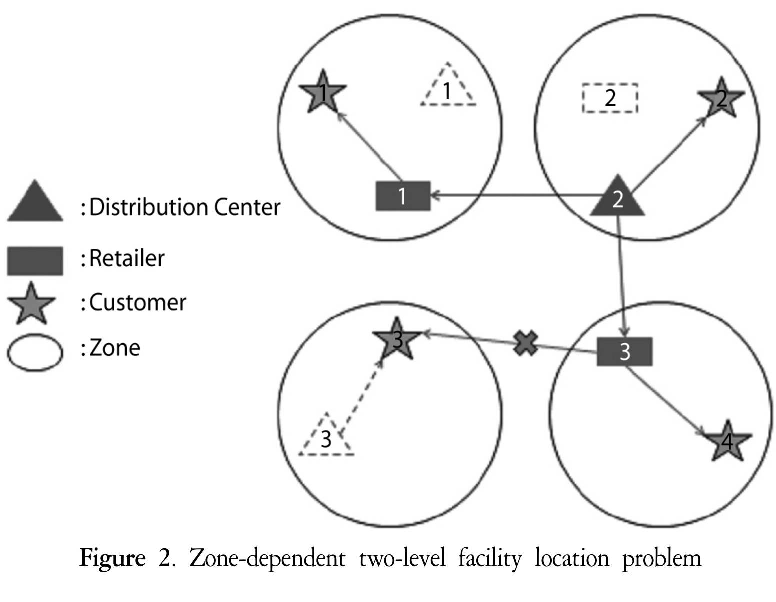

Figure 2. Zone-dependent two-level facility location problem

2. Literature Review

Multi-level facility location problems have been widely studied in the literature. For example, Daskin (1995) have considered multi-level facility location problems where the relations between levels were considered explicitly. Two review papers, Narula SC (1986) and Church R and Eaton DJ (1987), have also discussed on multi-level facility location problems. A classification of multi-level p-median problems have been proposed in Narula (1984) where the multi-level relations among the different facility types and the flow of goods/services allowed among them were involved. Narula (1984) has reviewed multi-level p-median problems applied to several multi-level networks with the classification scheme. Sahin *et al*. (2007) have considered a two-level multi-flow assignment-based model for regionalization of blood services. Gao and Robinson (1992) have related fixed costs with linked pairs of facilities from both levels, instead of associating with lower-level facilities being capacitated and served by only one higher-level facility, where each demand was single-sourced by a lower-level facility.

The above studies have commonly derived near-optimal solutions based on lagrangean or LP relaxations. Branch-and-bound procedures with specific branching rules have also been proposed. Tcha and Lee (1984) have used a dual ascent procedure to solve the lagrangean dual, a primal descent heuristic, and node simplication rules in their branch-and-bound algorithm. Barros and Labbe (1994) have connected the solution methods in Tcha and Lee (1984) regarding the node simplication procedure. Ro and Tcha (1984) have used a greedy heuristic to obtain upper and lower bounds and proposed node simplication rules. In Gao and Robinson (1992), the primal and dual procedures preceded by the branch-and-bound algorithm have been applied to solve the problems of 35 demand sites and 25 location sites. Barros and Labbe (1994) have considered a general solution approach for multi-flow versions of the model, based on lagrangean relaxation of the weak and strong formulations of the problem, where the upper bounds were attained by a greedy heuristic. Aardal *et al*. (1996) have employed a polyhedral theory where single-level problem facets were adapted into the two-level problem in addition to the new facets. Tragantalerngsak *et al*. (1997) have considered a capacitated version by developing several lagrangean relaxations to find both lower and upper bounds, which were composed with Barros and Labbe (1994). Chardaire (1999) have considered a capacitated version of the concentrator problem in Chardaire *et al*. (1999), using lagrangean relaxation of the assignment-based formulation.

In summary, most of the references in the literature have derived lagrangean relaxations, branch-and-bound procedures and conventional heuristic methods for multi-level location problems. In addition, facility location problem with zone constraints has not been studied.

3. Problem Description and Formulation

The zone-dependent two-level uncapacitated facility location problem (ZTUFLP) is described as follows. Given a set of locations where distribution centers may be built, a set of locations where retailers may be built, a known demand from a given set of customers of each zone which must be satisfied, either a distribution center or a retailer should be located in each zone, and customer demands of each zone should be satisfied directly from either its own distribution center or its own retailer being supplied from a distribution center of another zone. The objective of the proposed problem is to minimize total cost consisting of distribution center/retailer setup costs and transportation costs. The following notation is introduced throughout the rest of this paper.

- f_i : The fixed cost associated with the distribution center i
- g_j : The fixed cost associated with the retailer j
- c_{ij} : The total transportation cost of zone $k(j \in J_k)$ when all demand of zone k is transported from the distribution center i to the retailer j
- x_{ij} : The fraction of the demand of zone k satisfied by the distribution center i through the retailer $j(j \in J_k)$
- y_i : If the distribution center i is open, y_i is 1; Otherwise, y_i is 0.
- z_j : If the retailer j is open, z_j is 1; Otherwise, z_j is 0.
- K : The index set of zones
- I_k : The index set of distribution center candidates in zone k
- J_k : The index set of retailer candidates in zone k
- TC : Total cost

The proposed problem can be formulated as follows;

Minimize

$$TC = \sum_{k\in K}\sum_{i\in I_k} f_i y_i + \sum_{k\in K}\sum_{j\in J_k} g_j z_j + \sum_{k\in K}\sum_{i\in I_k}\sum_{l\in K|l\neq k}\sum_{j\in J_l} c_{ij}x_{ij} \quad (1)$$

Subject to

$$\sum_{i\in I_k} y_i + \sum_{j\in J_k} z_j = 1 \qquad \forall k \quad (2)$$

$$x_{ij} \le y_i \qquad \forall i\in I_k,\ \forall j\in J_l,\ \forall l\in K\backslash\{k\},\ \forall k \quad (3)$$

$$\sum_{l\in K\backslash\{k\}}\sum_{i\in I_l} x_{ij} = z_j \qquad \forall j\in J_k,\ \forall k \quad (4)$$

$$y_i,\ z_j\in\{0,1\},\ 0\le x_{ij}\le 1 \qquad \forall i\in I_k,\ \forall j\in J_l,\ \forall l\in K\backslash\{k\},\ \forall k \quad (5)$$

In the formulation, the objective function (1) is to minimize the sum of the fixed costs and delivery costs. Constraints (2) require that either a distribution center or a retailer should be located in each zone. Constraints (3) require that if the demand is transported from the distribution center i to the retailer j, the distribution center i will be required to be open. Constraints (4) require that if the retailer j is open, the sum of the fractions of the demands of zone k which includes the retailer j should be 1. And constraints (5) indicate that distribution centers and retailers are located entirely or not at all, and that the fractions of the demands are non-negative.

4. Problem Analysis

The objective of the proposed problem is to find the optimal locations of distribution centers and retailers which minimize the total cost. It is noted that the ZTUFLP is NP-hard. Suppose that the number of candidate sites of each zone is 1, and it is possible to locate a distribution center or a retailer in that candidate site, and the fixed costs of all retailers are 0. Then, the problem ZTUFLP is the same as the uncapacitated facility location problem (UFLP). This means that UFLP is a special case of ZTUFLP and any instance of UFLP can be reduced to an instance of ZTUFLP. It has been known that the problem UFLP is NP-hard(Francis and Mirchandani (1990)). Accordingly, the problem ZTUFLP is NP-hard, which implies that the proposed problem cannot be solved by any commercial software including CPLEX in polynomial time. Therefore, a branch-and-bound algorithm is proposed to solve the problem, for small-size cases, based on some branching and bounding rules.

Several efficient branch-and-bound algorithms have been proposed for a general two-level uncapacitated facility location problem. For example, Kaufman L, Eede V and Hansen P (1977) has considered an efficient branch-and-bound algorithm for a general problem. Ro and Tcha (1984) have been considered an branch-and-bound algorithm for a two-level uncapacitated facility location problem with some side constraints. Let $\left(x^*_{pj},\ y^*_p,\ z^*_j\right)$ be the optimal solution of ZTUFLP.

Lemma 1 : x^*_{pj} has a binary value.

Proof) Suppose that x^*_{pj} is a fractional value. Then, there exists q such that x^*_{qj} has a fractional value, since $\sum_k\sum_{i\in I_k} x_{ij} = 1$. Then, it follows that supplies from any two distribution centers at the same time are not optimal in comparison with one from the single distribution center, because no supply capacity is considered. This implies the contradiction to the hypothesis. □

By Lemma 1, if some of x_{ij}'s has fractional values, then the association solution cannot be optimal. Therefore, the solution set of x_{ij}'s are reduced from $0\le x_{ij} \le 1$ to $x_{ij}\in\{0,\ 1\}$.

Lemma 2 : If $x^*_{pj} = 1$, then $y^*_p = 1,\ z^*_j = 1$.

Proof) Suppose that $x^*_{pj} = 1$ and $y^*_p = 0$. This implies that the solution $x^*_{pj} > y^*_p$ holds. This is the contradiction to the relaxation $x_{ij} \le y_i$. In the same way, suppose that $x^*_{pj} = 1$ and $z^*_j = 0$. Then, it follows that $\sum_{l\in K\backslash\{k\}}\sum_{i\in I_l} x_{ij} > z_j$, which is contradiction to the relation $\sum_{l\in K\backslash\{k\}}\sum_{i\in I_l} x_{ij} = z_j$. □

Lemma 3 : If $z^*_j = 1$, then $x^*_{pj} = 1$ such that

$$p = \arg\min_i\{c_{ij}|y_i = 1\} \qquad \forall i.$$

Proof) According to Lemma 1, x^*_{pj} has binary value. Suppose that $z^*_j = 1,\ x^*_{pj} = 0$ and $x^*_{qj} = 1$ such that $y_q = 1,\ \ p\neq q$. There are no capacities in this problem. Therefore, if the solution is changed as $x^*_{pj} = 1$ and $x^*_{qj} = 0$, then the total cost is reduced by $c_{qj} - c_{pj}\,(>0)$. This implies that the corresponding solution cannot be optimal. □

By Lemma 2 and Lemma 3, if all of y_i^* and z_j^* are found, then x_{ij}^* can be found without any extra procedure. Therefore, the optimal solution can be found by only searching the solution set of y_i and z_j.

5. Solution Approach

The proposed problem is NP-hard, so that a branch-and-bound algorithm is derived for small-size problem.

5.1 Formulation

The following notation is introduced to explain the some associated mathematical formulation and the conditions of each variable at each node, needed for a branch-and-bound procedure.

P : Total set of distribution centers
Q : Total set of retailers
K : Total set of zones
$P_0 = \{i|y_i = 0,\ \ i \in I_k,\ k \in K\}$
$P_1 = \{i|y_i = 1,\ \ i \in I_k,\ k \in K\}$
$P_2 = P \setminus (P_0 \cup P_1) : free\ y_i$
$Q_0 = \{j|z_j = 0,\ \ j \in J_k,\ k \in K\}$
$Q_1 = \{j|z_j = 1,\ \ j \in J_k,\ k \in K\}$
$Q_2 = Q \setminus (Q_0 \cup Q_1) : free\ z_j$
K_1 : Zone set which has a distribution center or a retailer fixed by 1
$K_2 = K \setminus K_1$: Zone set which does not have a distribution center or a retailer fixed by 1

The following formulation is introduced to find the total cost at each node during a branch-and-bound procedure.

Minimize

$$TC = \sum_{i \in P_1} f_i + \sum_{j \in Q_1} g_j + \sum_{i \in P_2} f_i y_i + \sum_{j \in Q_2} g_j z_j + \sum_{i \in P_1 \cup P_2} \sum_{j \in Q_1 \cup Q_2} c_{ij} x_{ij} \quad (6)$$

Subject to

$$\sum_{i \in I_k \setminus P_0} y_i + \sum_{j \in J_k \setminus Q_0} z_j = 1 \qquad \forall k \in K_2 \quad (7)$$

$$x_{ij} \le y_i \qquad \forall i \in P_1 \cup P_2,\ \forall j \in Q_1 \cup Q_2 \quad (8)$$

$$\sum_{l \in K \setminus \{k\}} \sum_{i \in P_1 \cup P_2, I_l} x_{ij} = z_j \quad \forall j \in Q_1 \cup Q_2, J_k, \forall k \quad (9)$$

$$y_i, z_j \in \{0, 1\}, 0 \le x_{ij} \le 1 \ \forall i \in P_1 \cup P_2,\ \forall j \in Q_1 \cup Q_2 \quad (10)$$

5.2 Branching Rules

This section proposes several branching rules to promote the efficiency of the branch-and-bound algorithm. Five branching rules in total are developed for use in the proposed branch-and-bound algorithm. One branching rule is for zone constraints(Either a distribution center or a retailer should be located in each zone). Two branching rules are for getting free retailers closed and one branching rules is for getting free distribution centers closed. The last branching rule is for getting free distribution centers opened.

Lemma 4 : (Branching Rule 1)
If $i \in I_k, P_1$, then $p \in P_0\ \forall p \in I_k \setminus \{i\}$ and $j \in Q_0\ \forall j \in J_k$. In the same way, if $j \in J_k, Q_1$, then $i \in P_0\ \forall i \in I_k$ and $n \in Q_0\ \forall n \in J_k \setminus \{j\}$.
Proof) Suppose the results are false. Then, constraints (7)$\left(\sum_{i \in I_k \setminus P_0} y_i + \sum_{j \in J_k \setminus Q_0} z_j = 1\right)$ cannot be satisfied. □

By Lemma 4, if one distribution center or retailer is fixed at the value 1, the other distribution centers and retailers which are located in the same zone can be fixed at the value 0 without any extra procedure.

Lemma 5 : (Branching Rule 2)
For $k \in K_2$, if $\min_{p \in I_k, P_2}\{f_p\} < [g_j + \min_{i \in (P_1 \cup P_2 \setminus I_k)}\{c_{ij}\}]_{j \in J_k, Q_2}$, then $j \in Q_0$.
Proof) Suppose that $j \in Q_1$. Then, the associated possible minimum cost of zone k can be computed as $g_j + \min_{i \in (P_1 \cup P_2) \setminus I_k}\{c_{ij}\}$, and it follows that $\min_{p \in I_k, P_2}\{f_p\} < g_j + \min_{i \in (P_1 \cup P_2 \setminus I_k)}\{c_{ij}\}$, so that instead of retailers, locating a new distribution center does not affect the total cost(except the cost of its own zone) negatively. This contradicts to the hypothesis. □

By Lemma 5, if the possible minimum cost related with the retailer j in zone k is greater than the minimum fixed cost among distribution centers in zone k, then z_j can be fixed at the value 0 without any extra procedure.

Lemma 6 : (Branching Rule 3)
For $k \in K_2$, if $[g_j + \min_{i \in (P_1 \cup P_2) \setminus I_k}\{c_{ij}\}]_{j \in J_k, Q_2} >$, $[g_l + \max_{i \in (P_1 \cup P_2) \setminus I_k}\{c_{il}\}]_{l \in J_k, Q_2}$ then $j \in Q_0$.
Proof) Suppose that $j \in Q_1$. Then, the associated

possible minimum cost of zone k can be computed as $g_j + \min_{i\in(P_1\cup P_2)\backslash I_k}\{c_{ij}\}$. Moreover, suppose that $l \in Q_1$. Then the associated possible maximum cost of zone k can be computed as $g_l + \min_{i\in(P_1\cup P_2)\backslash I_k}\{c_{il}\}$. Then, it follows that $g_j + \min_{i\in(P_1\cup P_2)\backslash I_k}\{c_{ij}\} > g_l + \min_{i\in(P_1\cup P_2)\backslash I_k}\{c_{il}\}$. This contradicts to the hypothesis. □

By Lemma 6, if the possible minimum cost related with the retailer j in zone k is greater than the possible maximum cost related with the retailer l in zone k, then z_j can be fixed at the value 0 without any extra procedure.

Lemma 7 : (Branching Rule 4)

For $k\in K_2$, if $f_i > f_p$, $c_{ij} \geq c_{pj}$ $i, p\in I_k$, $\forall j\in (Q_1\cup Q_2)\backslash J_k$, then $i\in P_0$.

Proof) In zone k, the cost of locating the distribution center i is greater than the cost of locating the distribution center j, because of $f_i > f_p$. Moreover, instead of distribution center p, locating the distribution center i cannot reduce the costs of other zones, because of the relations, $c_{ij} \geq c_{pj}$ $i, p\in I_k$, $\forall j\in(Q_1\cup Q_2)\backslash J_k$. Then, it can be concluded that the solution including $i\in P_1$ cannot be optimal. □

By Lemma 7, if the fixed cost of the distribution center i is greater than the fixed cost of the distribution center p, and all of the possible transportation costs from the distribution center i than those from the distribution center p. Then, y_i can be fixed at the value 0 without any extra procedure.

Lemma 8 : (Branching Rule 5)

For $k\in K_2$, if $f_i > f_p, c_{ij} \geq c_{pj}$ $p\in I_k$, $\forall i\in I_k\backslash\{p\}$, $\forall j\in(Q_1\cup Q_2)\backslash J_k$ and $f_p < \min_{n\in J_k, Q_2}[g_n + \min_{m\in(P_1\cup P_2\backslash I_k)}\{c_{mn}\}]$, then $p\in P_1$.

Proof) According to Lemma 5 and the solution $f_p < \min_{n\in J_k, Q_2}[g_n + \min_{m\in(P_1\cup P_2\backslash I_k)}\{c_{mn}\}]$, it follows that $n\in Q_0$ $\forall n\in J_k$. According to Lemma 7 and the relations, $f_i > f_p, c_{ij} \geq c_{pj}$ $p\in I_k, \forall i\in I_k\backslash\{p\}, \forall j\in (Q_1\cup Q_2)\backslash J_k$, it follows that $i\in P_0$ $\forall i\in I_k\backslash\{p\}$. This leads to the relation $p\in P_1$. □

By Lemma 8, if the possible cost related with the distribution center i is smaller than the possible minimum cost related with any other distribution centers and retailers in the same zone, then y_i can be fixed at the value 1.

5.3 Bounding Rules

This section proposes three bounding procedures to promote the efficiency of the branch-and-bound algorithm. Let Lower Bound 1 correspond to LP relaxation which can be obtained by replacing $y_i, z_j\in\{0,1\}$ by $0 \leq y_i \leq 1, 0 \leq z_j \leq 1$ in constraints (10).

Lemma 9 : (Lower Bound 2)

$$TC_{LB2} = \sum_{i\in P_1} f_i + \sum_{j\in Q_1}\left[g_j + \min_{i\in P_1\cup P_2}\{c_{ij}\}\right] + \sum_{k\in K_2}\min\left[\min_{i\in I_k}\{f_i\}, \min_{\substack{i\in P_1\cup P_2\\ j\in J_k, Q_2}}\{g_j + c_{ij}\}\right]$$

Proof) TC_{LB2} is the objective function derived by eliminating the constraints (8) including $x_{ij} \leq y_i$ $\forall i \in P_2$, $\forall j\in Q_1\cup Q_2$. Then, clearly the value of TC_{LB2} is less than or equal to the objective value. □

Lemma 10. (Lower Bound 3)

$$TC_{LB3} = \sum_{i\in P_1} f_i + \sum_{j\in Q_1} g_j + \sum_{\substack{i\in P_1\\ j\in Q_1}} c_{ij} + \sum_{k\in K_2} C_k$$

where

$$C_k = \min\begin{cases} g_j + c_{ij} & i\in(P_1\cup P_2)\backslash I_k, j\in Q_2, J_k \\ f_i + \sum_{q\in Q_1}\min\left[\left(c_{iq} - \min_{r\in P_1}\{c_{rq}\}\right), 0\right] & i\in P_2, I_k \end{cases}$$

Proof) In case of the relation $g_j + c_{ij} \leq f_i + \sum_{q\in Q_1}\min[(c_{iq} - \min_{r\in P_1}\{c_{rq}\}), 0]$, the total cost of zone k is less than or equal to the optimal total cost of zone k, since it is same process as Lower Bound 2. In case of the relation $g_j + c_{ij} > f_i + \sum_{q\in Q_1}\min[(c_{iq} - \min_{r\in P_1}\{c_{rq}\}), 0]$, the total cost of zone k is equal to the optimal total cost of zone k, since it is the same process as finding the objective value. Therefore, TC_{LB3} is less than or equal to the optimal total cost. □

5.4 The Heuristic

The proposed heuristic algorithm is valuable itself to solve the problem efficiently, and it can also be used to find the upper bound of a branch-and-bound algorithm.

The heuristic (Upper Bound 1)

Step 1) If there is at least one distribution center in any existing fixed area, go to Step 2. Otherwise, select a distribution center whose fixed cost is minimum. Then, fix the zone which contains the selected distribution center.

Step 2) Except the fixed area, find a distribution center whose fixed cost is minimum. Then, find a retailer whose sum of fixed cost and transportation cost(from the distribution center fixed at the value 1) is minimum. Comparing the two costs, select a distribution center or a retailer whose cost is smaller. Then, fix the zone which contains the newly selected distribution center or retailer.

Step 3) Repeat Step 2 until all zones contain the fixed distribution center or retailer.

Step 4) If all zones contain their associated fixed distribution center or retailer, compute the total cost. Then, substitute the value for TC_{UB1}.

5.5 Branch-and-bound Algorithm

A branch-and-bound algorithm makes an intelligently structured search on the space of all feasible solutions. The efficiency of a branch-and-bound algorithm depends strongly on the branching rules and on the upper and lower bounds. The algorithm stops when all nodes of the search tree are either fathomed or solved. At that point, all non-fathomed subregions may have their upper and lower bounds equal to the global minimum of the function. The proposed branch-and- bound algorithm adapts the idea of Kaufman L, Eede V and Hansen P(1977) to apply to the zone-dependent two-level facility location problem. The overall procedure of a branch-and-bound is derived as follows;

Step 1) Initialization. Set z_{opt}, the value of the best solution found so far, equal to an arbitrarily large constant.

Step 2) The first direct optimality test : Compute TC_{LB1}, TC_{LB2}, and TC_{LB3}. If $\max(TC_{LB1}, TC_{LB2}, TC_{LB3}) \geq z_{opt}$, go to Step 8.

Step 3) The second direct optimality test : Compute TC_{UB1}. If $TC_{UB1} \leq z_{opt}$, go to Step 8.

Step 4) The resolution test : If all variables y_i and z_j are fixed at the value 0 or 1, keep the associated solution, update z_{opt} and go to Step 8.

Step 5) The first conditional optimality test : For $\forall k \in K_1$, apply Lemma 4 to the last variables fixed at the value 1.

Step 6) The second conditional optimality test : For $\forall k \in K_2$, apply Lemma 5, 6, 7 and 8.

Step 7) Choice step. If one variable at least has been fixed during the last application of the test in Steps 5 and 6, go to Step 2. Otherwise, let

$$F = \min_{i \in I_k, P_2, k \in K_2} \left[f_i + \sum_{q \in Q_1} \min\left\{ \left(c_{iq} - \min_{r \in P_1}\{c_{rq}\}\right), 0 \right\} \right]$$

$$p = \arg\min_{i \in I_k, P_2, k \in K_2} \left[f_i + \sum_{q \in Q_1} \min\left\{ \left(c_{iq} - \min_{r \in P_1}\{c_{rq}\}\right), 0 \right\} \right]$$

$$G = \min_{i \in (P_1 \cup P_2)\setminus I_k, j \in J_k, Q_2, k \in K_2}\{g_j + c_{ij}\}$$

$$q = \arg\min_{i \in (P_1 \cup P_2)\setminus I_k, j \in J_k, Q_2, k \in K_2}\{g_j + c_{ij}\}$$

If $F \leq G$, then fix y_p at the value 1. Otherwise, fix z_q at the value 1. Then go to Step (b).

Step 8) Backtracking. Seek the last variable y_i or z_j fixed at the value 1. If no such variable remains, end. Otherwise, fix that variable at the value 0, and free all variables fixed at the value 0 after y_i or z_j is fixed and go to Step 2.

6. Numerical Experiments

A set of numerical experiments are performed to evaluate the performance of the proposed algorithm. The proposed branch-and-bound algorithm is coded in C language and run on a 3.00GHz pentium processor with 3.00GB RAM with CPLEX version 9.

A set of test problem instances are constructed in order to test the proposed branch-and-bound algorithm. All of the distribution centers and the retailers are assigned to arbitrary coordinates at a 2-dimensional space(30×30) which is divided by $|k|$ zones previously. The fixed costs for locating distribution centers have been estimated by the formula $f_i = 50 \times r(i) + 30$ where $r(i)$ is the random integer number from 0 to 9. Fixed costs for locating retailers have been estimated by the formula $g_j = 30 \times r(j) + 10$ where $r(j)$ is the random integer number from 0 to 9. If the distribution centers and the retailers are located in the same zone, transportation costs from distribution centers to retailers are 0. Otherwise, they have been estimated by the formula $c_{ij} = 0.5 \times r(j) \times d(ij)$ where $d(ij)$ is the distance between the distribution center i and the retailer j. $|P|$ means the number of total distribution centers with $|P| \in \{200, 400, 800, 1600, 3200\}$. $|Q|$ means the number of total retailers with $|Q| \in \{400, 800, 1600, 3200\}$. $|K|$ means the number of total zones with $|K| \in \{100, 256, 400\}$. The dimensions $(|P| \times |Q| \times |K|)$ of the test problem are 200×400×100, 400×400×100, 400×800×100, 800×800×100, 400×800×256, 800×800×256, 800×1600×256, 1600×1600×256, 800×1600×400, 1600×1600×400, 1600×3200×400.

<Table 1> shows the performance of the branch-and-bound algorithm. The forth and fifth columns are the number of selected distribution centers $(= N_{DC})$ and retailers $(= N_R)$. The sum of the forth and fifth columns should be the same as the number of zones. The sixth column is the optimal objective value of the problem solved by branch-and-bound and CPLEX $(= TC^*)$. And the last two columns of the table specify the CPU times which is measured to compare the efficiency of the proposed branch-and- bound algorithm and CPLEX.

Table 1. The performance of the branch-and-bound algorithm

$\|P\|$	$\|Q\|$	$\|K\|$	N_{DC}	N_R	TC^*	CPU times(B&B)	CPU times(CPLEX)
200	400	100	16	84	9256.21	2.00	2.67
400	400	100	19	81	9090.22	2.42	6.42
400	800	100	17	83	8209.69	3.59	29.39
800	800	100	19	81	8139.22	9.88	43.21
400	800	256	36	220	23932.69	10.64	19.72
800	800	256	42	214	23554.60	18.55	57.14
800	1600	256	38	218	21548.66	34.52	95.20
1600	1600	256	41	215	21612.84	49.20	245.96
800	1600	400	72	328	36785.01	48.27	87.63
1600	1600	400	89	311	34859.22	85.20	310.42
1600	3200	400	80	320	32094.28	97.01	1682.30
3200	3200	400	94	306	31974.98	125.69	4702.38

Table 2. The analysis of the branch-and-bound algorithm

$\|P\|$	$\|Q\|$	$\|K\|$	case	N_{DC}	N_R	TC^*	CPU times(B&B)
200	400	100	1	16	84	9256.21	2.00
			2	11	89	9589.75	1.05
			3	43	57	15314.63	0.84
400	400	100	1	19	81	9090.22	2.42
			2	17	83	9917.57	1.84
			3	45	55	14260.12	1.44
400	800	100	1	17	83	8209.69	3.59
			2	15	85	8654.19	3.55
			3	30	70	13180.97	2.52
800	800	100	1	19	81	8139.22	9.88
			2	19	81	8316.94	6.86
			3	40	60	12683.09	4.41
400	800	256	1	36	220	23932.69	10.64
			2	22	234	25323.40	10.75
			3	80	176	39498.69	9.09
800	800	256	1	42	214	23554.60	18.55
			2	42	214	29987.68	15.92
			3	58	198	36633.02	13.63
800	1600	256	1	38	218	21548.66	34.52
			2	23	233	23896.47	30.27
			3	77	179	36163.74	28.36
1600	1600	256	1	41	215	21612.84	49.20
			2	39	217	24085.33	48.96
			3	51	205	38021.95	39.95
800	1600	400	1	72	328	36785.01	48.23
			2	64	336	39186.33	47.21
			3	82	318	58831.20	42.01
1600	1600	400	1	89	311	34859.22	85.20
			2	80	320	37665.84	80.21
			3	104	296	55438.96	76.20
1600	3200	400	1	80	320	32094.28	97.01
			2	77	323	33562.96	95.20
			3	81	319	50423.10	82.43
3200	3200	400	1	94	306	31974.98	125.69
			2	79	321	34581.35	119.87
			3	101	299	51528.71	111.24

As seen in <Table 1>, the branch-and-bound algorithm gives the optimal solutions and is more efficient than CPLEX, while the proposed branch-and-bound algorithm is suitable for solving ZTUFLP.

<Table 2> shows the analysis of the branch-and-bound algorithm. The first case is same as <Table 1>. The second case is changing the fixed costs for locating distribution centers to $f_i = 100 \times r(i) + 60$ from the first case($f_i = 50 \times r(i) + 30$). And the third case is changing the fixed costs for locating retailers to $g_j = 60 \times r(j) + 20$ from the first case($g_j = 30 \times r(j) + 10$).

As seen in <Table 2>, if the average fixed costs for locating distribution centers is doubled, the number of selected distribution centers and the optimal objective value are not serious affected(by first and second case). But doubling the average fixed costs for locating retailers affect the number of selected distribution centers and the optimal objective value. The number of selected distribution centers is increased and the optimal objective value is about one and a half times as large as the value of first and second case. And the CPU times is not affected by the average fixed costs for locating distribution centers and retailers.

7. Conclusion

This paper considers a problem of locating distribution centers and retailers together in a zone-dependent two-level distribution network where either a distribution center or a retailer should be located in each zone. The proposed problem is NP-hard, so that a branch and bound algorithm is derived for small-size problems with some branching rules and bounding rules derived for the algorithm efficiency promotion. And the heuristic which is also used for upper bound is proposed for large-size problems. The results of numerical experiments show that the proposed algorithm solves the zone-dependent two-level facility location problem for small-size cases within a reasonable amount of time and produces optimal solutions. The results of this study may immediately be applied for not only locating service centers or for locating public facilities but also foreign trades between different countries.

For further study, it may be interested in an extension of the proposed model to consider capacities of distribution centers, and also to decide the number and the range of zones. Any further consideration about more effective heuristic(e.g. Tabu Search) would be interesting to solve large-size problems.

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