Outage Probability Analysis of Multiuser MISO Systems Exploiting Joint Spatial Diversity and Multiuser Diversity with Outdated Feedback

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Abstract

In this paper, the outage performance of multiuser multiple-input single-output (MISO) systems exploiting joint spatial and multiuser diversities is investigated for Rayleigh fading channels with outdated feedback. First, we derive closed-form exact outage probabilities for the joint diversity schemes that combine user scheduling with different spatial diversity techniques, including: 1) transmit maximum-ratio combining (TMRC); 2) transmit antenna selection (TAS); and 3) orthogonal space-time block coding (OSTBC). Then the asymptotic outage probabilities are analyzed to gain more insights into the effect of feedback delay. It is observed that with outdated feedback, the asymptotic diversity order of the multiuser OSTBC (M-OSTBC) scheme is equal to the number of transmit antennas at the base station, while that of the multiuser TMRC (M-TMRC) and the multiuser TAS (M-TAS) schemes reduce to one. Further by comparing the asymptotic outage probabilities, it is found that the M-TMRC scheme outperforms the M-TAS scheme, and the M-OSTBC scheme can perform best in the outage regime of practical interest when the feedback delay is large. Theoretical analysis is verified by simulation results.

Keywords: Multiple-input single-output (MISO) systems, spatial diversity, multiuser diversity, outage probability, outdated feedback

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1. Introduction

Recently, multiple-input multiple-output (MIMO) has been well acknowledged as an essential technology to enhance the performance of wireless networks. In point-to-point MIMO systems, the multiple antennas can be exploited to increase the data rate through spatial multiplexing or to provide reliable transmissions over fading channels through spatial diversity techniques [1]. In multiuser MIMO downlink, another advantage of multiple antennas is the capability of simultaneously supporting multiuser transmissions via spatial division multiple access (SDMA) schemes, see, e.g., [2], [3]. On the other hand, because of the high complexity of SDMA implementation, time division multiple access with joint spatial diversity and multiuser diversity serves as a low complexity alternative. In a joint diversity scheme, an appropriate spatial diversity technique is employed for each link, and the user with the best link quality will be scheduled to extract the multiuser diversity gain [4][5][6][7][8][9][10][11][12][13].

Many existing works have focused on the joint diversity schemes for the multiuser MIMO downlink. In particular, for open-loop orthogonal space-time block coding (OSTBC) together with multiuser diversity, the cumulative distribution function (CDF) of the effective received signal-to-noise ratio (SNR) was analyzed in [4] for the Rayleigh fading channels. Then in [5], the authors extended the analysis of [4] by considering not only open-loop OSTBC, but also some closed-loop spatial diversity techniques, such as transmit maximum-ratio combining (TMRC) and transmit antenna selection (TAS). Unlike the above-mentioned works which only analyzed the CDF of the received SNR, [6] derived a closed-form expression of the system capacity for the generalized Nakagami fading channels. Moreover, for the asymptotic analysis allowing the number of users to approach infinity, the asymptotic system capacity was investigated in the literature [7], [8]. It has been established in [6] and [7] that in systems with joint diversities, the closed-loop spatial diversity techniques generally outperform the open-loop OSTBC in terms of the system capacity. In [9][10][11], the outage probabilities and error rates of the multiuser MIMO systems with joint diversities were studied. It was shown that the diversity orders at high SNRs of all the adopted joint diversity schemes are improved in proportion to the number of users, the number of transmit antennas as well as the number of receive antennas.

The results in [4][5][6][7][8][9][10][11] are all derived based on an essential assumption of delay-free channel state information (CSI) feedback. In practical applications, however, feedback delay always exists and the obtained information via feedback becomes outdated. In order to take the effect of outdated feedback into account, [12] and [13] examined the system capacity of the joint diversity schemes in the presence of feedback delay. Different from delay-free cases, it was found in [13] that the open-loop OSTBC may outperform the closed-loop spatial diversity techniques in terms of the system capacity when the feedback delay grows large. So far as we know, the outage probabilities of the joint diversity schemes with outdated feedback have not been studied.

In this paper, we present a comprehensive analysis of the outage probabilities for a multiuser multiple-input single-output (MISO) system with outdated feedback over Rayleigh fading channels. In order to exploit both the spatial and multiuser diversities, we consider three joint diversity schemes, including multiuser TMRC (M-TMRC), multiuser TAS (M-TAS), and multiuser OSTBC (M-OSTBC). The main contributions of this paper are summarized as follows.

- Exact closed-form outage probability expressions are derived for the joint diversity schemes with outdated feedback.
- Based on the derived exact results, a unified high SNR asymptotic analysis is developed. An interesting conclusion under outdated feedback is drawn from the asymptotic results that the asymptotic diversity orders of the M-TMRC, M-TAS and M-OSTBC schemes are 1, 1 and the transmit antenna number at the base station (BS), respectively. This conclusion is new and different from the delay-free feedback cases, where the three schemes have the same asymptotic diversity order that is equal to the product of the user number and the transmit antenna number [11].
- The effects of the user number and transmit antenna number on the asymptotic outage probabilities are analyzed. It is found that the more the users, the better the outage performance for the three joint diversity schemes. In addition, as the transmit antenna number increases, the outage performance of M-TMRC and M-TAS improves; and for M-OSTBC, the same observation is made in the low outage regime and with relatively large feedback delay.
- The three joint diversity schemes are compared in terms of their asymptotic outage probabilities. M-TMRC is shown to outperform M-TAS, and M-OSTBC is able to achieve the best asymptotic outage performance in the outage regime of practical interest when the feedback delay is large.

The remainder of this paper is organized as follows. Section 2 introduces the channel model and the joint diversity schemes under consideration. Section 3 studies the exact outage probabilities of the joint diversity schemes with outdated feedback. In Section 4, the high SNR asymptotic outage probabilities of the joint diversity schemes are calculated and discussed. Section 5 provides the numerical results. Concluding remarks are made in Section 6.

Notation: Bold lowercase letters denote vectors. $\mathrm{E}\{\cdot\}$ stands for the expectation operator, and $\Pr\{\cdot\}$ is the probability of a given event. $(\cdot)^{\mathrm{H}}$, $\|\cdot\|$ denote the conjugate transpose and the two-norm of a vector, respectively. $\mathbf{I}_M$ is the $M \times M$ identity matrix and $\mathbf{0}_{M\times P}$ denotes an all-zero matrix of size $M \times P$. $X \sim N\left(0,\sigma^2\right)$ means that X is Gaussian distributed with zero mean and variance σ^2. The special notation $\mathbf{x} \sim CN\left(\bar{\mathbf{x}}, \Sigma_{\mathbf{x}}\right)$ indicates that $\mathbf{x}$ is complex Gaussian distributed with mean $\bar{\mathbf{x}}$ and covariance matrix $\Sigma_{\mathbf{x}}$. Finally, $f(x) = o\left(g(x)\right)$, $x \to x_0$, if $\lim_{x\to x_0} \frac{f(x)}{g(x)} = 0$.

2. System Model

2.1 Channel Model

Consider the downlink of a multiuser MISO system with an M_T-antenna BS and K single-antenna users. Denote the channel vector of user k at time t as $\mathbf{h}_k(t) = \left[h_1^k(t), \cdots, h_{M_T}^k(t)\right]$, where $h_i^k(t)$, $1 \le i \le M_T$, are the fading coefficients between the i-th transmit antenna and the k-th user at time t. The following assumptions are made throughout this paper:

- Users experience independent and identically distributed (i.i.d) flat Rayleigh fading, i.e., $\mathbf{h}_k(t) \sim CN(\mathbf{0}_{1\times M_T}, \mathbf{I}_{M_T}), \forall k, t$.

- The channels are time-varying according to the Jake's model with Doppler spread f_d.
- Perfect channel estimation is performed at the users.
- The information, which is required by the BS for spatial diversity transmission and user scheduling, is correctly fed back from each user with a time delay τ.

Hence, if the channel estimation of user k is performed at time $t-\tau$, the feedback information will be obtained from $\mathbf{h}_k(t-\tau)$, whereas the actual channel at the instant of data transmission will be $\mathbf{h}_k(t)$. $\mathbf{h}_k(t)$ and $\mathbf{h}_k(t-\tau)$ are jointly complex Gaussian with correlation coefficient $\rho = J_0(2\pi f_d \tau)$, where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. For notational brevity, we omit the time indices, and denote $\mathbf{h}_k = \left[h_1^k, \cdots, h_{M_T}^k\right]$ and $\tilde{\mathbf{h}}_k = \left[\tilde{h}_1^k, \cdots, h_{M_T}^k\right]$ as the channel vector of user k at time t and $t-\tau$, respectively.

2.2 Joint Spatial and Multiuser Diversity Schemes

In this paper, the multiuser MISO downlink employs joint diversity schemes, where appropriate spatial diversity techniques are used for each link, and the user with the largest effective link SNR is chosen for data transmission to exploit the multiuser diversity. With regard to the spatial diversity techniques, we will focus on three well-known schemes. The first two are closed-loop TMRC and TAS, and the last one is the open-loop OSTBC. Below, we present the system model and the effective received SNR for each joint diversity scheme.

1) M-TMRC Scheme: The system model of the M-TMRC scheme with outdated feedback is depicted in **Fig. 1**.

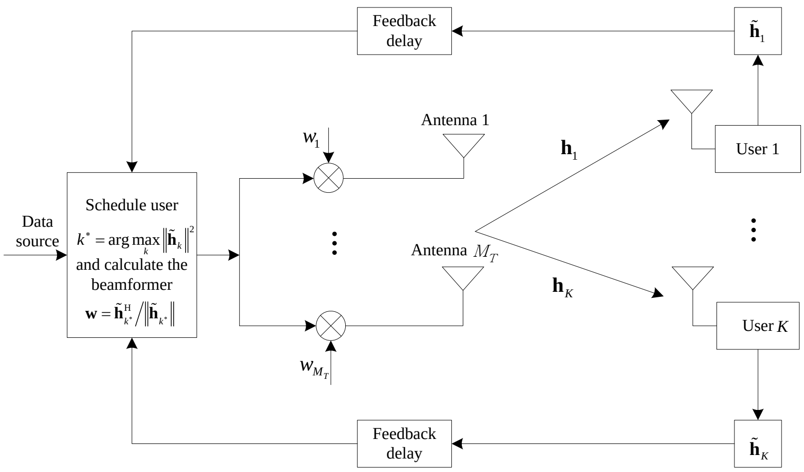

Fig. 1. System model of the M-TMRC scheme with outdated feedback.

To begin with, we first consider TMRC in a single user MISO system, whose channel vector is given by $\mathbf{h}$. Given perfect CSI $\mathbf{h}$ at the BS, TMRC maximizes the received SNR by applying the specific beamforming vector $\mathbf{w} = \mathbf{h}^{\mathrm{H}} / \|\mathbf{h}\|$ at the transmitter [14]. The received signal of TMRC is given by

$$y=\sqrt{E_s}\mathbf{h}\mathbf{w}s+n=\sqrt{E_s}\frac{\mathbf{h}\mathbf{h}^{\mathrm{H}}}{\|\mathbf{h}\|}s+n\,, \tag{1}$$

where E_s is the transmit power, s is the transmit signal with unit average energy, n is the complex additive white Gaussian noise (AWGN) with zero mean and variance N_0. And the received SNR of TMRC is

$$\gamma=\frac{E_s}{N_0}\frac{\left\|\mathbf{h}\mathbf{h}^{\mathrm{H}}\right\|^2}{\|\mathbf{h}\|^2}=\bar{\gamma}\|\mathbf{h}\|^2\,, \tag{2}$$

where $\bar{\gamma}=\frac{E_s}{N_0}$ can be interpreted as the average received SNR in a single-input single-output (SISO) fading channel.

In the M-TMRC scheme, the BS allocates the radio resource to the user that can achieve the largest received SNR to obtain the multiuser diversity gain. If no feedback delay exists, the scheduled user will be $k^*=\arg\max_k\|\mathbf{h}_k\|^2$ and the resultant SNR is $\bar{\gamma}\max_k\|\mathbf{h}_k\|^2$. However, as to the outdated feedback considered in this paper, the situation becomes different. From previous subsection, we know that the channel vector measured at user k and fed back to the BS is $\tilde{\mathbf{h}}_k$, whereas the current channel vector of user k is $\mathbf{h}_k$. The BS treats the outdated channel $\tilde{\mathbf{h}}_k$ as the actual channel and selects the target user $k^*=\arg\max_k\|\tilde{\mathbf{h}}_k\|^2$. Accordingly, the beamformer at the transmitter is $\mathbf{w}=\tilde{\mathbf{h}}_{k^*}^{\mathrm{H}}/\|\tilde{\mathbf{h}}_{k^*}\|$, and the received SNR of the target user k^* becomes

$$\gamma_{k^*,\,\text{M-TMRC}}=\bar{\gamma}\frac{\left\|\mathbf{h}_{k^*}\tilde{\mathbf{h}}_{k^*}^{\mathrm{H}}\right\|^2}{\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2}\,. \tag{3}$$

2) M-TAS Scheme: The system model of the M-TAS scheme with outdated feedback is shown in **Fig. 2**. In this paper, we consider selecting only one antenna of the BS that leads to the largest channel gain for data transmission [15].

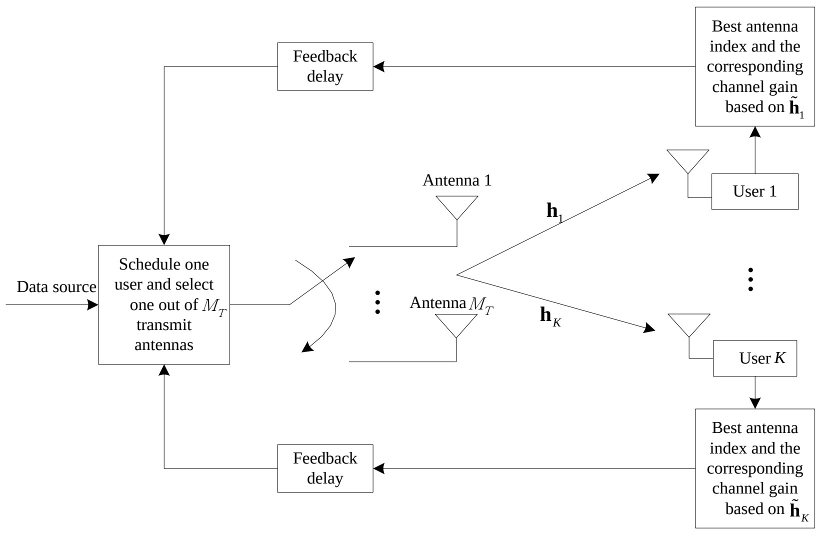

Fig. 2. System model of the M-TAS scheme with outdated feedback.

Without loss of generality, we first assume that antenna i of the BS is selected for data transmission. Under the assumption that transmit antenna i is selected, the received signal of user k is

$$y_i^k = \sqrt{E_s} h_i^k s + n_k, \tag{4}$$

with the corresponding received SNR given by $\gamma_i^k = \bar{\gamma} \left\| h_i^k \right\|^2$. To maximize the received SNR, the scheduled user and the selected antenna will be that maximize the channel gain $\left\| h_i^k \right\|^2$.

Therefore, in the M-TAS scheme with outdated feedback, user k needs to estimate its channel vector $\tilde{\mathbf{h}}_k$ and inform the BS about its best antenna index $A_k = \arg\max_i \left\| \tilde{h}_i^k \right\|^2$ as well as the corresponding channel gain $\left\| \tilde{h}_{A_k}^k \right\|^2$. Based on the feedback information from all users, the BS schedules the user

$$k^* = \arg\max_k \left\| \tilde{h}_{A_k}^k \right\|^2. \tag{5}$$

As the current channel vector of user k^* is $\mathbf{h}_{k^*}$, the received SNR of user k^* is then given as

$$\gamma_{k^*,\text{M-TAS}} = \bar{\gamma} \left\| h_{A_{k^*}}^{k^*} \right\|^2. \tag{6}$$

3) M-OSTBC Scheme: The system model of the M-OSTBC scheme with outdated feedback is presented in **Fig. 3**. OSTBC is an attractive low complexity spatial diversity scheme with full diversity order [16], [17]. With an OSTBC, the MISO channel is transformed into an equivalent AWGN scaled channel, and the effective received SNR of user k is given by [18]

$$\gamma_k = \frac{E_s}{M_T N_0 r_c} \left\| \mathbf{h}_k \right\|^2 = \frac{\bar{\gamma}}{M_T r_c} \left\| \mathbf{h}_k \right\|^2, \tag{7}$$

where $r_c \le 1$ is the code rate of OSTBC.

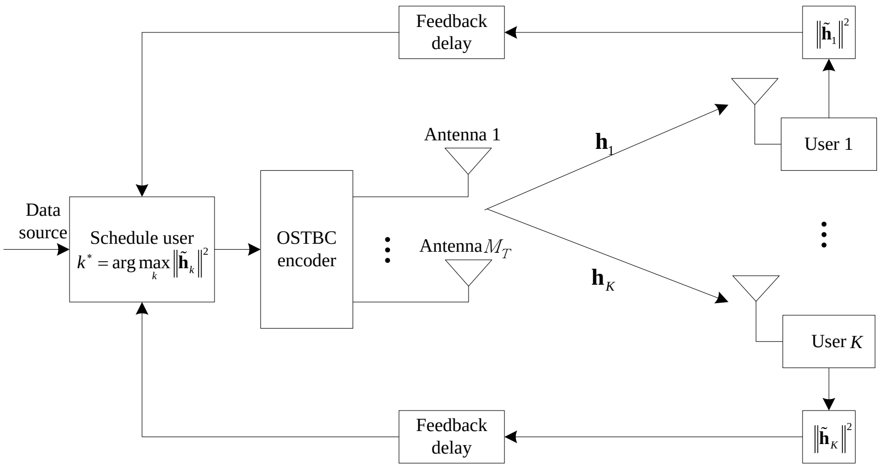

Fig. 3. System model of the M-OSTBC scheme with outdated feedback.

In the M-OSTBC scheme with feedback delay, user k estimates its channel vector $\tilde{\mathbf{h}}_k$ and sends $\left\|\tilde{\mathbf{h}}_k\right\|^2$ to the BS. Based on $\left\|\tilde{\mathbf{h}}_k\right\|^2 \left(k=1,\cdots,K\right)$, the scheduler selects the user with the largest effective SNR. Mathematically, the selected user can be expressed as

$$k^* = \arg\max_k \left\|\tilde{\mathbf{h}}_k\right\|^2 . \tag{8}$$

As the current channel vector of user k^* is $\mathbf{h}_{k^*}$, its received SNR is

$$\gamma_{k^*,\,\text{M-OSTBC}} = \frac{\bar{\gamma}}{M_T r_c}\left\|\mathbf{h}_{k^*}\right\|^2 . \tag{9}$$

3. Exact Outage Probabilities of the Joint Diversity Schemes with Outdated Feedback

In this section, we analyze the exact outage probabilities of the three joint diversity schemes one by one. To investigate the effect of feedback delay on the outage performance, we relate $\mathbf{h}_k$ and $\tilde{\mathbf{h}}_k$ by the following channel feedback error model [19]:

$$\mathbf{h}_k = \rho\tilde{\mathbf{h}}_k + \sqrt{1-\rho^2}\,\mathbf{v}_k , \tag{10}$$

where $\mathbf{v}_k \sim CN\left(\mathbf{0}_{1\times M_T}, \mathbf{I}_{M_T}\right)$ is independent of $\tilde{\mathbf{h}}_k$.

3.1 Outage Probability of the M-TMRC Scheme

By utilizing (10), the received SNR of the scheduled user k^* in (3) can be decomposed as

$$\begin{aligned}\gamma_{k^*,\,\text{M-TMRC}} &= \bar{\gamma}\frac{\left\|\left(\rho\tilde{\mathbf{h}}_{k^*} + \sqrt{1-\rho^2}\,\mathbf{v}_{k^*}\right)\tilde{\mathbf{h}}_{k^*}^{\mathrm{H}}\right\|^2}{\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2} \\ &= \frac{1-\rho^2}{2}\bar{\gamma}\left\|\sqrt{\frac{2\rho^2}{1-\rho^2}}\left\|\tilde{\mathbf{h}}_{k^*}\right\| + \sqrt{2}\frac{\mathbf{v}_{k^*}\tilde{\mathbf{h}}_{k^*}^{\mathrm{H}}}{\left\|\tilde{\mathbf{h}}_{k^*}\right\|}\right\|^2 .\end{aligned} \tag{11}$$

For convenience, let $\zeta_{\text{M-TMRC}} = \left\|\sqrt{\frac{2\rho^2}{1-\rho^2}}\left\|\tilde{\mathbf{h}}_{k^*}\right\| + \sqrt{2}\frac{\mathbf{v}_{k^*}\tilde{\mathbf{h}}_{k^*}^{\mathrm{H}}}{\left\|\tilde{\mathbf{h}}_{k^*}\right\|}\right\|^2$. Then the conditional outage probability of the M-TMRC scheme for a target rate R ,[1] $P_{o,\text{M-TMRC}}\left(R\middle|\tilde{\mathbf{h}}_{k^*}\right)$, can be calculated by

$$\begin{aligned}P_{o,\text{M-TMRC}}\left(R\middle|\tilde{\mathbf{h}}_{k^*}\right) &= \Pr\left\{\log_2\left(1+\frac{1-\rho^2}{2}\bar{\gamma}\zeta_{\text{M-TMRC}}\right) < R\right\} \\ &= \Pr\left\{\zeta_{\text{M-TMRC}} < \frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right\},\end{aligned} \tag{12}$$

which will be determined by the CDF of $\zeta_{\text{M-TMRC}}$.

From (11), we know that conditioned on $\tilde{\mathbf{h}}_{k^*}$, $\zeta_{\text{M-TMRC}}$ is noncentral chi-square distributed with 2 degrees of freedom, variance 1 for each degree of freedom and noncentrality parameter

[1] The conditional outage probability for a target rate R is the probability that the conditional instantaneous capacity is less than R .

$\frac{2\rho^2}{1-\rho^2}\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2$.

Definition 1: Let $F_{nc-\chi^2,\,2L,\alpha x}(\cdot)$ be the CDF of a noncentral chi-square distributed random variable with $2L$ (L is a positive integer) degrees of freedom, variance 1 for each degree of freedom and noncentrality parameter αx.

Based on the above definition, the conditional outage probability in (12) is

$$P_{o,\text{M-TMRC}}\left(R\middle|\tilde{\mathbf{h}}_{k^*}\right)=F_{nc-\chi^2,\,2,\,\frac{2\rho^2}{1-\rho^2}\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2}\left(\frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right). \tag{13}$$

With the conditional outage probability, now we proceed to derive the average outage probability of the M-TMRC scheme, which will be obtained by averaging the conditional outage probability (13) over the probability density function (PDF) of $\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2$. Note that $\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2=\max\left\{\left\|\tilde{\mathbf{h}}_1\right\|^2,\cdots,\left\|\tilde{\mathbf{h}}_K\right\|^2\right\}$, where $\left\|\tilde{\mathbf{h}}_k\right\|^2$, $k=1,\cdots,K$, are central chi-square distributed with $2M_T$ degrees of freedom and variance 0.5 for each degree of freedom.

Definition 2: Denote $f_{\max,Z,Q}(x)$ as the PDF of the maximum of Z i.i.d random variables, each of which follows a central chi-square distribution with $2Q$ (Q is a positive integer) degrees of freedom and variance 0.5 for each degree of freedom.

Utilizing Definition 2, the PDF of $\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2$ is obtained as $f_{\max,\,K,\,M_T}(x)$. Hence, the average outage probability of M-TMRC for the target rate R, $P_{o,\text{M-TMRC}}(R)$, is calculated as

$$P_{o,\text{M-TMRC}}(R)=\int_0^\infty F_{nc-\chi^2,\,2,\,\frac{2\rho^2}{1-\rho^2}x}\left(\frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right)f_{\max,\,K,\,M_T}(x)\,\mathrm{d}x. \tag{14}$$

Next, defining

$$P_o(L,\alpha,\beta,Z,Q)=\int_0^\infty F_{nc-\chi^2,\,2L,\,\alpha x}(\beta)\,f_{\max,\,Z,\,Q}(x)\mathrm{d}x,\ \ (L\le Q), \tag{15}$$

we eventually get

$$P_{o,\text{M-TMRC}}(R)=P_o\left(1,\frac{2\rho^2}{1-\rho^2},\frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)},K,M_T\right). \tag{16}$$

The detailed calculation of $P_o(L,\alpha,\beta,Z,Q)$ is presented in Appendix B with the final result given in (45). Thus, the average outage probability of the M-TMRC scheme in (16) can be evaluated by substituting the corresponding parameters into (45).

The subsequent outage probability analysis for M-TAS and M-OSTBC follows the similar procedure as presented in this subsection.

3.2 Outage Probability of the M-TAS Scheme

Using the feedback error model (10), the received SNR (6) of the selected user k^* in the M-TAS scheme can be rewritten as

$$\begin{aligned}\gamma_{k^*,\,\text{M-TAS}} &= \bar{\gamma}\left\|\rho\tilde{h}_{A_{k^*}}^{k^*} + \sqrt{1-\rho^2}\,v_{k^*}\right\|^2 \\ &= \frac{1-\rho^2}{2}\bar{\gamma}\left\|\sqrt{\frac{2\rho^2}{1-\rho^2}}\tilde{h}_{A_{k^*}}^{k^*} + \sqrt{2}v_{k^*}\right\|^2.\end{aligned} \tag{17}$$

Let $\zeta_{\text{M-TAS}} = \left\|\sqrt{\frac{2\rho^2}{1-\rho^2}}\tilde{h}_{A_{k^*}}^{k^*} + \sqrt{2}v_{k^*}\right\|^2$. It is easy to find that conditioned on $\left\|\tilde{h}_{A_{k^*}}^{k^*}\right\|^2$, $\zeta_{\text{M-TAS}}$ is noncentral chi-square distributed with 2 degrees of freedom, variance 1 for each degree of freedom and noncentrality parameter $\frac{2\rho^2}{1-\rho^2}\left\|\tilde{h}_{A_{k^*}}^{k^*}\right\|^2$. Hence, the conditional outage probability of the M-TAS scheme for a target rate R, $\mathrm{P}_{o,\text{M-TAS}}\left(R\middle|\left\|\tilde{h}_{A_{k^*}}^{k^*}\right\|^2\right)$, is given by

$$\begin{aligned}\mathrm{P}_{o,\text{M-TAS}}\left(R\middle|\left\|\tilde{h}_{A_{k^*}}^{k^*}\right\|^2\right) &= \Pr\left\{\log_2\left(1+\frac{1-\rho^2}{2}\bar{\gamma}\zeta_{\text{M-TAS}}\right) < R\right\} \\ &= \Pr\left\{\zeta_{\text{M-TAS}} < \frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right\} \\ &= F_{nc-\chi^2,\,2,\,\frac{2\rho^2}{1-\rho^2}\left\|\tilde{h}_{A_{k^*}}^{k^*}\right\|^2}\left(\frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right).\end{aligned} \tag{18}$$

Note that $\left\|\tilde{h}_{A_{k^*}}^{k^*}\right\|^2 = \max_{1\le k\le K,\,1\le i\le M_T}\left\|\tilde{h}_i^k\right\|^2$, where $\left\|\tilde{h}_i^k\right\|^2$, $1\le k\le K$, $1\le i\le M_T$, are central chi-square distributed with 2 degrees of freedom and variance 0.5 for each degree of freedom. Using Definition 2, the PDF of $\left\|\tilde{h}_{A_{k^*}}^{k^*}\right\|^2$ is $f_{\max,\,KM_T,\,1}(x)$. As a result, the average outage probability of the M-TAS scheme for the target rate R, $\mathrm{P}_{o,\text{M-TAS}}(R)$, is calculated as

$$\begin{aligned}\mathrm{P}_{o,\text{M-TAS}}(R) &= \int_0^\infty F_{nc-\chi^2,\,2,\,\frac{2\rho^2}{1-\rho^2}x}\left(\frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right) f_{\max,\,KM_T,\,1}(x)\,\mathrm{d}x \\ &= P_o\left(1, \frac{2\rho^2}{1-\rho^2}, \frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}, KM_T, 1\right),\end{aligned} \tag{19}$$

where $P_o(.,.,.,.,.)$ has been defined in (15). The above expression (19) can be evaluated analytically by letting $L=1$, $\alpha = \frac{2\rho^2}{1-\rho^2}$, $\beta = \frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}$, $Z = KM_T$ and $Q=1$ in (45).

When $M_T = 1$, the M-TAS scheme reduces to the special case of a multiuser SISO (M-SISO) system exploiting the multiuser diversity only. Therefore, the average outage probability of the M-SISO system for the target rate R with outdated feedback, $\mathrm{P}_{o,\text{M-SISO}}(R)$, can be obtained by letting $M_T = 1$ in (19), that is,

$$\mathrm{P}_{o,\text{M-SISO}}(R) = P_o\left(1, \frac{2\rho^2}{1-\rho^2}, \frac{2\left(2^R-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}, K, 1\right). \tag{20}$$

3.3 Outage Probability of the M-OSTBC Scheme

Similar to the above two joint diversity schemes, the received SNR of the target user k^* in the M-OSTBC scheme is obtained from (9) and (10) as

$$\begin{aligned}\gamma_{k^*,\,\text{M-OSTBC}} &= \frac{\bar{\gamma}}{M_T r_c}\left\|\rho\tilde{\mathbf{h}}_{k^*} + \sqrt{1-\rho^2}\,\mathbf{v}_{k^*}\right\|^2 \\ &= \frac{1-\rho^2}{2}\frac{\bar{\gamma}}{M_T r_c}\left\|\sqrt{\frac{2\rho^2}{1-\rho^2}}\tilde{\mathbf{h}}_{k^*} + \sqrt{2}\,\mathbf{v}_{k^*}\right\|^2.\end{aligned} \tag{21}$$

Let $\zeta_{\text{M-OSTBC}} = \left\|\sqrt{\frac{2\rho^2}{1-\rho^2}}\tilde{\mathbf{h}}_{k^*} + \sqrt{2}\,\mathbf{v}_{k^*}\right\|^2$. Then conditioned on $\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2$, $\zeta_{\text{M-OSTBC}}$ is noncentral chi-square distributed with $2M_T$ degrees of freedom, variance 1 for each degree of freedom and noncentrality parameter $\frac{2\rho^2}{1-\rho^2}\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2$. It follows that the conditional outage probability of the M-OSTBC scheme for a target rate R, $\mathrm{P}_{o,\text{M-OSTBC}}\left(R\middle|\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2\right)$, is given by

$$\begin{aligned}\mathrm{P}_{o,\text{M-OSTBC}}\left(R\middle|\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2\right) &= \Pr\left\{r_c\log_2\left(1+\frac{1-\rho^2}{2}\frac{\bar{\gamma}}{M_T r_c}\zeta_{\text{M-OSTBC}}\right) < R\right\} \\ &= \Pr\left\{\zeta_{\text{M-OSTBC}} < \frac{2M_T r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right\} \\ &= F_{nc-\chi^2,\,2M_T,\,\frac{2\rho^2}{1-\rho^2}\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2}\left(\frac{2M_T r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right).\end{aligned} \tag{22}$$

Based on (8), $\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2 = \max\left\{\left\|\tilde{\mathbf{h}}_1\right\|^2,\cdots,\left\|\tilde{\mathbf{h}}_K\right\|^2\right\}$, which is the same as that in the M-TMRC scheme. Therefore, we know that the PDF of $\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2$ is $f_{\max,\,K,\,M_T}(x)$. Averaging the conditional outage probability (22) over the PDF of $\left\|\tilde{\mathbf{h}}_{k^*}\right\|^2$, the average outage probability of the M-OSTBC scheme for the target rate R, $\mathrm{P}_{o,\,\text{M-OSTBC}}(R)$, is obtained as

$$\begin{aligned}\mathrm{P}_{o,\,\text{M-OSTBC}}(R) &= \int_0^\infty F_{nc-\chi^2,\,2M_T,\,\frac{2\rho^2}{1-\rho^2}x}\left(\frac{2M_T r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right) f_{\max,\,K,\,M_T}(x)\,\mathrm{d}x \\ &= P_o\left(M_T, \frac{2\rho^2}{1-\rho^2}, \frac{2M_T r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}, K, M_T\right),\end{aligned} \tag{23}$$

where the closed-form expression for calculating $P_o(.,.,.,.,.)$ is given in (45).

4. High SNR Asymptotic Analysis

Given the exact analysis presented in the above section, we are able to offer some more insights into the system performance by further studying the asymptotic behavior at high average SNRs. The effects of the user number K and the antenna number M_T on the asymptotic performance will be investigated, and the asymptotic outage probabilities of the different joint diversity schemes will be compared.[2]

4.1 A Unified Asymptotic Outage Probability Analysis

It is worth noting from (16), (19), (20) and (23) that the exact outage probabilities have the same format of $P_o\left(L,\alpha,\frac{\lambda}{\bar{\gamma}},Z,Q\right)$. Therefore, a unified asymptotic analysis can fortunately be conducted. From (45), $P_o\left(L,\alpha,\frac{\lambda}{\bar{\gamma}},Z,Q\right)$ is given by

$$P_o\left(L,\alpha,\frac{\lambda}{\bar{\gamma}},Z,Q\right) = \frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}\sum_{i=0}^{Q-L+n}(-1)^t\binom{Z-1}{t}\binom{Q-L+n}{i}\frac{\eta_n^t(Q+n-1)!}{\left(t+1+\frac{\alpha}{2}\right)^{Q+n-L}}\frac{\left(\frac{\alpha}{2}\right)^i}{(t+1)^{L+i}}\Gamma_{L+i}\left(\frac{\lambda(t+1)}{\bar{\gamma}(\alpha+2t+2)}\right), \quad (24)$$

where η_n^t is given in (37).

Utilizing $\Gamma_m(y)\approx\frac{y^m}{m!}$ for a small y, the outage probability under high average SNR $\bar{\gamma}$ can be closely approximated by

$$P_{o,\,\mathrm{asy}}\left(L,\alpha,\frac{\lambda}{\bar{\gamma}},Z,Q\right) = \left[\sum_{t=0}^{Z-1}(-1)^t\binom{Z-1}{t}\sum_{n=0}^{t(Q-1)}\frac{\eta_n^t(Q+n-1)!}{\left(t+1+\frac{\alpha}{2}\right)^{Q+n}}\right]\frac{Z\left(\frac{\lambda}{2}\right)^L}{(Q-1)!L!}\bar{\gamma}^{-L}+o\left(\bar{\gamma}^{-L}\right). \quad (25)$$

It is easy to see from (25) that the asymptotic diversity order, i.e., the slope of the outage probability vs. average SNR curve on a log-log scale at high SNRs, is obtained as L. Moreover, by comparing (25) with (16), (19) and (23), the following Proposition can be stated.

Proposition 1: With outdated feedback, the asymptotic diversity orders of the M-TMRC and M-TAS schemes reduce to one, while that of the M-OSTBC scheme is M_T.

It should be mentioned that with instantaneous feedback, all the above joint diversity schemes can achieve a full asymptotic diversity order of KM_T [11]. Comparing this result with Proposition 1, one may conclude that with outdated feedback, it is the closed-loop spatial diversity order and multiuser diversity order that asymptotically reduce to one. In contrast, the open-loop spatial diversity order is not affected by the feedback delay. This phenomenon is mainly due to the reason that both the closed-loop spatial diversity and multiuser diversity depend on the feedback information. At very high SNRs, even a little mismatch of the CSI at the transmitter will make the diversities that depend on it ineffective.[3]

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[2] Both the investigation of the effects of K and M_T on the outage performance and the comparison of the different joint diversity schemes are based on the asymptotic outage probabilities at sufficiently high SNRs. However, as seen from the numerical results presented in the next section, the conclusions made for sufficiently high SNRs are also valid for relatively high SNRs.

[3] In [19][20][21], the closed-loop spatial diversity order with outdated feedback has also been reported to asymptotically reduce to one. Both Eq. (26) of [19] and Eq. (14) of [20] state that the asymptotic diversity order of the TMRC scheme for a single user MISO system with outdated feedback is 1. In Eq. (33) of [21], the asymptotic

4.2 Effects of K and M_T on the Asymptotic Outage Probabilities

From Proposition 1, it is obvious that the user number K has no impact on the asymptotic diversity order for all the joint diversity schemes, and the transmit antenna number M_T only affects the asymptotic diversity order of the M-OSTBC scheme.

An interesting question naturally arises: *how do K and M_T influence the asymptotic outage probabilities of the joint diversity schemes with outdated feedback?* This question can be answered by examining the asymptotic outage probability $P_{o,\text{asy}}(L,\alpha,\lambda/\bar{\gamma},Z,Q)$.

Lemma 1: $P_{o,\text{asy}}(L,\alpha,\lambda/\bar{\gamma},Z,Q)$ is a decreasing function of both Z and Q.

Proof: Let

$$\Phi(Z,Q,\alpha)=\frac{Z}{(Q-1)!}\left[\sum_{t=0}^{Z-1}(-1)^t\binom{Z-1}{t}\sum_{n=0}^{t(Q-1)}\frac{\eta_n^t(Q+n-1)!}{\left(t+1+\frac{\alpha}{2}\right)^{Q+n}}\right], \tag{26}$$

then the asymptotic outage probability becomes

$$P_{o,\text{asy}}\left(L,\ \alpha,\ \frac{\lambda}{\bar{\gamma}},\ Z,\ Q\right)\approx\frac{1}{L!}\left(\frac{\lambda}{2}\right)^L\bar{\gamma}^{-L}\Phi(Z,Q,\alpha). \tag{27}$$

From (40), we have

$$\Phi(Z,Q,\alpha)=B(0)=\int_0^\infty f_{\max,Z,Q}(x)e^{-\alpha x/2}\,dx=\mathrm{E}\left\{e^{-\alpha X/2}\right\}, \tag{28}$$

where the PDF of X is $f_{\max,Z,Q}(x)$. According to the definition of $f_{\max,Z,Q}(x)$, X can be expressed as $X=\max(X_1,\cdots,X_Z)$, where X_i , $i=1,\cdots,Z$, follow a central chi-square distribution with $2Q$ degrees of freedom and variance 0.5 for each degree of freedom. It is well-known that the central chi-square distributed random variable X_i can be expressed as a sum of squares of i.i.d Gaussian random variables, that is, $X_i=Y_{i,1}^2+\cdots+Y_{i,2Q}^2$ with $Y_{i,j}\sim N(0,0.5)$, $j=1,\cdots,2Q$ [22]. Therefore, X can be further expressed as $X=\max_{1\le i\le Z}\left(Y_{i,1}^2+\cdots+Y_{i,2Q}^2\right)$. Based on these discussions, we conclude that $\Phi(Z,Q,\alpha)=\mathrm{E}\left\{e^{-\alpha X/2}\right\}$ is a decreasing function of both Z and Q as $\alpha>0$, and so is $P_{o,\text{asy}}(L,\alpha,\lambda/\bar{\gamma},Z,Q)$.

■

Making use of Lemma 1, the following observations can be made readily from (16), (19), (20) and (23):

i) *The asymptotic outage probabilities of the M-SISO system and the three joint diversity schemes decrease as K increases.*

ii) *The asymptotic outage probabilities of the M-TMRC and M-TAS schemes decrease as M_T increases.*

How the asymptotic outage probability of the M-OSTBC scheme,

---

diversity order of the TAS scheme for a single user MIMO system is shown to be the receive antenna number M_R. Accordingly, when $M_R=1$, the asymptotic diversity order of the TAS scheme for a single user MISO system will be 1. In this paper, we generalize these conclusions to the multiuser case.

$\frac{M_T^{M_T}}{M_T!}\left(\frac{r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right)^{M_T}\Phi\left(K, M_T, \frac{2\rho^2}{1-\rho^2}\right)$, varies with M_T is much more complicated. Let $\frac{M_T^{M_T}}{M_T!}\left(\frac{r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}\right)^{M_T}=\varphi\left(M_T\right)$, then $\frac{\varphi\left(M_T+1\right)}{\varphi\left(M_T\right)}=\left(1+\frac{1}{M_T}\right)^{M_T}\frac{r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}<e\frac{r_c\left(2^{R/r_c}-1\right)}{\bar{\gamma}\left(1-\rho^2\right)}$. In practice, we are interested in small outage probabilities (i.e., 0.01, 0.001, ...), which correspond to small values of R. Therefore, in the low outage regime and with relatively large feedback delay (i.e., small ρ), we will have $\frac{\varphi\left(M_T+1\right)}{\varphi\left(M_T\right)}<1$. In other words, $\varphi\left(M_T\right)$ is a decreasing function of M_T. Recalling that $\Phi\left(K, M_T, \frac{2\rho^2}{1-\rho^2}\right)$ is also a decreasing function of M_T, as a result, *the asymptotic outage probability of the M-OSTBC scheme decreases as* M_T *increases in the low outage regime and with relatively large feedback delay.*

4.3 Comparison of the Joint Diversity Schemes

1) Comparison of M-TMRC, M-TAS and M-SISO: The M-TMRC, M-TAS and M-SISO schemes have the same parameters L, α, λ. From (27), it can be seen that the difference of their asymptotic outage probabilities lies in the item $\Phi(Z, Q, \alpha)=\mathrm{E}\left\{e^{-\alpha X/2}\right\}$. Below, we will compare $\Phi_{\text{M-TMRC}}$, $\Phi_{\text{M-TAS}}$ and $\Phi_{\text{M-SISO}}$.

For the M-TMRC scheme, $\Phi_{\text{M-TMRC}}=\mathrm{E}\left\{e^{-\frac{\alpha}{2}X_{\text{M-TMRC}}}\right\}$ with $X_{\text{M-TMRC}}$ given by

$$X_{\text{M-TMRC}}=\max\left\{\left\|\tilde{h}_1^1\right\|^2+\cdots+\left\|\tilde{h}_{M_T}^1\right\|^2,\cdots,\left\|\tilde{h}_1^k\right\|^2+\cdots+\left\|\tilde{h}_{M_T}^k\right\|^2,\cdots,\left\|\tilde{h}_1^K\right\|^2+\cdots+\left\|\tilde{h}_{M_T}^K\right\|^2\right\}. \quad (29)$$

For the M-TAS scheme, $\Phi_{\text{M-TAS}}=\mathrm{E}\left\{e^{-\frac{\alpha}{2}X_{\text{M-TAS}}}\right\}$ with

$$X_{\text{M-TAS}}=\max\left\{\left\|\tilde{h}_1^1\right\|^2,\cdots,\left\|\tilde{h}_{M_T}^1\right\|^2,\cdots,\left\|\tilde{h}_1^k\right\|^2,\cdots,\left\|\tilde{h}_{M_T}^k\right\|^2,\cdots,\left\|\tilde{h}_1^K\right\|^2,\cdots,\left\|\tilde{h}_{M_T}^K\right\|^2\right\}. \quad (30)$$

And for M-SISO, $\Phi_{\text{M-SISO}}=\mathrm{E}\left\{e^{-\frac{\alpha}{2}X_{\text{M-SISO}}}\right\}$ with

$$X_{\text{M-SISO}}=\max\left\{\left\|\tilde{h}_1^1\right\|^2,\left\|\tilde{h}_1^2\right\|^2,\cdots,\left\|\tilde{h}_1^K\right\|^2\right\}. \quad (31)$$

Obviously, for any realization of $\left\|\tilde{h}_1^1\right\|^2,\cdots,\left\|\tilde{h}_{M_T}^1\right\|^2,\cdots,\left\|\tilde{h}_1^k\right\|^2,\cdots,\left\|\tilde{h}_{M_T}^k\right\|^2,\cdots,\left\|\tilde{h}_1^K\right\|^2,\cdots,\left\|\tilde{h}_{M_T}^K\right\|^2$, we have $X_{\text{M-SISO}}<X_{\text{M-TAS}}<X_{\text{M-TMRC}}$, and thus $e^{-\frac{\alpha}{2}X_{\text{M-SISO}}}>e^{-\frac{\alpha}{2}X_{\text{M-TAS}}}>e^{-\frac{\alpha}{2}X_{\text{M-TMRC}}}$ as $\alpha>0$. Therefore, $\Phi_{\text{M-SISO}}>\Phi_{\text{M-TAS}}>\Phi_{\text{M-TMRC}}$ and eventually $P_{o,\text{ M-SISO, asy}}>P_{o,\text{ M-TAS, asy}}>P_{o,\text{ M-TMRC, asy}}$. In other words, *the M-TMRC scheme outperforms the M-TAS scheme, and the latter outperforms the M-SISO system.*

Specifically, for the extreme case of $\rho=0$, from (28), we have

$$\Phi\big|_{\rho=0}=\Phi\big|_{\alpha=0}=\int_0^\infty f_{\max, Z, Q}(x)\mathrm{d}x=1. \quad (32)$$

Consequently, *when* $\rho = 0$, *the three schemes have the same asymptotic outage probability* $P_{o,\mathrm{asy}} = \frac{2^R - 1}{\bar{\gamma}}$, *which is independent of* K, M_T.

2) Comparison of M-TMRC and M-OSTBC: The M-TMRC and M-OSTBC schemes have the same parameters Z, Q and α. Hence, it is seen from (27) that the difference of their asymptotic outage probabilities comes from the item $\frac{1}{L!}\left(\frac{\lambda}{2}\right)^L \bar{\gamma}^{-L}$. Let $\psi = \frac{1}{L!}\left(\frac{\lambda}{2}\right)^L \bar{\gamma}^{-L}$, then the ratio of $\psi_{\text{M-TMRC}}$ to $\psi_{\text{M-OSTBC}}$ is given by

$$\frac{\psi_{\text{M-TMRC}}}{\psi_{\text{M-OSTBC}}} = \left[\frac{(1-\rho^2)\bar{\gamma}}{M_T}\right]^{M_T-1} \frac{2^R - 1}{\left(2^{R/r_c} - 1\right)^{M_T}} \frac{(M_T - 1)!}{(r_c)^{M_T}} \geq \left[\frac{(1-\rho^2)\bar{\gamma}}{M_T}\right]^{M_T-1} \frac{2^R - 1}{\left(2^{R/r_c} - 1\right)^{M_T}}. \quad (33)$$

Clearly, the above ratio depends on multiple parameters. In the low outage regime and with relatively large feedback delay (i.e., small R and ρ), we will have $\frac{(1-\rho^2)\bar{\gamma}}{M_T} > 1$, $\frac{2^R - 1}{\left(2^{R/r_c} - 1\right)^{M_T}} > 1$, and therefore, $\frac{\psi_{\text{M-TMRC}}}{\psi_{\text{M-OSTBC}}} > 1$. That is to say, in this case, the M-OSTBC scheme achieves better asymptotic outage performance than the M-TMRC scheme. Recalling that the M-TMRC performs better than the M-TAS scheme, we conclude that *the M-OSTBC scheme has the best asymptotic outage performance in the low outage regime and with relatively large feedback delay*.

5. Numerical Results

In this section, we present some numerical results of (16), (19), (20) and (23) for the joint diversity schemes mentioned above, and Monte Carlo simulation results are also presented for comparison. From **Fig. 4** to **Fig. 8**, it is found that the analytical results conincide well with the simulation results, which validates the theoretical derivations in this paper. In the following figures, $\bar{\gamma}$ denotes the average SNR, K is the user number, M_T is the transmit antenna number of the BS, ρ is the correlation coefficient and R is the target rate in bps/Hz. The code rate r_c of OSTBC is 1 if $M_T = 2$, and $3/4$ if $M_T = 3$ or 4 [17].

Fig. 4 shows the outage probabilities vs. $\bar{\gamma}$, where $K = 10$, $M_T = 2$, $\rho = 0.7$ and $R = 0.5\,\text{bps/Hz}$. The asymptotic results calculated from (25) are also presented. It is observed that for $\bar{\gamma} \geq 1.2\,\text{dB}$, M-OSTBC outperforms the other schemes, and at 10^{-3} outage probability, the gain is 5.3 dB, 8.7 dB, and 11.3 dB over M-TMRC, M-TAS and M-SISO, respectively. At relatively high SNRs, the asymptotic results are very close to the exact analytical results, so the former can be regarded as a good approximation of the latter. It is also found, from the asymptotic results, that the asymptotic diversity orders of M-OSTBC, M-TMRC and M-TAS are 2, 1 and 1, respectively, which supports the conclusion of Proposition 1.

In **Fig. 5**, the outage probabilities of the joint diversity schemes vs. K are presented with $\bar{\gamma} = 10\,\text{dB}$, $M_T = 2$, $\rho = 0.7$ and $R = 0.5\,\text{bps/Hz}$. It is clear that the exact outage probabilities satisfy $P_{o,\text{M-OSTBC}} < P_{o,\text{M-TMRC}} < P_{o,\text{M-TAS}} < P_{o,\text{M-SISO}}$ and the performance of all the schemes improves as K increases. We obtain the same conclusions as above from the asymptotic analysis made in Section 4 in the case of sufficiently high SNRs, however, from the figure, it is shown that the conclusions are also valid for relatively high SNRs.

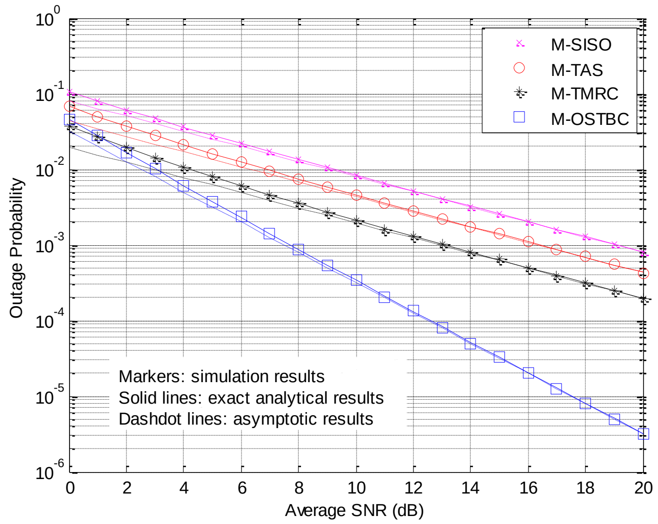

Fig. 4. Outage probabilities of the joint diversity schemes vs. $\bar{\gamma}$, with $K=10,\ M_T=2,\ \rho=0.7$ and $R=0.5\,\text{bps/Hz}$.

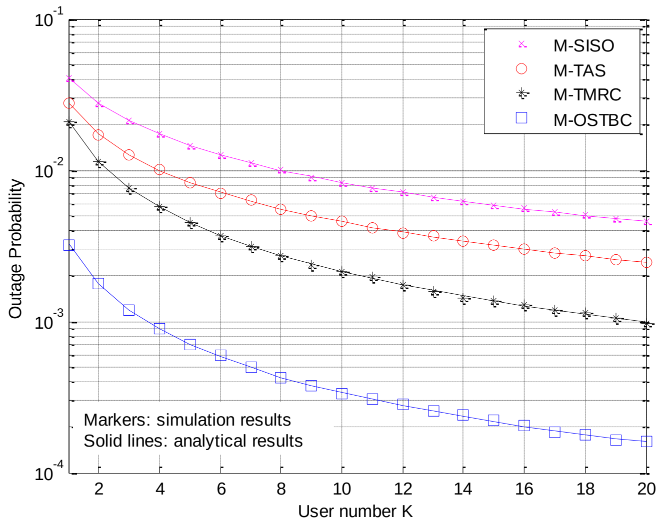

Fig. 5. Outage probabilities of the joint diversity schemes vs. K , with $\bar{\gamma}=10$ dB, $M_T=2,\ \rho=0.7$ and $R=0.5$ bps/Hz .

Fig. 6 depicts the outage probabilities of the joint diversity schemes vs. M_T with $\bar{\gamma}=10\,\text{dB}$, $K=10$, $\rho=0.7$ and $R=0.5$ bps/Hz , where $M_T=1$ corresponds to an M-SISO system. As seen from the figure, the more the transmit antennas, the better the outage performance for all the schemes, which is consistent with the observations made from the asymptotic outage probabilities in Section 4. Specifically, the performance of M-OSTBC improves much more rapidly with the increase of M_T mainly due to the increased diversity order, and this clearly demonstrates the superiority of M-OSTBC under the considered conditions.

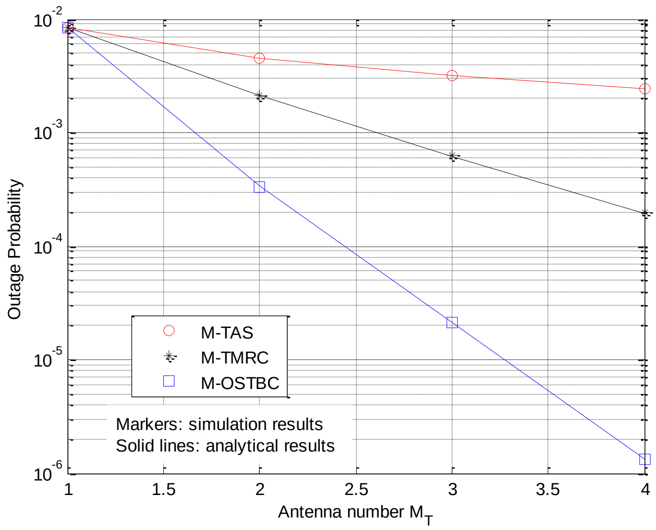

Fig. 6. Outage probabilities of the joint diversity schemes vs. M_T, with $\bar{\gamma} = 10$ dB, $K = 10$, $\rho = 0.7$ and $R = 0.5$ bps/Hz.

Fig. 7 shows the outage probabilities vs. ρ with $\bar{\gamma} = 10\,\text{dB}$, $K = 10$, $M_T = 2$ and $R = 0.5\,\text{bps/Hz}$. As expected, as ρ increases, the outage performance of all the schemes improves owing to the increased accuracy of the CSI obtained at the BS. Specifically, when $\rho = 0$, the outage probability of M-TMRC is the same as that of M-TAS and M-SISO, and larger than that of M-OSTBC. The reason for the results of $\rho = 0$ is that no useful user CSI is obtained at the BS, so the closed-loop spatial diversity in M-TMRC and M-TAS is invalid, and, at the same time, the open-loop spatial diversity in M-OSTBC can still be exploited to improve the system performance. As ρ increases, the superiority of M-OSTBC over M-TMRC decreases, while that of M-TMRC over M-TAS and M-SISO increases. It is concluded that M-OSTBC is more robust to the channel uncertainty arising from feedback delay.

In **Fig. 8**, the outage probabilities vs. R are plotted with $\bar{\gamma} = 10\,\text{dB}$, $K = 10$, $M_T = 2$ and $\rho = 0.7$. In accord with intuition, all the outage probabilities approach 1 as R increases. Furthermore, it is seen that M-OSTBC outperforms the other schemes in a wide range of the outage probabilities (in this case, for outage probabilities < 0.02). It is also interesting to observe that the opposite occurs for high outage levels. This figure can also be used to determine the outage capacity for a target outage probability. For example, at 10^{-2} outage probability, M-OSTBC, M-TMRC and M-TAS achieve the outage capacity of 1.6, 1.4 and 0.9 bps/Hz, respectively. M-OSTBC achieves larger outage capacity than M-TMRC/ M-TAS when the target outage probability is less than 0.02/0.15, but exhibits smaller outage capacity when the target outage probability is larger than the threshold. This implies that which one of these joint diversity schemes should be used depends on the system parameters and the desired outage probability.

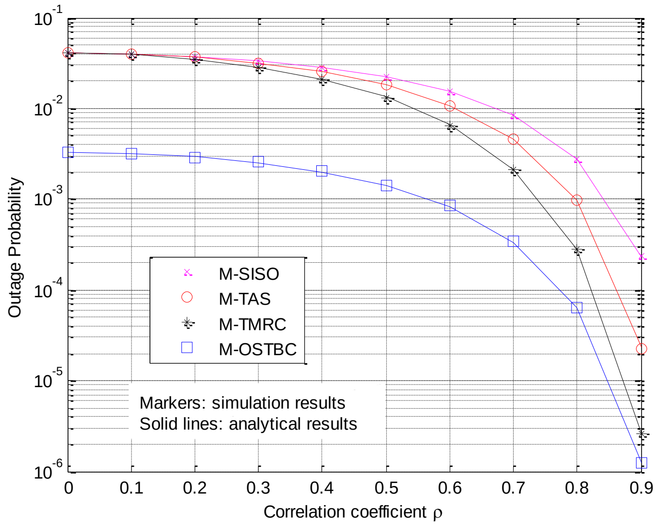

Fig. 7. Outage probabilities of the joint diversity schemes vs. ρ , with $\bar{\gamma} = 10$ dB, $K = 10$, $M_T = 2$ and $R = 0.5$ bps/Hz .

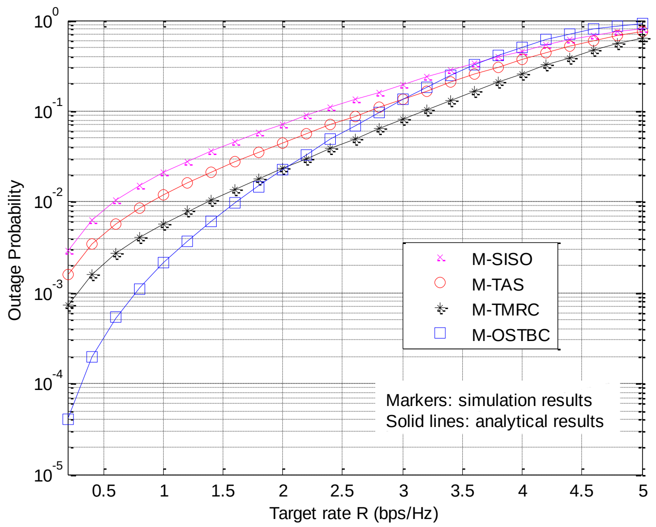

Fig. 8. Outage probabilities of the joint diversity schemes vs. R , with $\bar{\gamma} = 10$ dB, $K = 10$, $M_T = 2$ and $\rho = 0.7$.

6. Conclusions and Future work

This paper studied the outage probabilities of the joint diversity M-TMRC, M-TAS and M-OSTBC schemes in Rayleigh fading channels with outdated feedback. Both the exact and the asymptotic outage probabilities were derived. The asymptotic results revealed that the asymptotic diversity orders of the M-TMRC, M-TAS and M-OSTBC schemes are 1, 1, and the number of transmit antennas at the BS, respectively. The asymptotic outage probabilities of the three schemes were further compared. It was shown that with outdated feedback, the M-TMRC scheme is superior to the M-TAS scheme, and the M-OSTBC scheme can achieve the best outage performance in the low outage regime when the feedback delay is large. Simulation results were presented to substantiate the analysis.

In this paper, we only focused on analyzing the *outage probabilities* of the joint diversity schemes in *Rayleigh* fading. It will be interesting and challenging to extend the analysis to the *Nakagami* fading or analyze the *bit error rate (BER)* of the joint diversity schemes with outdated feedback. More research is needed in this area.

Appendix A. Some Preliminaries for the Calculation of $P_o(L,\alpha,\beta,Z,Q)$

In this appendix, we provide some preliminary results, which will be useful in the calculation of $P_o(L,\alpha,\beta,Z,Q)$ in Appendix B.

A.1 Lemma 2

Lemma 2: For positive a and non-negative integers M and N, we have

$$\sum_{k=0}^{\infty}\frac{a^k}{(k+N)!}\binom{k+M+N}{k}=e^a\sum_{t=0}^{M}\binom{M}{t}\frac{a^t}{(N+t)!}. \tag{34}$$

The above expression transforms an infinite summation into a finite summation, enabling an accurate calculation. It should be mentioned that the left side of (34) is in fact an extension of Eq. (41) in [20]. The following proof follows the same idea as that in the calculation of Eq. (41) in [20].

Proof:

According to Eq. (42) of [20], we have

$$\binom{k+M+N}{k}=\sum_{t=0}^{\min(M,k)}\binom{M}{t}\binom{k+N}{k-t}. \tag{35}$$

Then, applying (35) to the left side of (34), it gives

$$\begin{aligned}&\sum_{k=0}^{\infty}\frac{a^k}{(k+N)!}\binom{k+M+N}{k}\\&=\sum_{k=0}^{\infty}\frac{a^k}{(k+N)!}\sum_{t=0}^{\min(M,k)}\binom{M}{t}\binom{k+N}{k-t}\\&=\sum_{t=0}^{M}\sum_{k=t}^{\infty}\frac{a^k}{(k+N)!}\binom{M}{t}\binom{k+N}{k-t}\\&=\sum_{t=0}^{M}\binom{M}{t}\sum_{k=t}^{\infty}\frac{a^k}{(k+N)!}\frac{(k+N)!}{(k-t)!(N+t)!}\\&=\sum_{t=0}^{M}\binom{M}{t}\frac{a^t}{(N+t)!}\sum_{k=t}^{\infty}\frac{a^{k-t}}{(k-t)!}\\&=e^a\sum_{t=0}^{M}\binom{M}{t}\frac{a^t}{(N+t)!},\end{aligned}$$

where the third line is obtained by interchanging the order of summation.

■

A.2 Expression of $f_{\max,Z,Q}(x)$

As the definition of $P_o(L,\alpha,\beta,Z,Q)$ in (15) contains $f_{\max,Z,Q}(x)$, we need the exact expression of $f_{\max,Z,Q}(x)$ to calculate $P_o(L,\alpha,\beta,Z,Q)$. From Eq. (40) of [6], we get

$$f_{\max,Z,Q}(x)=\frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}(-1)^t\binom{Z-1}{t}\eta_n^t\, x^{Q-1+n}e^{-(t+1)x}, \tag{36}$$

with η_n^t given in Eq. (16) of [6] as

$$\eta_n^t=\begin{cases}1 & ,n=0\\ t & ,n=1\\ \dfrac{1}{n}\displaystyle\sum_{j=1}^{\min(n,Q-1)}\frac{j(t+1)-n}{j!}\eta_{n-j}^t & ,2\le n\le t(Q-1)-1\\ \left[\dfrac{1}{(Q-1)!}\right]^t & ,n=t(Q-1)\end{cases}. \tag{37}$$

Appendix B. Calculation of $P_o(L,\alpha,\beta,Z,Q)=\int_0^\infty F_{nc-\chi^2,2L,\alpha x}(\beta)\, f_{\max,Z,Q}(x)\mathrm{d}x,\ L\le Q$

According to [22], $F_{nc-\chi^2,2L,\alpha x}(\beta)$ can be derived as

$$F_{nc-\chi^2,2L,\alpha x}(\beta)=\sum_{k=0}^{\infty}\frac{\left(\frac{\alpha x}{2}\right)^k e^{-\frac{\alpha x}{2}}}{k!}\frac{\int_0^{\frac{\beta}{2}}y^{L+k-1}e^{-y}\,\mathrm{d}y}{(L+k-1)!}. \tag{38}$$

Applying (38) to the definition of $P_o(L,\alpha,\beta,Z,Q)$, it follows that

$$\begin{aligned}P_o(L,\alpha,\beta,Z,Q)&=\int_0^\infty\sum_{k=0}^{\infty}\frac{(\alpha x/2)^k e^{-\alpha x/2}}{k!}\frac{\int_0^{\frac{\beta}{2}}y^{L+k-1}e^{-y}\mathrm{d}y}{(L+k-1)!}f_{\max,Z,Q}(x)\mathrm{d}x\\ &=\sum_{k=0}^{\infty}\frac{(\alpha/2)^k}{k!}\frac{\int_0^{\frac{\beta}{2}}y^{L+k-1}e^{-y}\mathrm{d}y}{(L+k-1)!}\int_0^\infty f_{\max,Z,Q}(x)x^k e^{-\alpha x/2}\mathrm{d}x.\end{aligned} \tag{39}$$

For easy notation, let $B(k)=\int_0^\infty f_{\max,Z,Q}(x)x^k e^{-\alpha x/2}\mathrm{d}x$. Substituting (36) into $B(k)$, we have:

$$\begin{aligned}B(k)&=\frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}(-1)^t\binom{Z-1}{t}\eta_n^t\int_0^\infty x^{k+Q+n-1}\exp\left[-\left(t+1+\frac{\alpha}{2}\right)x\right]\mathrm{d}x\\ &=\frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}(-1)^t\binom{Z-1}{t}\eta_n^t\frac{(k+Q+n-1)!}{\left(t+1+\frac{\alpha}{2}\right)^{k+Q+n}},\end{aligned} \tag{40}$$

where η_n^t is given in (37), and $\int_0^\infty x^d e^{-\mu x}\mathrm{d}x=d!\mu^{-d-1}$ is used to obtain the last equation [23].

Then, substituting (40) into (39), $P_o(L,\alpha,\beta,Z,Q)$ can be further expressed by

$$\begin{aligned} P_o(L,\alpha,\beta,Z,Q) &= \frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}(-1)^t\binom{Z-1}{t}\eta_n^t\sum_{k=0}^{\infty}\frac{(\alpha/2)^k}{k!}\frac{(k+Q+n-1)!}{\left(t+1+\frac{\alpha}{2}\right)^{k+Q+n}}\frac{\int_0^{\frac{\beta}{2}}y^{L+k-1}e^{-y}\mathrm{d}y}{(L+k-1)!} \\ &= \frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}(-1)^t\binom{Z-1}{t}\frac{\eta_n^t}{\left(t+1+\frac{\alpha}{2}\right)^{Q+n}}\int_0^{\frac{\beta}{2}}y^{L-1}e^{-y}g(y)\mathrm{d}y, \end{aligned} \quad (41)$$

where $g(y)$ is given as

$$\begin{aligned} g(y) &= \sum_{k=0}^{\infty}\frac{(\alpha/2)^k}{k!}\frac{(k+Q+n-1)!}{\left(t+1+\frac{\alpha}{2}\right)^k}\frac{y^k}{(L+k-1)!} \\ &= (Q+n-1)!\sum_{k=0}^{\infty}\left(\frac{\alpha y/2}{t+1+\alpha/2}\right)^k\frac{1}{[k+(L-1)]!}\binom{k+(Q-L+n)+(L-1)}{k}. \end{aligned} \quad (42)$$

Using Lemma 2 in the above expression, $g(y)$ is simplified as:

$$g(y) = (Q+n-1)!\exp\left(\frac{\alpha y/2}{t+1+\alpha/2}\right)\sum_{i=0}^{Q-L+n}\frac{\binom{Q-L+n}{i}}{(L-1+i)!}\left(\frac{\alpha y/2}{t+1+\alpha/2}\right)^i. \quad (43)$$

Then, substituting (43) into (41), it yields:

$$\begin{aligned} &P_o(L,\alpha,\beta,Z,Q) \\ &= \frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}\sum_{i=0}^{Q-L+n}(-1)^t\binom{Z-1}{t}\frac{\eta_n^t(Q+n-1)!}{(t+1+\alpha/2)^{Q+n}}\frac{\binom{Q-L+n}{i}}{(L-1+i)!} \\ &\times\left(\frac{\alpha/2}{t+1+\alpha/2}\right)^i\int_0^{\frac{\beta}{2}}y^{L-1+i}\exp\left(-\frac{t+1}{t+1+\alpha/2}y\right)\mathrm{d}y \\ &= \frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}\sum_{i=0}^{Q-L+n}(-1)^t\binom{Z-1}{t}\frac{\eta_n^t(Q+n-1)!}{(t+1+\alpha/2)^{Q+n}}\frac{\binom{Q-L+n}{i}}{(L-1+i)!} \\ &\times\frac{\left(\frac{\alpha}{2}\right)^i\left(t+1+\frac{\alpha}{2}\right)^L}{(t+1)^{L+i}}\int_0^{\frac{\beta(t+1)}{\alpha+2t+2}}y^{L-1+i}e^{-y}\mathrm{d}y. \end{aligned} \quad (44)$$

Finally, by defining $\Gamma_m(x) = \frac{1}{(m-1)!}\int_0^x y^{m-1}e^{-y}\,\mathrm{d}y$, $P_o(L,\alpha,\beta,Z,Q)$ can be calculated in a closed-form as

$$\begin{aligned} P_o(L,\alpha,\beta,Z,Q) &= \frac{Z}{(Q-1)!}\sum_{t=0}^{Z-1}\sum_{n=0}^{t(Q-1)}\sum_{i=0}^{Q-L+n}(-1)^t\binom{Z-1}{t}\binom{Q-L+n}{i} \\ &\times\frac{\eta_n^t(Q+n-1)!}{\left(t+1+\frac{\alpha}{2}\right)^{Q+n-L}}\frac{\left(\frac{\alpha}{2}\right)^i}{(t+1)^{L+i}}\Gamma_{L+i}\left(\frac{\beta(t+1)}{\alpha+2t+2}\right). \end{aligned} \quad (45)$$

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