J. KSIAM Vol.15, No.2, 151–160, 2011

# CLOSED-FORM SOLUTIONS OF AMERICAN PERPETUAL PUT OPTION UNDER A STRUCTURALLY CHANGING ASSET

DONG-HOON SHIN[1]

[1]THE INSTITUTE OF BASIC SCIENCE, KOREA UNIVERSITY, ANAM-DONG SEONGBUK-GU, SEOUL, 136-701, KOREA
*E-mail address*: theater7@korea.ac.kr

ABSTRACT. Typically, it is hard to find a closed form solution of option pricing formula under an asset governed by a change point process. In this paper we derive a closed-form solution of the valuation function for an American perpetual put option under an asset having a change point. Structural changes are formulated through a change-point process with a Markov chain. The modified smooth-fit technique is used to obtain the closed-form valuation function. We also guarantee the optimality of the solution via the proof of a corresponding verification theorem. Numerical examples are included to illustrate the results.

## 1. INTRODUCTION

We consider American perpetual put option pricing under an asset governed by a change point process. We start with a set of geometric Brownian motions :

$$dX(t) = X(t)\mu(\alpha(t))dt + X(t)\sigma(\alpha(t))dw(t) \tag{1}$$

where $\alpha(t) \in \{1, 2\}$ is a two state Markov chain and $w(t)$ is a standard Wiener process. Here, $\mu(i)$ and $\sigma(i)$ are constants for $i \in \{1, 2\}$. An American option is a derivative giving its holder the right of exercising a share of stock at one's choice of time $\tau \in [0, T]$ with payoff of $(K - X(\tau))^+ = \max(0, K - X(\tau))$ where $T$ is the expiration date and $K$ is the strike price. Pricing an American option can be formulated as an optimal stopping problem. We consider the corresponding optimal stopping problem under the framework of this change point process.

Without any changing, the problem becomes a Black-Scholes model. The famous formula gives a closed-form solution for the option prices under a GBM. Nevertheless, because the real market is fickleness, the underlying asset usually does not follow a strict stationary log-normal process with fixed parameter such as constant volatility and rate of return. Therefore, we assume that the underlying asset follows a set of GBMs. Prior to this paper, McKean [7] had studied the Black-Scholes case with HJB equation and smooth fit principle. Guo and Zhang

Received by the editors February 1, 2011; Accepted June 21, 2011.
2000 *Mathematics Subject Classification.* 91G80.
*Key words and phrases.* option pricing, american perpetual options, change point process, structuaral changes, optimal stopping time.

[4] studied the regime-switching model with an irreducible chain and found a closed form solution of the perpetual case.
In this work, structural changes are modeled through a change-point process, rather than a Markov switching process. In a Markov switching process, a regime once occupied can be visit again. However, this assumption is not reasonable if one believes that parameters determining a regime are unique and never repeated. Otherwise, in a change point specification, a regime once occupied never comes again. Chib [1] formulated the change point process as a specific unidirectional Markov process.
In this paper, we consider the case when there are two states and one of them is an absorbing state which describes a regime after a change point. So if there is a jump of $\alpha(t)$, it may have only jump in its lifetime. Without loss of generality, we call a state to state 1 and the absorbing state to state 2. It is easy to see that if $\alpha(0) = 2$, it will stay there and if $\alpha(0) = 1$, it may jump to state 2 at a later time.
Next, we formulate the problems and study the value functions dependent on the initial state. In section 3, we use a smooth fit technique to find the value functions. Then in section 4, we show that these functions are indeed the optimal reward functions. In Section 5, we report numerical simulations. Section 6 concludes this work.

## 2. PROBLEM FORMULATION

We consider perpetual options, i.e., $T = \infty$. In this case, our optimal stopping problem becomes the evaluation of

$$V^*(x,i) = \sup_{0\le\tau\le\infty} E[e^{-r\tau}(K - X_\tau)^+ | X(0) = x, \alpha(0) = i] \tag{2}$$

where $r > 0$ is the discount factor, and $\tau$ is an $\mathcal{F}_+ = \sigma\{(w(s), \alpha(s)) | s \le t\}$-stopping time.
As a first step, we derive an optimal stopping rule for (2) with a restriction that the regime switching can occur at most once. In addition, we suppose the initial state $\alpha(0) = 1$ and it jumps to state 2, and if $\alpha(0) = 2$, then there is no jump afterward. We derive the corresponding value function of (2) under these conditions.
Let $\lambda$ be a rate to jump from state 1 to state 2, then our generator of the Markov chain has the form
$\begin{pmatrix} -\lambda & \lambda \\ 0 & 0 \end{pmatrix}$ with $\lambda > 0$, because the state 2 is absorbing.
Recall that when there is no regime switching, this problem becomes a McKean's problem: There exists a threshold $x^*$ such that the optimal stopping rule is $\tau^* = \inf\{t > 0 : X(t) \notin (x^*, \infty)\}$, and the corresponding value function

$$\begin{aligned} V^*(x) &= \sup_{0\le\tau\le\infty} E[e^{-r\tau}(K - X(\tau))^+ | X(0) = x] \\ &= E[e^{-r\tau^*}(K - X(\tau^*))^+ | X(0) = x] \end{aligned}$$

is given by

$$V^*(x) = \begin{cases} (K - x^*)(x/x^*)^\gamma & \text{if } x > x^*, \\ K - x, & \text{if } x \le x^* \end{cases}$$

for some $\gamma < 0$, and $x^* > 0$. In our problem, $V_2(x)$ follows the McKean's rule since state 2 is absorbing. Therefore, $V_2(x)$ can be given as follows:

$$V_2(x) = \begin{cases} (K - x_2)(x/x_2)^\gamma & \text{if } x > x_2, \\ K - x, & \text{if } x \le x_2 \end{cases}$$

where the $\gamma$ is the negative solution of $r = \mu_2\gamma + \frac{1}{2}\sigma(2)^2\gamma(\gamma - 1)$, i.e. $\gamma = \dfrac{-(\mu(2) - \frac{1}{2}\sigma(2)^2) - \sqrt{(\mu(2) - \frac{1}{2}\sigma(2)^2)^2 + 2\sigma(2)^2 r}}{\sigma(2)^2}$, and $x_2 = \dfrac{\gamma K}{\gamma - 1} > 0$. from McKean[7].

With a two-state Markov chain and with $\sigma(1) \neq \sigma(2)$, it is easy to see that $(X(t), \alpha(t))$ is a joint Markov process [3]. We expect that the optimal stopping rule is also of threshold type, except that the threshold levels should vary depending on the state $\alpha(t)$. In other words, we expect the existence of two thresholds $x_1, x_2 \le K$, so that the optimal stopping rule is given as

$$\tau^* = \inf\{t \ge 0 | (X(t), \alpha(t)) \notin D\},$$

where

$$D = \{(x, i) | V^*(x, i) > (K - x)^+\}.$$

The set $D$ is referred to as the *continuation region*. Using $\tau^*$, the corresponding value functions are

$$V^*(x, i) = E[e^{-r\tau^*}(K - X(\tau^*))^+ | X(0) = x, \alpha(0) = i].$$

We consider the case when $D$ can be represented by two threshold levels $x_1$ and $x_2$, i.e.,

$$D = \{(x, 1) | x \in (x_1, \infty)\} \cup \{(x, 2) | x \in (x_2, \infty)\}.$$

Notice that $x_1$ and $x_2$ should depend on $r, K, \mu(i), \sigma(i), \lambda$. For any $x_1$ and $x_2$, there are only three possibilities, $x_1 < x_2, x_1 > x_2$ and $x_1 = x_2$. In the next sections we discuss each of these cases and derive the values of these thresholds $x_i$ as well as the corresponding reward functions (denoted as $V_i(x)$) under this type of stopping rule.

## 3. Solving the problem

At any given time $t$, if $\alpha(t) = 1$ and $X(t) \le x_1$, then one should stop immediately and obtain a payoff of $(K - X(t))^+$; this follows from the definition of $x_1$ and $x_2$. In view of Ito's differential rule, this is translated into a set of differential equations. $V_1$ satisfies :

$$\begin{cases} V_1(x) = K - x & \text{if } x \in [0, x_1), \\ (\lambda + r)V_1(x) = x\mu(1)V_1'(x) + \frac{1}{2}x^2\sigma(1)^2 V_1''(x) + \lambda V_2(x) & \text{if } x \in [x_1, \infty). \end{cases} \quad (3)$$

Let $\beta_1 > 0 > \beta_2$ be the solutions of $(\lambda+r) = (\mu(1)-\frac{\sigma(1)^2}{2})\beta+\frac{1}{2}\sigma(1)^2\beta^2$. Then the general solution of 3 is given by $V_1(x) = C_1x^{\beta_1} + C_2x^{\beta_2} + \phi(x)$, where $\phi(x)$ is the special solution of $V_1(x)$. Therefore, we need to determine the special solution for this problem.

For the solution of $V_1(x)$ of (3) where $x \in [x_1, \infty)$, we need a change of variable. Substituting $V_1(x) = \phi_1(\log x) = \phi_1(y)$, $\log(x) = y$ and $\log(x_1) = y_1$, we have $V_1'(x) = \frac{1}{x}\phi_1'(y)$ and $V_1''(x) = \frac{1}{x^2}\phi_1''(y)$. The equation for $V_1(x)$ in (3) is changed in terms of $y$ as

$$(\lambda + r)\phi_1(y) = (\mu(1) - \frac{\sigma(1)^2}{2})\phi_1'(y) + \frac{1}{2}\sigma(1)^2\phi_1''(y) + \lambda V_2(e^y) \tag{4}$$

To find the general equation for $\phi_1(y)$, we use the follow characteristic function. Let $\beta_1 > 0 > \beta_2$ are the solutions of $(\lambda + r) = (\mu(1) - \frac{\sigma(1)^2}{2})\beta + \frac{1}{2}\sigma_1^2\beta^2$. It is easy to see that $\phi_1(y) = C_1e^{\beta_1 y} + C_2e^{\beta_2 y} + \phi(y)$, where $\phi(y)$ is the special solution of $\phi_1(y)$.
Next, we find the special solution $\phi(y)$ on $[\log x_1, \infty)$. From (4), we have

$$\phi''(y) = \frac{2}{\sigma(1)^2}(\lambda + r)\phi(y) + (-\frac{2}{\sigma(1)^2}\mu(1) + 1)\phi'(y) - \frac{2}{\sigma(1)^2}\lambda V_2(e^y).$$

Construct a matrix ODE

$$\frac{d}{dy}\begin{pmatrix} \phi(y) \\ \phi'(y) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{2}{\sigma(1)^2}(\lambda + r) & -\frac{2}{\sigma(1)^2}\mu(1) + 1 \end{pmatrix}\begin{pmatrix} \phi(y) \\ \phi'(y) \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{2}{\sigma(1)^2}\lambda V_2(e^y) \end{pmatrix}. \tag{5}$$

Therefore, the special solution is

$$\begin{pmatrix} \phi(y) \\ \phi'(y) \end{pmatrix} = \int_{y_1}^{y} \exp(A(y - s)) \begin{pmatrix} 0 \\ -\frac{2}{\sigma(1)^2}\lambda V_2(e^y) \end{pmatrix} ds \tag{6}$$

where the matrix $A = \begin{pmatrix} 0 & 1 \\ \frac{2}{\sigma(1)^2}(\lambda + r) & -\frac{2}{\sigma(1)^2}\mu(1) + 1 \end{pmatrix}$.
Let $y_m = \max\{y_1, y_2\}$. The integration (6) gives

$$\begin{aligned} \phi(y) &= \frac{1}{\beta_2 - \beta_1}\int_{y_1}^{y}(e^{\beta_1(y-s)} - e^{\beta_2(y-s)})(\frac{2}{\sigma_1^2}\lambda V_2(e^s))ds \\ &= \frac{1}{\beta_2 - \beta_1}[\int_{y_1}^{y} -e^{\beta_2(y-s)}(\frac{2}{\sigma_1^2}\lambda V_2(e^s))ds + \int_{y_1}^{y_m} e^{\beta_1(y-s)}(\frac{2}{\sigma_1^2}\lambda V_2(e^s))ds \ . \\ &+\frac{2}{\sigma(1)^2}\lambda\frac{K - x_2}{(x_2)^\gamma}\int_{y_m}^{y} e^{\beta_1(y-s)}e^{\gamma s}ds] \end{aligned}$$

Consider the last term that

$$\begin{aligned}\int_{y_m}^{y} e^{\beta_1(y-s)}(e^{\gamma s})ds &= e^{\beta_1 y}\int_{y_m}^{y} e^{(\gamma-\beta_1)s}ds\\ &= e^{\beta_1 y}\frac{1}{\gamma-\beta_1}(e^{(\gamma-\beta_1)y}-e^{(\gamma-\beta_1)y_m})\\ &= \frac{1}{\gamma-\beta_1}(e^{\gamma y}-e^{\beta_1 y+(\gamma-\beta_1)y_m}).\end{aligned}$$

We note that $-e^{\beta_1 y+(\gamma-\beta_1)y_m}$ is unbounded part as $y\to\infty$. Choose $C_1 = \frac{2\lambda}{(\beta_2-\beta_1)\sigma(1)^2}\times (\int_{y_1}^{y_m} e^{-\beta_1 s}V_2(e^s)ds + \frac{K-x_2}{(x_2)^\gamma}\frac{1}{\gamma-\beta_1}e^{(\gamma-\beta_1)y_m})$ and recall $V_1(x) = C_1x^{\beta_1} + C_2x^{\beta_2} + \phi(\log x)$. Then $C_1x^{\beta_1}$ and some unbounded parts of $\phi(\log x)$ cancel each other, so that $C_1x^{\beta_1}+\phi(\log x)$ becomes bounded as $y\to\infty$. Finally, we attain

$$V_1(x) = C_2x^{\beta_2} - \frac{2\lambda x^{\beta_2}}{(\beta_2-\beta_1)\sigma(1)^2}(\int_{x_1}^{x} t^{-\beta_2-1}V_2(t)dt - \frac{(K-x_2)x^\gamma}{x_2^\gamma(\gamma-\beta_1)}). \tag{7}$$

where $x\in[x_1,\infty)$. In order to determine $C_2$ and $x_1$, use the smooth fit condition at $x = x_1$. Then we obtain

$$\begin{cases} K - x_1 = C_2x_1^{\beta_2} + \dfrac{2\lambda x^{\beta_2}(K-x_2)x^\gamma}{(\beta_2-\beta_1)\sigma(1)^2x_2^\gamma(\gamma-\beta_1)},\\ -x_1 = C_2\beta_2x_1^{\beta_2+1} - \dfrac{2\lambda}{(\beta_2-\beta_1)\sigma(1)^2}(V_2(x_1) - \dfrac{(K-x_2)x_1^\gamma}{x_2^\gamma(\gamma-\beta_1)}).\end{cases} \tag{8}$$

Eliminating $C_2$, we have

$$\frac{2\lambda}{(\beta_2-\beta_1)\sigma(1)^2}(V_2(x_1) + \frac{(\beta_2-\gamma)(K-x_2)x_1^\gamma}{x_2^\gamma(\gamma-\beta_1)}) = \beta_2(K-x_1) + x_1.$$

The above equation gives $x_1$, so $C_2$ can be calculated by (8). In conclusion, the solution for $V_1(x)$ is

$$\begin{cases} V_1(x) = K - x, & \text{if } x\in[0,x_1)\\ V_1(x) = C_2x^{\beta_2} - \dfrac{2\lambda x^{\beta_2}}{(\beta_2-\beta_1)\sigma_1^2}(\displaystyle\int_{x_1}^{x} t^{-\beta_2-1}V_2(t)dt - \dfrac{(K-x_2)x^\gamma}{x_2^\gamma(\gamma-\beta_1)}), & \text{if } x\in[x_1,\infty)\end{cases} \tag{9}$$

with $\beta_1,\beta_2$ and $C_2$ given above.

## 4. Optimality of the solution

We give a verification theorem to show that $V_1(x), V_2(x)$ are indeed the value function.

**Theorem 4.1.** *Suppose that (3) has a solution $x_1^*$ such that $0 < x_1^* \le K$. Assume $V_1(x) > (K-x)^+$ on $(x_1^*,\infty)$ and $\mu(1)\ge 0$. Define*

$$D = \{(x,i)|V_i(x) > (K-x)^+\}, i = 1,2,$$

*and let*

$$\tau^* = \inf\{t \geq 0 | (X(t), \alpha(t)) \notin D\}$$

*Then* $\tau^*$ *is an optimal stopping time, and* $V_1(x), V_2(x)$ *are the value functions.*

Since $V_2(x)$ is derived by McKean's rule, we prove the theorem only about $V_1(x)$. We need a proposition before the proof of the theorem. For $V_1(x) \in C^2$, define

$$LV_1(x) = x\mu(1)\frac{\partial V_1(x)}{\partial x} + \frac{x^2\sigma(1)^2\partial^2 V_1(x)}{2\partial x^2} + \lambda(V_2(x) - V_1(x)) - rV_1(x).$$

and

$$D = \{(x, 1)|x \in (x_1, \infty)\} \cup \{(x, 2)|x \in (x_2, \infty)\}.$$

**Proposition 4.2.** $LV_1(x) \leq 0$ *on D.*

*Proof.* It is obvious by the definition of $LV_1(x)$ that $LV_1(x) = 0$ where $x \geq x_1$. Consider $LV_1(x)$ where $x \in [x_2, x_1]$. Since $V_1(x) = K - x$, $V_2(x) = (K - x_2)(x/x_2)^\gamma$ on $[x_2, x_1]$,

$$\begin{aligned} LV_1(x) &= x\mu(1)V_1'(x) + \frac{1}{2}x^2\sigma(1)^2 V_1''(x) + \lambda(V_2(x) - V_1(x)) - rV_1(x) \\ &= -x\mu(1) + \lambda((K - x_2)(x/x_2)^\gamma - (K - x)) - r(K - x) \end{aligned}$$

We have $LV_1(x_2) < 0$ from the above and we know that $LV_1(x)$ is continuous on $[x_2, x_1]$ because $V_1(x) \in C^2$. In addition, since the second derivative is positive, i.e.,

$$(LV_1(x))'' = \lambda\gamma(\gamma - 1)(K - x_2)x_2^{-\gamma}x^{\gamma-2} > 0$$

on $[x_2, x_1]$, $LV_1(x)$ is concave on $[x_2, x_1]$. So, if $LV_1(x_1) < 0$, the proposition is true. To show $LV_1(x_1) < 0$, we use the fact that

$$\lim_{x \to x_1^-} [x\mu(1)V_1'(x) + \lambda(V_2(x) - V_1(x)) - rV_1(x)] = \lim_{x \to x_1^+} [x\mu(1)V_1'(x) + \lambda(V_2(x) - V_1(x)) - rV_1(x)] \tag{10}$$

by the smooth fit principle around $x = x_1^+$, and

$$\lim_{x \to x_1^+} x\mu(1)V_1'(x) + \frac{1}{2}x^2\sigma(1)^2 V_1''(x) + \lambda(V_2(x) - V_1(x)) - rV_1(x) = 0 \tag{11}$$

because $LV_1(x) = 0$ on $[x_1, \infty)$. Combining (10) with (11) implies that

$$\lim_{x \to x_1^-} x\mu(1)V_1'(x) + \lambda(V_2(x) - V_1(x)) - rV_1(x) = \lim_{x \to x_1^+} -\frac{1}{2}x^2\sigma(1)^2 V_1''(x).$$

We know that $\lim_{x \to x_1^+} V_1''(x) > 0$ because $V_1(x)$ is generally concave on $[x_1, \infty]$. It completes the proof of the proposition. Additionally, we see the $\lim_{x \to x_1^-} LV_1(x)$ is indeed negative at the next numerical analysis section. □

Now we start the proof of the theorem.

*Proof of the theorem.* It is easy to see that $V_1(\infty) = 0$ and $LV_1(x) \leq 0$ on $x \in D$ from the above proposition. Using Dynkin's formula, we have

$$d(e^{-rt}V_1(X(t))) = e^{-rt}LV_1(X(t))dt + d(martingale).$$

A smooth approximation approach for variational inequalities in Øksendal[9] [p224] implies that for any stopping time $\tau$ we obtain

$$V_1(x) \geq E[e^{-r\tau}V_1(X_\tau)] \geq E[e^{-r\tau}(K - X_\tau)^+]. \tag{12}$$

To show the optimality of $\tau^*$, note that if $\tau^* < \infty$, then $V_1(X(\tau^*)) = (K - X(\tau^*))^+$. In this case, Dynkin's formula yields $V_1(x) = E[e^{-r\tau^*}(K - X(\tau^*))^+]$. Otherwise, let $D_k = D \cap \{x < k\}$, for $k = 1, 2, ....$ Let $\tau_k = \inf\{t \geq 0 | (X(t), \alpha(t)) \notin D_k\}$. Then we can show that $\tau_k \to \tau^*$ a.s. Moreover, as in [10, Theorems 4.5 and 4.6], we can show that $\tau_k < \infty$ a.s. for each $k$. Using the definition of $\tau_k$ with some large number $k$, we have

$$V_1(X(\tau_k)) = V_1(X(\tau_k))I_{\{X(\tau_k)=k\}} + V_1(X(\tau_k))I_{\{X(\tau_k)<k\}}.$$

Note that

$$V_1(X(\tau_k))I_{\{X(\tau_k)<k\}} = (K - X(\tau_k))^+I_{\{X(\tau_k)<k\}} \leq (K - X(\tau_k))^+.$$

Moreover, not that $0 \leq V_1(x) \leq K$ and $e^{r\tau_k}I_{\{X(\tau_k)=k\}} \to 0$, as $k \to \infty$, a.s. It follows that $E[e^{-r\tau_k}V_1(X(\tau_k))I_{\{X(\tau_k)=k\}}] \to 0$. Therefore, as $k \to \infty$, we have

$$V_1(x) \leq E[e^{-r\tau_k}V_1(X(\tau_k))] \leq E[e^{-r\tau^*}(K - X(\tau_k))^+]. \tag{13}$$

From (12) and (13),

$$V_1(X(\tau_k)) = E[e^{-r\tau^*}(K - X(\tau^*))^+].$$

This completes the proof. □

## 5. Numerical simulation

In this simulation, we observe the change of thresholds with varying several parameters. First, we choose $K = 2, r = 0.04, \lambda = 4, \mu(1) = 0.07, \sigma(1) = 1, \mu(2) = 0.09, \sigma(2) = 1$. The thresholds from the illustrated method in section 3 and McKean's method are given by $(x_1, x_2) = (0.1602, 0.1619)$.

In Figure 1, $V_1$ and $V_2$ denote the value functions from the illustrated method, and McKean's method, respectively. The difference between $V_1$ and $V_2$ in Figure 1 represents the basic structure of the optimal stopping policy in terms of threshold levels $(x_1, x_2)$.

As the second, we examine the monotonicity of these threshold levels in terms of $\sigma(1), \lambda$ and $K$.

As the first, we vary only $\sigma(1)$. The results for $(x_1, x_2)$ are listed in Table 1. All levels of $x_1$ increase with the increasing of $\sigma(1)$. This means that a larger $\sigma(1)$ leads to a wider continuation region, and then the threshold level is lower in put options.

Second, we vary $K$. Table 2 represent that $(x_1, x_2)$ increase in $K$ due to the fact that a larger

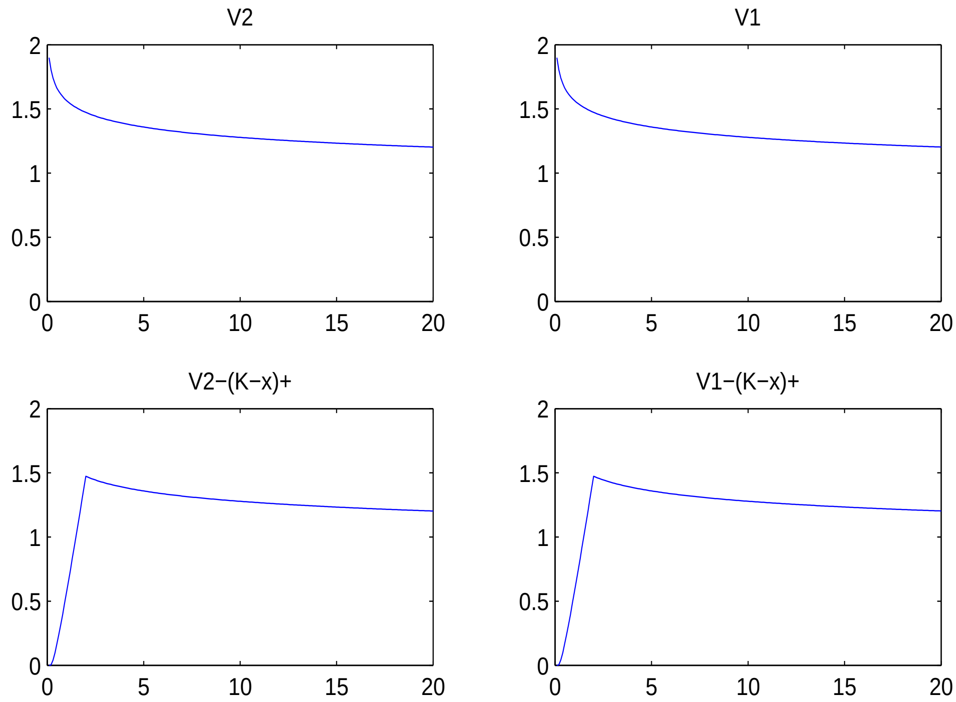


FIGURE 1. Horizontal axis–Strike price; Vertical axis–Value of the function

$K$ implies a higher transaction cost which in turn needs to be compensated by a higher return level.

Finally, we vary $\lambda$. Table 3 implies that if $\lambda$ increase, all values of $x_1$ increase to the value of $x_2$. This is because a larger $\lambda$ reads to a shorter period for $\alpha(t)$ to stay at $\alpha(t) = 1$, and it pursues $x_1$ being closer to $x_2$. As the last analysis, we check that $\lim_{x \to x_1^-} LV_1(x) < 0$ in Table 4 under the condition $x_2 < x_1$. In our table, the condition $x_2 < x_1$ is hold when $\sigma(1)$ =0.6 or 0.8 under the same condition of other variables.

## 6. CONCLUSION

In this paper, we have obtained closed-form solutions of American perpetual put option where the associated stochastic asset dynamics have a form of a change-point process; geometric Brownian motions with a two state Markov chain with one absorbing state. These results can be used as an approximation to American options when the underlying asset has a change point of the market mode on the option period, and with finite horizon $T$ when $T$ is large.

TABLE 1. Dependency on $\sigma(1)$

| $\sigma(1)$ | 0.6 | 0.8 | 1 | 1.2 | 1.4 |
|---|---|---|---|---|---|
| x1 | 0.1983 | 0.1793 | 0.1602 | 0.1427 | 0.1277 |
| x2 | 0.1619 | 0.1619 | 0.1619 | 0.1619 | 0.1619 |

TABLE 2. Dependency on $K$

| $K$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| x1 | 0.0801 | 0.1602 | 0.2402 | 0.3203 | 0.4004 |
| x2 | 0.0810 | 0.1619 | 0.2429 | 0.3239 | 0.4048 |

TABLE 3. Dependency on $\lambda$

| $\lambda$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| x1 | 0.1596 | 0.1602 | 0.1604 | 0.1606 | 0.1607 |
| x2 | 0.1619 | 0.1619 | 0.1619 | 0.1619 | 0.1619 |

TABLE 4. Negativeness of $LV_1(x_1 - h)$ when $x_2 < x_1$

| $h$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ |
|---|---|---|---|---|
| $\sigma(1)$=0.8 | -0.0844 | -0.0821 | -0.0818 | -0.0817 |
| $\sigma(1)$=0.6 | -0.0789 | -0.0734 | -0.0728 | -0.0727 |

## REFERENCES

[1] Chib, S., *Estimation and comparison of multiple change-point models.*, Journal of Economics 86, 221-241, 1998.

[2] Cox, J.C., and Ross, S., *The valuation of options for alternative stochastic process.* , Journal of Financial Economics 3, 145-166, 1976.

[3] Guo, X., *An explicit solution to an optimal stopping problem with regime switching*, J. Appl. Prob. , 38, pp. 464-481, 2001

[4] Guo, X. and Zhang, Q., *Closed-form solutions for perpetual american put options with regime switching*, SIAM J. Appl. Math. , 64, pp. 2034-2049, 2004

[5] Hamilton, J.D., *A new approach to the economic analysis of nonstationary time series*, Econometrica, 57, 357-384, 1989.

[6] Hull, J.C., *Options, Futures, and Other Derivatives*,4th Ed., Prentice-Hall, Upper Saddle River, NJ, 2000.

[7] McKean, H.P., *A free boundary problem for the heat equation arising from a problem of mathematical economics.* , Inderstrial Managem. review 61, 32-39, 1965 Spring.

[8] Merton, R.C., *Option pricing when underlying stock returns are discontinuous.* , Journal of Financial Economics 3, 125-144, 1976.

[9] Øksendal, B., *Stochastic differential Equations*, 6th ed., Springer-Verlag, New York, 2005.

[10] ZHANG, Q., *Stock trading: An optimal selling rule*, SIAM J. Control Optim., 40(2001), pp. 67-84, 2001.