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AN EFFICIENT BINOMIAL TREE METHOD FOR CLIQUET OPTIONS

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ABSTRACT. This work proposes a binomial method for pricing the cliquet options, which provide a guaranteed minimum annual return. The proposed binomial tree algorithm simplifies the standard binomial approach, which is problematic for cliquet options in the computational point of view, or other recent methods, which may be of intricate algorithm or require pre- or post-processing computations. Our method is simple, efficient and reliable in a Black-Scholes framework with constant interest rates and volatilities.

1. INTRODUCTION

Financial derivatives have become extremely popular, which provide a form of insurance in hedging or speculate on the value of the underlying asset. A cliquet option or ratchet option consists of financial derivatives, called forward start options, which start at-the-money or proportionally in- or out-of-the-money after a known reset date in the future. Since cliquet options may be globally and locally floored and capped, they provide a guaranteed minimum return in exchange for capping the maximal return earned each period over the life of the contract [1]. After experiencing a rapid falling financial market, investors demand products that reduce downside risk while still offering upside potential. The structure of cliquet options meets this demand and attracts large pension funds, portfolio insurers and retail investors as well.

For an European forward start option, there exist closed form solutions [2] under Black-Scholes model and [3] under Heston's stochastic volatility model. However, there exists no closed form solution for cliquets, and therefore one may rely on numerical approximations. There have been studies to valuate cliquets. Wilmott [4] introduced a finite difference method by solving a set of one dimensional PDEs embedded in a higher dimensional space. Windcliff et al. [5] studied PDE methods for jump diffusion models, calibrated volatility surface models and uncertain volatility models. Den Iseger and Oldenkamp [6] presented a Laplace transform inversion technique for different models. These works focus on reducing the model

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risk due to the difficulties of correct pricing of the forward volatility smile, such as studying the sensitivity of a cliquet option under local volatility, stochastic volatility, jump diffusion model etc.

Cliquet options are path-dependent contracts, and therefore it is not easy to make an efficient binomial tree method. Nevertheless, it is important to design an accurate and efficient binomial method, since binomial method is quite simple to implement and particularly useful for American and Bermudan type options. There has been not much work of binomial method for cliquet options. However there are recent studies to adapt the classical binomial tree model to cliquets. Gaudenzi and Zanette [8] compute a continuous representation of the cliquet option price as a piecewise linear function of the sum of the returns based on the singular points techniques. But the number of singular points grows rapidly at every monitoring date which eventually needs complicated pre- and post-processing to reduce this number.

Hull and White [7] extend the classical binomial method to valuate path-dependent Asian options by pricing the options for a predetermined representative set of values for average asset prices. Inspiring from this idea, we propose a new binomial tree method by introducing a suitable representative set of values for the cliquet option. In particular, this representative set can be reduced to one element due to the special structure of cliquet option. This method is quite simple and computational examples illustrate its convergence to the continuous value.

The outline of the paper is as follows. Since a cliquet option consists of forward starting options, we first describe and illustrate the binomial model for both European and American forward start options in Section 2. In Section 3 we derive a new binomial method for pricing cliquet options with cap and floor type by introducing a representative set of values for the returns of the option along the path named Middle. Numerical experiments illustrate the efficiency and accuracy of the new method in Section 4. A summary and some concluding remarks are given in Section 5.

2. Forward Start Options

Let us first consider a forward start option which commences at some specified future date with an expiration further in the future. An application of forward start options is employee stock options, which typically has a forward start feature since its strike price is not fixed when the employee begins to work.

Let us consider the price of the underlying asset as a stochastic process $\{S_t\}_{t\in[0,T]}$, which satisfies the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,\ 0 < t < T, \tag{2.1}$$

where μ is an expected rate of return, σ is a volatility, T is an expiration date, and W_t is a Brownian motion. If we consider a continuous dividend yield q, the drift rate becomes $\mu - q$ instead of μ in (2.1).

A forward start option with time to maturity T starts at-the-money or proportionally in- or out-of-the-money after a known elapsed time t^* in the future. The strike is set equal to a positive constant α times the asset price S_{t^*} after the known time t^*. Under the Black-Scholes model, European forward start option can be priced using the Rubinstein formula, see [2, 1].

However, the price of an American forward start option needs to be numerically approximated and we propose a simple and efficient binomial tree method.

2.1. **The binomial model.** Let us partition the interval $[0, T]$ into N cells of uniform length $\Delta t = T/N$ and let $0 = t_0 < t_1 < ... < t_N = T$. Assume that the known elapsed time t^* in the future is one of the nodes, $t^* = t_m$ for some m with $0 < m < N$.

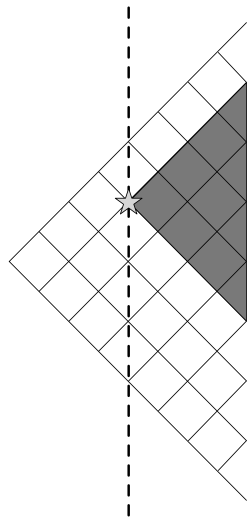

FIGURE 1. The structure of a binomial tree method for a forward start option.

The binomial method by Cox et al. [9, 10, 11] assumes that the asset price $S(t_n)$ at $t = t_n$ moves either up to $uS(t_n)$ for $u = \exp(\sigma\sqrt{\Delta t})$ or down to $dS(t_n)$ for $d = \exp(-\sigma\sqrt{\Delta t})$ and $d < \exp(r\Delta t) < u$ with probabilities $p = (\exp(r\Delta t) - d)/(u - d)$ or $1 - p$, respectively. From the present asset price S_0^0 at t_0, the price S_j^n at t_n can be computed by

$$S_j^n = d^{n-j} u^j S_0^0, \quad 0 \le j \le n,\ 0 \le n \le N\ .$$

Then the standard binomial method calculates the payoffs of the option at expiry, $V_j^N = \Lambda(S_j^N)$ with a given payoff $\Lambda(\cdot)$ of the derivatives for $j = 0, 1, ..., N$, and computes the option price $V_0^0 = V(S_0^0, 0)$ by backward averaging,

$$V_j^n = e^{-r\Delta t}\left(p\, V_{j+1}^{n+1} + (1-p)\, V_j^{n+1}\right),\ 0 \le j \le n\ , \tag{2.2}$$

for $n = N-1, N-2, \cdots, 0$.

For an European forward start option, if the dashed line in Figure 1 represents $t = t^*$ when the strike price is determined, there are $m + 1$ values on the dashed line. Once the strike priced is set, the domain of dependence for the strike price can be found. For instance, if the asset price at the star node in Figure 1 is S^*, the path through the star remains within the shaded region in the figure for the remaining of the contract and the strike price is αS^* there. Therefore we have $m+1$ different vanilla option problems starting at $t^* = t_m$ and maturing at t_N with strike price $\alpha S^* = \alpha S_j^m$ for $0 \le j \le m$, respectively. We solve these vanilla option problems using the standard backward averaging in (2.2) for $n = N-1, N-2, \cdots, m$ with the payoffs at expiry, for instance with $V_j^N = \max(S_j^N - \alpha S^*, 0)$ for the call option. Once we compute the option values at $t^* = t_m$, we again apply the backward averaging (2.2) in order to compute the option value, V_0^0, at present. When the European forward start option price is of interest, the computation can be simplified further as follows. The option price V_j^m at the node corresponding to S_j^m for $0 \le j \le m$ is obtained by

$$V_j^m = e^{-(N-m)r\Delta t} \sum_{k=0}^{N-m} \binom{N-m}{k} p^k (1-p)^{N-m-k} \max\{S_0 u^k d^{N-m-k} - \alpha S_j^m,\ 0\}\ . \quad (2.3)$$

Then the option price V_0^0 at $t=0$ is computed by

$$V_0^0 = e^{-mr\Delta t} \sum_{k=0}^{m} \binom{m}{k} p^k (1-p)^{m-k} V_j^m\ . \quad (2.4)$$

For American options, it is similar to the European case except an extra comparison step during the backward averaging. Since the American case allows early exercise, one may choose the better between two possibilities, retaining (2.2) or exercising $\Lambda(S_j^n)$, the payoff at time t_n and asset price S_j^n. Therefore this leads to the following averaging process

$$V_j^n = \max\left\{\Lambda(S_j^n), e^{-r\Delta t}\left(p\, V_{j+1}^{n+1} + (1-p)\, V_j^{n+1}\right)\right\},\ 0 \le j \le n\ , \quad (2.5)$$

for $n = N-1, N-2, \cdots, 0$. For the American forward start option, we apply the above averaging relation (2.5) from expiry to time $t^* = t_m$ for $m+1$ different American vanilla options with the strike $\alpha S^* = \alpha S_j^m$ for $0 \le j \le m$, respectively. After we calculate the option values at $t^* = t_m$, we apply (2.5) once again from time t_m to present t_0 with the strike price αS_0^0. If one allows early exercise only after the reset date t^*, this gives a so-called Bermudan forward start option.

2.2. **Numerical examples.** The left graph in Figure 2 shows the convergence of the binomial method for European forward start call option when the initial stock price $S_0^0 = 60$, the time to maturity $T = 1$, the risk-less interest rate $r = 8\%$, the stock volatility $\sigma = 30\%$ and the instantaneous dividend yield rate $q = 4\%$. The option starts 10% out-of-the-money at three months from today, namely $\alpha = 1.1, t^* = 0.25$ and the strike is $\alpha S(t^*)$. It shows that compared to the Black-Scholes price in [1], the error $|V_0^0 - C(S_0^0, 0)|$ in the new binomial method tends to zero as $N \to \infty$. We can see a typical saw-tooth pattern in the sequence of approximations as N increases. The reference curve $f(x) = \alpha e^{-\beta x}$ with $\alpha = 0.0297$ and $\beta = 0.0033$ is given in the figure to estimate the approximate convergence rate. The right graph in Figure 2 shows the comparison of the option values for European, American and Bermudan forward start options. Though it is not possible to compare with exact formulas for American and Bermudan cases,

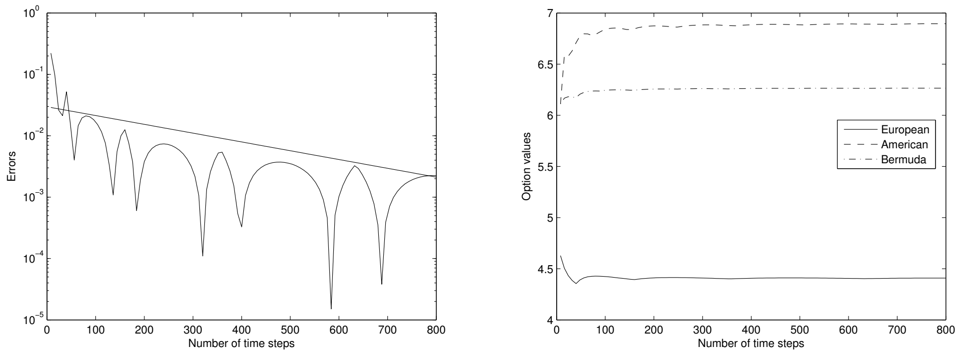

FIGURE 2. Convergence of the binomial method for an European forward start put option (left) and an American forward start put option (right).

we can see that the option values converge to slightly higher values than that from the European case as $N \to \infty$.

3. CLIQUET OPTIONS

Let us divide the interval $[0,\ T]$ into N_{obs} subintervals called reset periods of length $\Delta T = T/N_{\text{obs}}$. $\{T_i = i\Delta T\}$ with $T_0 = 0$ and $T_{N_{\text{obs}}} = T$ are called reset days. The return of an asset with the price S is defined as

$$R_i = \frac{S_i - S_{i-1}}{S_{i-1}} \ .$$

Truncated return over a reset period $[T_{i-1},\ T_i)$ is

$$\bar{R}_i = \max\left(F_{\text{loc}}, \min\left(C_{\text{loc}}, R_i\right)\right) \ ,$$

where C_{loc} and F_{loc} are local cap and local floor, respectively. The cliquet option is a series of forward start option with a single premium determined up front, which lock in any gains on reset dates. The payoff V is the sum of truncated returns bounded by global cap or global floor.

$$V = e^{-rT} B\ E\left[\min\left(\max\left(\sum_{i=1}^{N_{\text{obs}}} \bar{R}_i, F_{\text{glob}}\right), C_{\text{glob}}\right)\right] \tag{3.1}$$

where B is the notional amount, which is set to 1 in this study.

The American cliquet option allows the holder of the option to exercise it at any time, then paying at t [12],

$$\Lambda(S(t)) = B \min\left(\max\left(z + \min\left(\max\left(S(t)/\bar{S} - 1, F_{\text{loc}}\right), C_{\text{loc}}\right), F_{\text{glob}}\right), C_{\text{glob}}\right) \ , \tag{3.2}$$

where $z = \sum_{i=1}^{m-1} \bar{R}_i$ is the sum of truncated returns up to the moment t assuming $t \in [T_{m-1}, T_m)$, $S(t)$ is the asset price at t and $\bar{S}$ is the asset price at T_{m-1}.

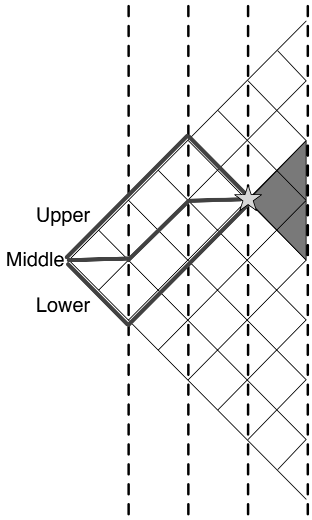

FIGURE 3. The structure of a binomial tree method for a cliquet option.

We propose a binomial tree method so that the prices of European and American cliquet options can be estimated. Suppose that the dashed lines in Figure 3 represent the reset days and backward averaging is performed at the node marked star. As in Figure 1 in Section 2, if the asset price path passes through the star node, the path remains within the shaded region in the figure for the remaining of the contract. But, there is a difference over the forward start option when the backward induction procedure is performed. In case of the forward start option in Section 2, how the asset price reaches the star node is not important, but only the value of the asset at the star node matters as in (2.3). In case of the cliquet option, however, the path from the initial node at $t = 0$ to the star node determines the option values to be used for the backward induction. In other words, the backward induction at one point may result in two different option prices if two paths of asset prices are different because $\sum_{i=1}^{N_{\text{obs}}} \bar{R}_i$ values in (3.1) are different for different paths. Thus, even though the cliquet option may be considered a generalization of the forward start option, the implementation algorithm of the binomial method for the forward start option cannot be directly applied to the cliquet option.

For the path-dependent options such as the cliquet option, it is impossible in practice to apply the standard binomial tree method [9] because the number of paths may increase exponentially. Thus, Hull and White [7] proposed to use two paths called Upper and Lower in Figure 3 for such a path-dependent option. For the Asian option with arithmetic average, for example, these paths are meaningful because Upper and Lower paths make the average asset price up to the star node maximum and minimum, respectively. These paths, however, neither result in maximum or minimum payoffs in case of the cliquet option, nor bound the payoff even when a path is within these paths. Thus the binomial method for path-dependent options by Hull and White needs to be adjusted.

We propose a binomial tree method assuming that when the backward induction is performed at a given node, asset price paths up to the node may be represented by one appropriate path. For example, if the backward induction is considered at the star node in Figure 3, all the possible paths after the star in the shaded region are considered during each reset period but only one appropriate path before the star will be considered. Various types of paths have been tested in various problems and the path right in the middle between Upper and Lower, called Middle in the Figure 3 has been selected as this appropriate path due to the accuracy and also simplicity of coding. In fact, it shows the accuracy comparable to that from the standard binomial method [9]. See the Appendix for the various types we considered and the results from several examples.

The backward induction procedure can be summarized as follows. Suppose that the backward induction is performed at the star node in Figure 3 and the star node corresponds to $t = t^*$. The asset price may increase by a factor of u during the next time-step Δt or decrease by a factor of d in the binomial method, where Δt denotes the time-step size of the binomial tree. Note that ΔT above represents the reset interval size of the cliquet option. If $\{S_0, S_1, \ldots, S_M\}$ denotes the asset prices following the path Middle, there may not be option prices at $t = t^* + \Delta t$ in the binomial tree corresponding to $\{S_0, S_1, \ldots, S_M, uS_M\}$ or $\{S_0, S_1, \ldots, S_M, dS_M\}$. Thus, the option prices corresponding to these paths are obtained by the interpolation, then the ordinary backward averaging procedure is performed on these interpolated values. This procedure is repeated up to the point $t = 0$ to derive the option price at present. When the early exercise is allowed as in the American option, the averaged option price is compared with the payoff (3.2) to check whether it is optimal to exercise or not.

Note that there is a modeling error when all the asset paths up to a node in the tree are represented by a single path, but this error seems to be quite negligible based on the results from numerical experiments in Section 4. The accuracy of the proposed binomial method is as accurate as those from the standard binomial method [9] or recent methods [8] and this computationally shows that the proposed algorithm is reliable and works well.

4. Numerical Experiments

The proposed binomial algorithm has been compared with the recent singular point method by Gaudenzi and Zanette in [8] and other well-known methods therein for the European cliquet options and with the method by Kjaer in [12] for the American cliquet options. Figures 4 and 5 and Tables 1 and 2 show that the proposed scheme is as accurate as the standard binomial method or recent singular point method, and eventually outperforms these because the standard binomial method is problematic in the computational point of view when the number of time steps is large and the singular point method requires additional pre- and post-processing work. Figure 6 and Table 3 show that the proposed method can be extended to cliquet options with early exercise.

4.1. **European option.** Figure 4 compares the results of the European cliquet option using the parameters from Gaudenzi and Zanette [8] with $F_{\text{loc}} = 0, C_{\text{loc}} = 0.08, F_{\text{glob}} = 0.16, C_{\text{glob}} = \infty, N_{\text{obs}} = 5, T = 5, r = 0.03, \sigma = 0.2$. The result from the proposed algorithm with path

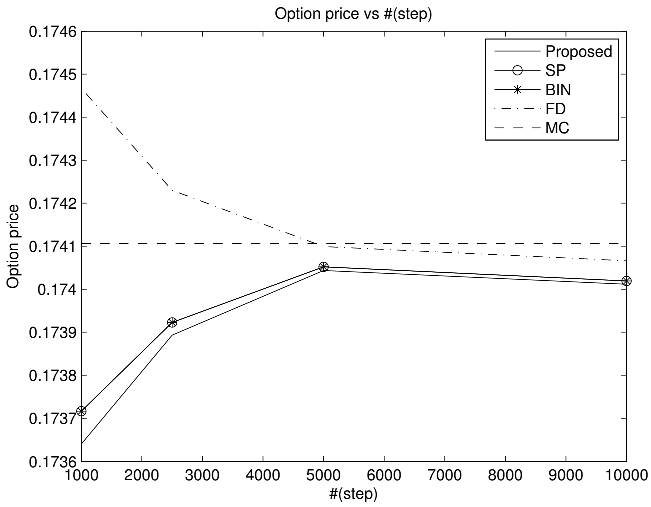

FIGURE 4. The first example of European cliquet option from Gaudenzi and Zanette [8].

Middle is labeled Proposed. SP and BIN represent result from the singular point method introduced in Gaudenzi and Zanette [8] and the result from the standard binomial method [9]. FD denotes the value from the finite difference algorithm with similarity reduction technique provided by Windcliff et al. [5]. MC corresponds to the result from the Monte Carlo method with 10^6 number of realizations. Since there is no known exact solution for this cliquet option, the value from the standard binomial method with 10^4 time steps is assumed to be exact and results from various methods are compared to this BIN value. The figure shows that our proposed algorithm is as accurate as SP by Gaudenzi and Zanette or the standard binomial method, and performs better than the finite difference method or the Monte Carlo method.

TABLE 1. Comparison of European cliquet option prices for the first example from Gaudenzi and Zanette [8].

N	Proposed	SP	BIN	FD	MC
200	0.173639948	0.173716366	0.173716366	0.174467	0.174106
500	0.173893240	0.173922671	0.173922597	0.174230	0.174106
1000	0.174043503	0.174051983	0.174051949	0.174099	0.174106
2000	0.174011443	0.174019094	0.174018925	0.174066	0.174106

Table 1 compares European cliquet option prices for several numbers of time steps of the tree, N. The proposed scheme gives the accuracy comparable to the SP method by Gaudenzi and Zanette [8].

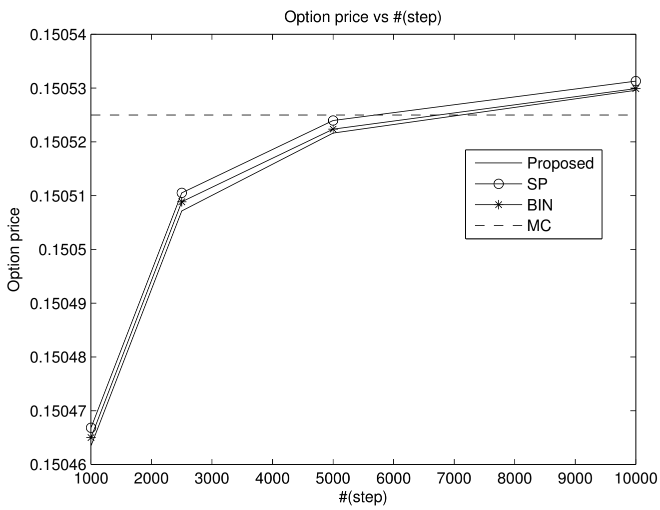

FIGURE 5. The second example of European cliquet option from Gaudenzi and Zanette [8].

Figure 5 compares the results of the European cliquet option using the parameters from Gaudenzi and Zanette [8] with $F_{\text{loc}} = 0, C_{\text{loc}} = 0.08, F_{\text{glob}} = 0.16, C_{\text{glob}} = \infty, N_{\text{obs}} = 5, T = 5, r = 0.03, \sigma = 0.02$. The exact solution for this cliquet option is not known as above and results from various methods are compared to the BIN value from the standard binomial method with 10^4 time steps. Similarly to the case of Figure 4, results from the proposed method and the singular point method are very close to the result from the standard binomial method and show better result than the Monte Carlo method when the number of time steps is sufficiently large. In this example, the proposed method performs slightly better than the singular point method.

TABLE 2. Comparison of European cliquet option prices for the second example from Gaudenzi and Zanette [8].

N	Proposed	SP	BIN	MC
200	0.150463462	0.150466827	0.150465004	0.150525
500	0.150507165	0.150510524	0.150508871	0.150525
1000	0.150521647	0.150523963	0.150522368	0.150525
2000	0.150529597	0.150531287	0.150529922	0.150525

Table 2 also compares European cliquet option prices, which shows again that the proposed scheme is reliable and performs moderately better than the singular point method.

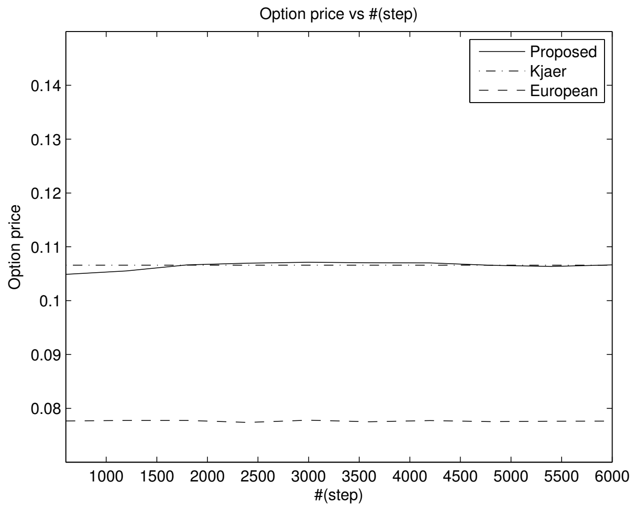

FIGURE 6. American cliquet option from Kjaer [12].

4.2. **American option.** Figure 6 compares the results for the American cliquet option using the parameters from Kjaer [12], $F_{\mathrm{loc}} = -0.1, C_{\mathrm{loc}} = 0.1, F_{glob} = 0, C_{glob} = \infty, N_{obs} = 6, T = 3, r = 0.05, \sigma = 0.3$. Proposed scheme gives very good accuracy, whose value converges to the value from Kjaer [12]. European cliquet option price is shown as a reference.

TABLE 3. Comparison of American cliquet option prices for the example from Kjaer [12].

Proposed					Kjaer
$N = 1200$	$N = 2400$	$N = 3600$	$N = 4800$	$N = 6000$	
0.105527653	0.106950983	0.107042909	0.106573191	0.106639224	0.1066

Table 3 compares American cliquet option prices, which illustrates that the proposed scheme is accurate even when the early exercise is allowed as in the American option.

5. CONCLUSIONS

A binomial method is proposed for pricing European and American cliquet options in a Black-Scholes framework. Recent methods for the cliquet option are of complicated algorithm or require supplementary pre- or post-processing computations. For instance, the singular point method by Gaudenzi and Zanette requires pre-processing calculations in order to find singular points and also requires elimination procedures. But, the proposed algorithm can be easily implemented while eliminating supplementary computations and maintaining the accuracy. In addition, it can be easily extended to American or Bermudan cliquet options.

6. APPENDIX

Given a node P of the binomial tree for the cliquet option, we considered and tested several paths between the initial node at time $t = 0$ and the node P. See Figure 7. First, we considered

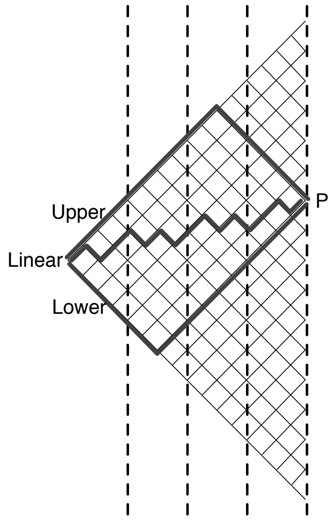

FIGURE 7. Various paths we tested.

two paths called Upper and Lower, which Hull and White [7] initially proposed to use for the path-dependent options. These made the average asset price up to the node P maximum and minimum, respectively, in case of the Asian option with arithmetic average. We then considered the path right in the middle between Upper and Lower, called Middle (shown in the Figure 3). We also considered the path called Linear, which is closest to the linear line from the initial node to the node P as in Figure 7. We then considered the average path among Upper, Linear and Lower and called it Avg. We then tested four examples, two examples from Gaudenzi and Zanette [8], one example from Kjaer [12] and one example with our own selection of parameter values.

Figure 8 shows the result when parameters from Gaudenzi and Zanette [8] are used, $F_{\text{loc}} = 0, C_{\text{loc}} = 0.08, F_{glob} = 0.16, C_{glob} = \infty, N_{\text{obs}} = 5, T = 5, r = 0.03, \sigma = 0.2$. Results from our algorithm with five different paths, Upper, Linear, Lower, Middle, and Avg, are compared with the results from the standard binomial method (BIN) [9], the finite difference algorithm with similarity reduction technique (FD) provided by Windcliff et al. [5] and the Monte Carlo method (MC) with 10^6 number of realizations. Our method with above mentioned paths except Linear show very good convergence to the result from the standard binomial method, and even better result than the finite difference method or the Monte Carlo method. As shown in Section 4, the results from the proposed method are as accurate as the method by Gaudenzi and Zanette [8]. We also tested another European cliquet option example in [8] using $F_{\text{loc}} =$

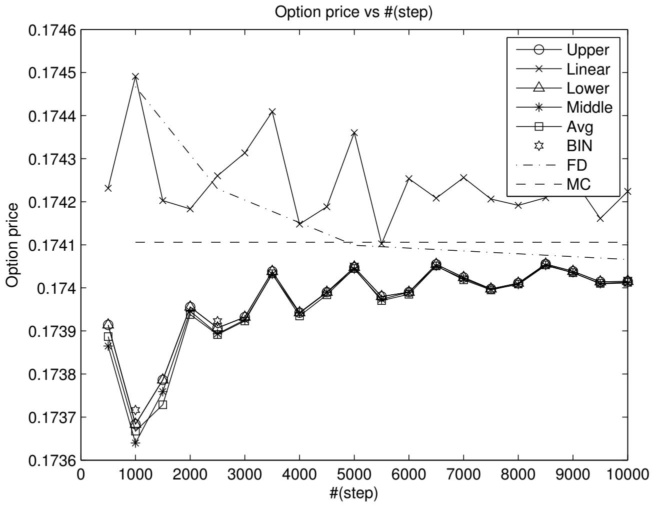

FIGURE 8. Results for the first example of European cliquet option from Gaudenzi and Zanette [8].

$0, C_{\text{loc}} = 0.08, F_{\text{glob}} = 0.16, C_{glob} = \infty, N_{\text{obs}} = 5, T = 5, r = 0.03, \sigma = 0.02$. The results were similar to the first example above and were omitted here.

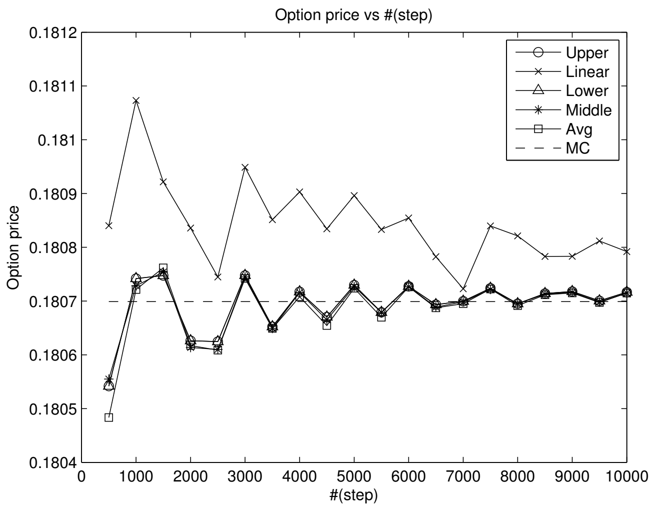

FIGURE 9. Results using parameters of our own

Figure 9 shows the result when $F_{\mathrm{loc}} = 0, C_{\mathrm{loc}} = 0.08, F_{glob} = 0.16, C_{glob} = \infty, N_{\mathrm{obs}} = 5, T = 1, r = 0.05, \sigma = 0.3$ are used. The results from the proposed method are very close to the price 0.18070 from the Monte Carlo method with 10^6 number of realizations except Linear path results in some error and Avg path gives marginal discrepancy.

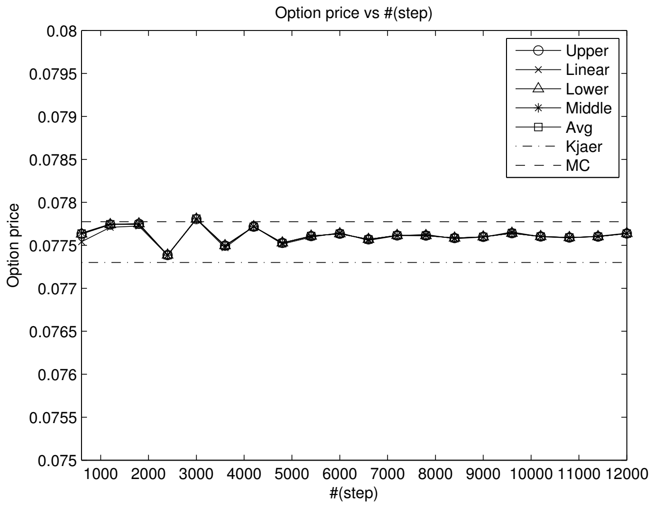

FIGURE 10. Results for the example of European cliquet option from Kjaer [12].

Figure 10 shows the result when parameters from Kjaer [12] are used, $F_{\mathrm{loc}} = -0.1, C_{\mathrm{loc}} = 0.1, F_{glob} = 0, C_{glob} = \infty, N_{\mathrm{obs}} = 6, T = 3, r = 0.05, \sigma = 0.3$. Values from our paths are very close to the value 0.0773 from Kjaer [12], or the value 0.0778 from the Monte Carlo method with 10^6 number of realizations.

Based on the examples above, our proposed method with path Middle performs quite well for the cliquet option and we propose to use it in our study. It is observed that taking any one of three paths, Upper, Lower, or Middle, results in indistinguishable behavior and the final option price. See Figures 8, 9 and 10. The authors are expecting that this results from the special structure of the cliquet option and performing in-depth analysis on this aspect.

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