DOI: 10.5370/JEET.2011.6.2.226

Robust Control of Induction Motor with H_∞ Theory based on Loopshaping

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Abstract - The H_∞ approach, adopted in this paper, is based on loop shaping using a normalized coprime factor combined with a field-oriented control to control induction motor. We develop two loops. The first one, the inner loop, controls the stator current by H∞ controller in order to obtain good performance. The second loop, the outer one, guarantees stability and tracking performance of speed and rotor flux using a proportional integral controller. When the rotor flux cannot be measured, we introduce a flux observer to estimate the rotor flux. Simulation and experimental results are presented to validate the effectiveness and the good performance of this control technique.

Keywords: Field-oriented control, H_∞ controller, Loop-shaping, Normalized coprime factorization, Robust control of induction machine

1. Introduction

The vector control technique has been widely used for high-performance induction motor drive. The progress in system control computing and in power electronics technology, and the availability of high-performance digital signal processors makes field-oriented control (FOC) a practical choice for a wide range of applications. The rotor flux orientation is generally preferred, owing to the high dynamic and steady-state performance obtained. Nevertheless, the induction motor has multivariable, nonlinear differential equations, and not all its states are measurable for feedback control; consequently, it is very difficult to control. Moreover, parametric variations can significantly affect the dynamic performance and the stability of the system [1]-[4].

Recently, many applications of FOC of the induction motor have been developed around H_∞ control theory, with many papers having been published in this field [5]-[6].

In this paper, an H_∞ controller based on loop shaping using a normalized coprime factor approach is designed to ameliorate the performance of an FOC for an induction motor in terms of decoupling between torque and flux components. Simulation and experimental results are provided to demonstrate the effectiveness of the proposed approach.

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Received: October 8, 2009; Accepted: January 5, 2011

2. Field-Oriented Control of Induction Motor

To design the H_∞ controller, it is necessary to perform the model of the open loop process. The induction motor is represented by the following equations expressed in a d-q synchronous reference frame:

$$\begin{aligned}
\dot{i}sd &= -\delta . i_{sd} + \omega_s i_{sq} + \beta . \Phi_{rd} + \mu\,\omega . \Phi_{rq} + \alpha . v_{sd} \\
\dot{i}sq &= -\omega_s i_{sd} - \delta\, i_{sq} - \mu\,\omega\,\Phi_{rd} + \beta . \Phi_{rq} + \alpha . v_{sq} \\
\dot{\Phi}_{rd} &= \frac{M}{T_r} i_{sd} - \frac{1}{T_r}\Phi_{rd} + (\omega_s - \omega)\,\Phi_{rq} \\
\dot{\Phi}_{rq} &= \frac{M}{T_r} i_{sq} - (\omega_s - \omega)\,\Phi_{rd} - \frac{1}{T_r}\Phi_{rq}
\end{aligned} \tag{1}$$

$$\text{With : } \delta = \frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r}, \quad \beta = \frac{1}{M\,T_r}\frac{1-\sigma}{\sigma}$$

$$\mu = \frac{1-\sigma}{M\sigma}, \quad \alpha = \frac{1}{\sigma L_s}, \quad T_r = \frac{L_r}{R_r}, \quad T_s = \frac{L_s}{R_s}$$

$$\sigma = 1 - \frac{M^2}{L_r L_s}$$

where V_s, ϕ_r, and i_s are stator voltage, rotor flux, and stator current, respectively, as expressed by their (d-q) orthogonal components; σ is total leakage coefficient; R_s and R_r are stator and rotor resistance, respectively; L_s and L_r denote stator and rotor inductance, respectively; M is mutual inductance; ω is a mechanical frequency of the electrical rotor speed; and ω_s is the frequency stator electrical speed.

A rotor field orientation in the synchronous reference frame is realized if we let $\phi_{rq} = 0$ and $\phi_{rd} = \phi_r$.

In order to decouple the system (1), we introduce two new input variables, U_{sd} and U_{sq}, as follows:

$$\begin{aligned} u_{sd} &= v_{sd} + a_d \\ u_{sq} &= v_{sq} + a_q \end{aligned} \quad (2)$$

where,

$$\begin{aligned} a_d &= \omega_s \sigma L_s i_{sq} + \frac{M}{T_r L_r}\Phi_r \\ a_q &= -\omega_s \sigma L_s i_{sd} - \frac{M}{L_r}\omega\Phi_r \end{aligned} \quad (3)$$

In the results, the linear system can be written as follows:

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \begin{bmatrix} G(s) & 0 \\ 0 & G(s) \end{bmatrix} \cdot \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} \quad (4)$$

where $G = \dfrac{\alpha}{s+\delta}$

G(s) represents the transfer function for the decoupled motor model.

3. H_∞ Control of Induction Motor

3.1 Normalized Coprime Factorization

A plant model G can be factorized into two stable transfer functions (M, N) so that

$$G(s) = M^{-1}(s)\,N(s) \quad (5)$$

This factorization is called a left coprime factorization of G if there exists stable transfer function matrices (U,V), such that

$$\begin{bmatrix} N(s) & M(s) \end{bmatrix} \cdot \begin{bmatrix} U(s) \\ V(s) \end{bmatrix} = I \quad (6)$$

A left coprime factorization of plant model G is normalized if and only if

$$M(s)\,M^*(s) + N(s)\,N^*(s) = I \quad (7)$$

where $M^*(s) = M^T(-s)$ and $N^*(s) = N^T(-s)$.

3.1.1 H_∞ Synthesis via Normalized Coprime Factorization Approach

Uncertainty in the plant is represented by stable additive perturbations on each factor in the plant coprime factorization. A perturbed plant model G_Δ shown in Fig. 1 is defined as

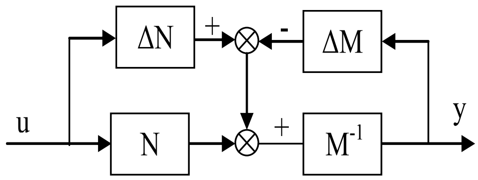

Fig. 1. Left coprime factorization of a perturbed model G_Δ.

$$G_\Delta(s) = (M(s) + \Delta M(s))^{-1}(N(s) + \Delta N(s)) \quad (8)$$

where ΔM and ΔN are stable unknown transfer functions. They represent the uncertainty in the nominal plant model.

The objective of robust control is to stabilize the nominal model G and the set of plants defined by

$$G_\Delta(s) = (M(s) + \Delta M(s))^{-1}(N(s) + \Delta N(s)) \quad (9)$$

With: $\left\| [\Delta M \quad \Delta N] \right\|_\infty < \varepsilon = \dfrac{1}{\gamma}\ ;\ \varepsilon > 0$

where ε is the stability margin. The maximum stability ε_{max} against system uncertainties is given by the lowest achievable value of γ:

$$\gamma_{min} = \varepsilon_{max}^{-1} = (1 + \lambda_{max}(XZ))^{1/2} \quad (10)$$

where λ_{max} denotes the maximum eigenvalue for a minimal state-space realization (A, B, C, D) of G_Δ, and the values X and Z are the unique positive definite solutions to algebraic Riccati equations:

$$\begin{aligned} &(A - BS^{-1}D^TC)^T X + X(A - BS^{-1}D^TC) \\ &- XBS^{-1}B^TX + C^TR^{-1}C = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} &(A - BS^{-1}D^TC)^T Z + Z(A - BS^{-1}D^TC) \\ &- ZC^TR^{-1}CZ + BS^{-1}B^T = 0 \end{aligned} \quad (12)$$

where $R = I + DD^T$, $S = I + D^T D$.

3.2 Loop-shaping Design Procedure

Robust stabilization on its own is not practical because the designer cannot specify the performance. To overcome this, the H_∞ loop shaping procedure is proposed. The desired closed loop performance is specified by shaping the singular values of the nominal plant G using pre and/or post compensators W_1 and W_2. The weighed plant G_Δ is defined as

$$G_\Delta(s) = W_1(s)G(s)W_2(s) \quad (13)$$

The weighting functions W_1 and W_2 should be chosen such that $G_\Delta(s)$ has a sufficiently large gain at low frequency, where good disturbance attenuation is required, and a sufficiently small gain at high frequency, where robust stability is required and does not roll off at a high rate near crossover.

The necessary condition of robust stabilization controller K_∞ for stabilizing the perturbed plant G_Δ is

$$\left\| \begin{bmatrix} I \\ K_\infty(s) \end{bmatrix} (I - G_\Delta(s) K_\infty(s))^{-1} \begin{bmatrix} I & G_\Delta(s) \end{bmatrix} \right\|_\infty \quad (14)$$
$$= \gamma_{min} = \varepsilon_{max}^{-1}$$

The largest value of the robust stability margin ε_{max} is always less than 1, giving good indication of the robustness to a wide class of unstructured perturbations [5]. The feedback controller for the plant G(s) is finally computed by letting $K_{final} = W_1 K_\infty W_2$.

3.3 Weighting Function Choice

The appropriate weighting functions are chosen as follows, taking into account the desirable characteristic of GW_1:

$$W_1 = 10^6 \frac{.s + 100}{s^3 + 900 s^2 + 90s} \quad (15)$$
$$W_2 = 1$$

In Fig. 2, the robust stability margin εmax can be satisfied because the gain margin and the phase margin are respectively equal to $\Delta G = 19.3$ dB and $\Delta\varphi = 47.2^0$. Thus, the closed loop achieves better performance than the open loop according to frequencies greater than the crossover frequency ω_c.

Synthesis of the K_∞ controller is computed from the available program in the robust control Matlab toolbox. According to the above choice, the derived H_∞ controller is computed as:

$$K_\infty = 2.3 \frac{(s+900)(s+188.6)(s+43.23)}{(s+746.1)(s+530.16)(s+97.75)} \quad (16)$$

With : $\varepsilon_{max} = 1/\gamma_{min} = 0.3994$.

The final controller K_{final} = KW is given as mentioned after reduction by:

$$K_{final} = 1.876.10^5. \left(1 + \frac{188.6}{s}\right) \left(\frac{1 + s/43.23}{1 + s/530.16} \right). \quad (17)$$
$$\frac{1}{s^2 + 746.2s + 74.61}$$

The controller appears as a proportional integral controller in cascade with a band pass filter and a second order filter in order to attenuate uncertainty higher frequencies like noise more steeply. The final H_∞ controller is therefore used to regulate both i_{sd} and i_{sq} currents as shown in Fig. 5.

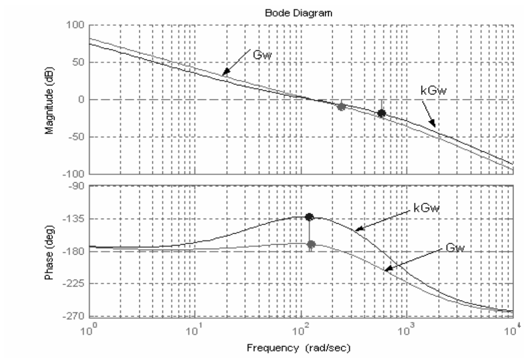

Fig. 2. Shaping transfer function GW and new open loop transfer function kGw.

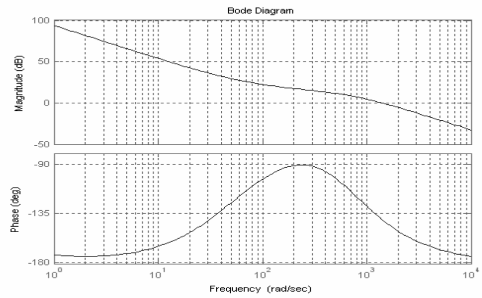

Fig. 3. Frequencies response of controller $KW = W_1 K_\infty W_2$.

4. Flux Observer

Direct FOC of induction machines is one of the effective techniques for high-performance control. However, its proper orientation requires a sufficiently accurate estimate of the rotor flux vector. The proposed observer, given by [10], is obtained by solving the stator and rotor equations separately in the stationary frame. Two common models can be easily derived. The first one, referred as the "current model,", is obtained using the rotor equation. Thus, the flux estimation is achieved as follows:

$$\frac{d\bar{\hat{\Phi}}_r}{dt} = -\frac{1}{T_r}\bar{\hat{\Phi}}_r + \frac{M}{T_r}\bar{i}_s + j\omega\bar{\hat{\Phi}}_r \quad (18)$$

In a similar manner, the stator equation can be manipulated to achieve the "voltage model" for flux estimation using stator voltage $V_{\alpha\beta}$ in the stator reference frame as follows:

$$\frac{d\,\bar{\tilde{\Phi}}_r}{dt} = \frac{L_r}{M}\left(\bar{V}_s - R_s \bar{i}_s - \sigma L_s \frac{d\,\bar{i}_s}{dt} \right) \quad (19)$$

The main drawback of the voltage model is that it requires an open loop integrator, which fails in low-speed operations. The current model fails at high-speed operations due to speed-dependent eigenvalues. We correct this

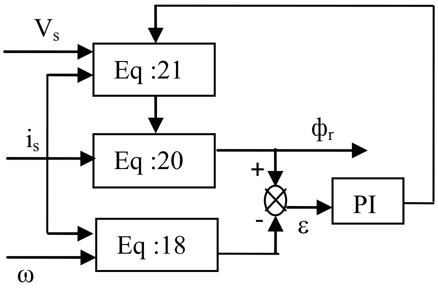

Fig. 4. Rotor flux observer.

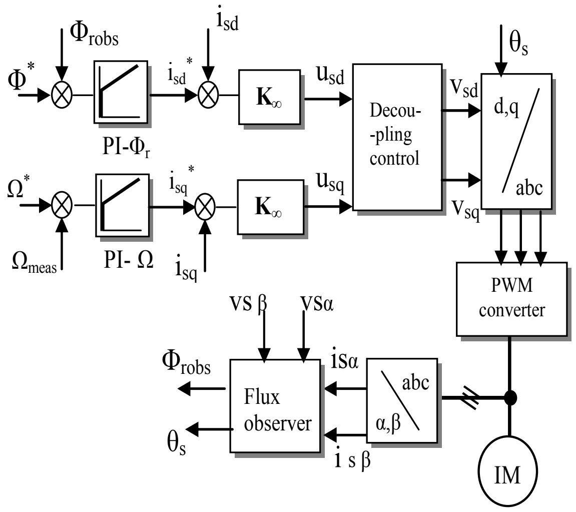

Fig. 5. Flux-oriented control of the induction motor.

problem with a "closed loop Gopinath-style observer" in which the bandwidth controller produces a smooth transition between the current and voltage models. The current model is used for low-velocity operations and the voltage model is used for high-velocity operations. This results in a closed loop estimation equation as follows:

$$\frac{d\overline{\hat{\Phi}}_r}{dt} = \frac{L_r}{M}\left(\frac{d\overline{\hat{\Phi}}_s}{dt} - \sigma L_s \frac{d\bar{i}_s}{dt}\right) \tag{20}$$

where

$$\frac{d\overline{\hat{\Phi}}_s}{dt} = \overline{V}_s - R_s\bar{i}_s + \left(\overline{\hat{\Phi}}_r - \overline{\tilde{\Phi}}_r\right)\left(k_p + \frac{k_i}{s}\right) \tag{21}$$

With:

k_i a proportional gain, $k_i > 0$.

k_p a integrator gain, $k_p > 0$.

The flux observer implementation is shown in Fig. 4.

5. Implementation Results

The proposed controller was tested according to the diagram given in Fig. 5. Direct FOC of the induction motor dealing with high-performance control techniques requires accurate observation of the rotor flux components. Such a problem is overcome by means of a flux observer as in [10], which can be considered in this work. The control scheme,

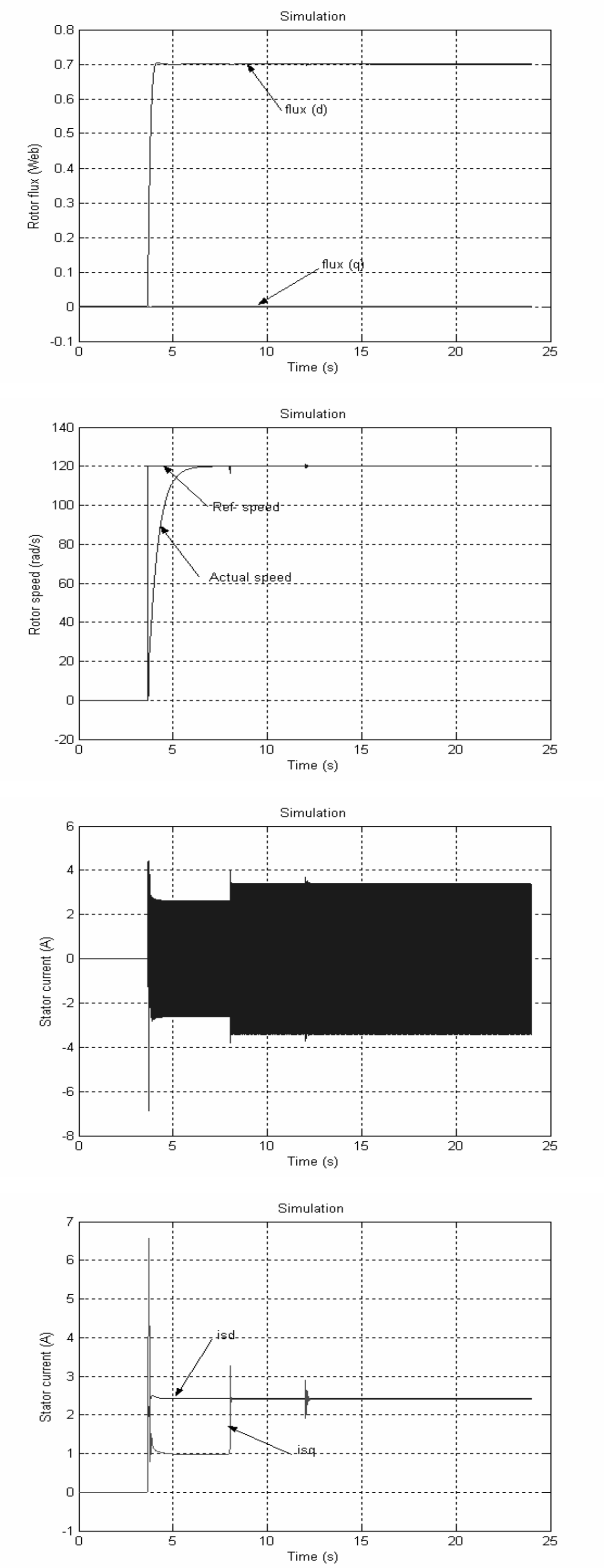

Fig. 6 Dynamic responses with H_∞ controller for load torque Tl=3 Nm at 8 s and 100% variation of Rr at 12 s.

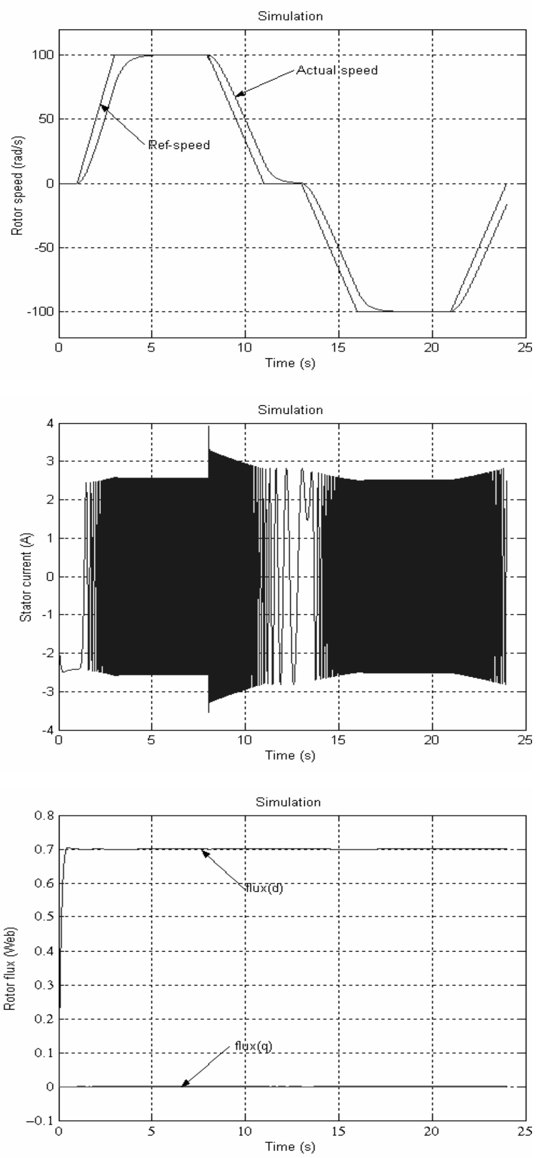

Fig. 7 Dynamic responses for a ramp variation form of the speed under load torque 3 N.m at 8 s and 100% variation of Rr at 12 s.

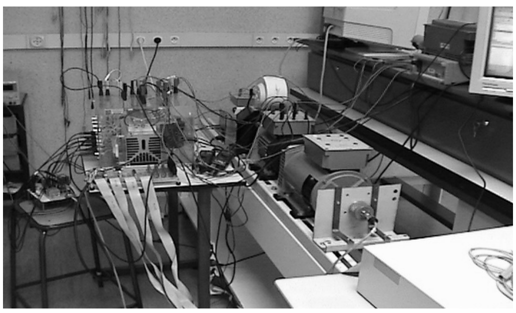

Fig. 8 Experimental system in the L.T.I laboratory, consisting of a 1.5 Kw induction motor, a voltage-source inverter, and a digital signal processor (DSP)

including FOC and H_∞ controller, was implemented on a DSPACE card 1104 with Matlab and Real Time Workshop

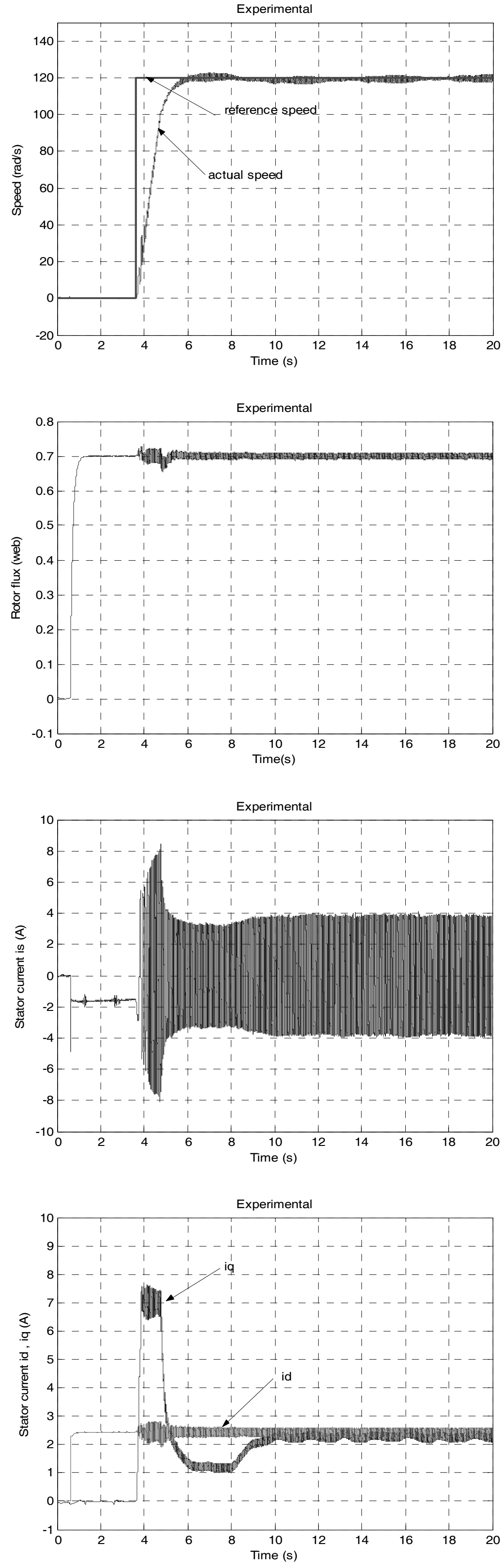

Fig. 9 Experimental dynamic responses of FOC with H∞ controller for load torque Tl=3 Nm at 8 s

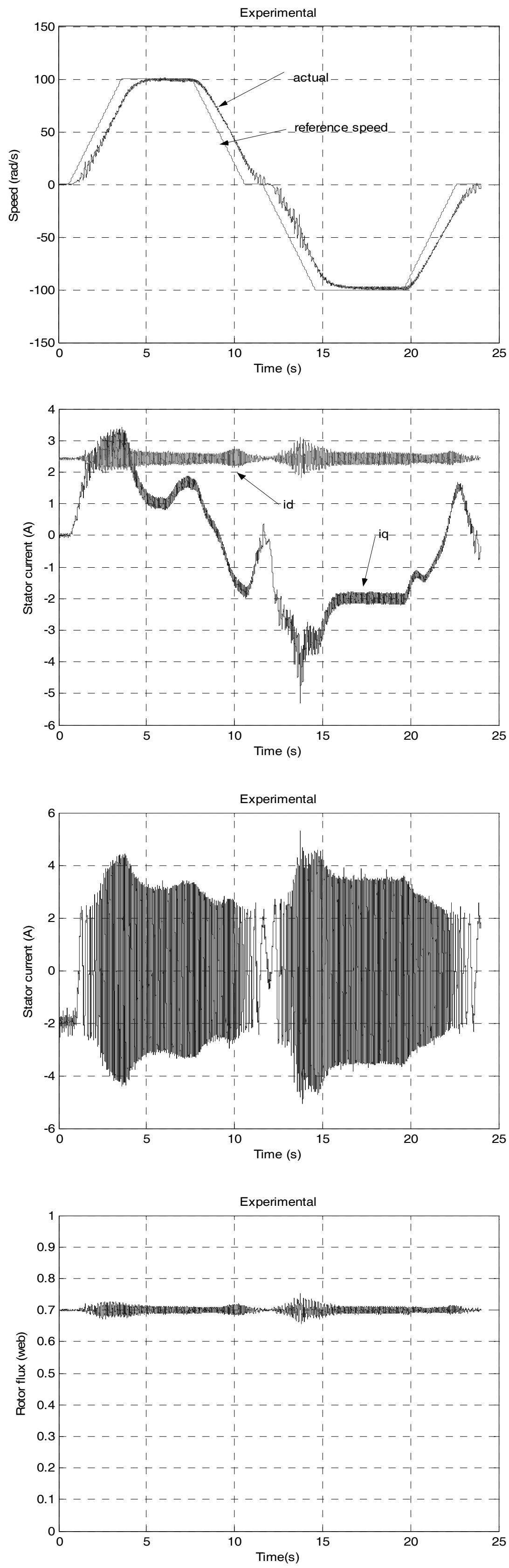

Fig. 10 Experimental responses for a ramp up and a ramp down variation of the speed command under load torque 3 N.m at 8 s

Software. The experimental setup shown in Fig. 8 was based on a 1.5 kW induction motor where the identification of the electrical parameter yielded the following: $R_s = 4.2$ Ω, $R_r = 2.9$ Ω, $L_s = 362$ mH, $L_r = 288$ mH, M = 288 mH, J = 0.0108 kg.m², and F = 0.0175 Nm.s / rad. Furthermore, we used a voltage-source inverter with a switching frequency equal to 10 KH.

The reference trajectory is in a step form varying from 0 to 120 rad/s. The motor is started at no load, and then a load torque equal to 3 N.m is applied at t= 8s. A 100% variation of Rr at t=12s is taken into account. Figs. 6 and 9 show that speed and flux response in the simulation and in the experiment converge to their references. Thus, a perfect orientation of the rotor flux on the d axis is noted. The comparative study between the two sets of figures confirms that the simulation results and the experimental results are similar. Considering the experimental flux response, no considerable variations occur from the reference value due to an unavoidable noise.

A second test was performed to validate the efficiency of the proposed method and to evaluate system performance and stability.

The reference trajectory is now in ramp form as shown in Figs. 7 and 10. A load torque is applied at t= 8s. Results given by the same figures show that the flux is well oriented on the d axis and no appreciable variations occur when the torque is increased. Motor speed also converges to the reference and achieves best speed tracking with a good reject of the load torque perturbation.

6. Conclusion

The main objective of this work is to design an H_∞ controller combined with FOC of an induction motor in order to compensate for the disturbance effect that happens from change of torque, noise, and incertitude of electrical parameter due temperature and saturation. All the simulation and experimental results achieved confirm good performance and good stability of the system.

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