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REALCOMPACTIFICATION OF A PRODUCT SPACE $X \times Y$

ChangIl Kim*

Abstract. Observing that $\upsilon X \times \upsilon Y$ is a Wallman realcompactification of $X \times Y$ if $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$, we show that $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$ if and only if $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$ and $\upsilon X \times \upsilon Y$ is a Wallman compactification of $X \times Y$.

1. Introduction

All spaces in this paper are assumed to be Tychonoff spaces and for any space X, $\beta X(\upsilon X$, resp.) denotes the Stone-Čech compactification (the Hewitt realcompactification, resp.) of X.

Glickberg([3]) showed that for any spaces X and Y, $\beta(X \times Y) = \beta X \times \beta Y$, that is, $X \times Y$ is C^*-embedded in $\beta X \times \beta Y$ if and only if the product space $X \times Y$ is a pseudo-compact space. An important open question in the theory of Hewitt realcompactifications of spaces concerns when the equality $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$(that is, $X \times Y$ is C-embedded in $\upsilon X \times \upsilon Y$) is valid([4]). Comfort([1]) showed that $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$ if and only if $X \times Y$ is C^*-embedded in $\upsilon X \times \upsilon Y$ and that if X and Y have non-measurable cardinal and $X \times Y$ is C^*-embedded in $X \times \beta Y$, then $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$. Moreover, Mcarthur([6]) showed that $X \times Y$ is C^*-embedded in $X \times \beta Y$ if and only if the first projection $\pi_X : X \times Y \longrightarrow X$ is a z-closed map and that if $\pi_X : X \times Y \longrightarrow X$ is a z-closed map, then X is a P-space or Y is a pseudo-compact space.

In this paper, we first show that $\pi_X : X \times Y \longrightarrow X$ is a z-closed map if and only if $X \times Y$ is z-embedded in $X \times \upsilon Y$ and $X \times \upsilon Y$ is C^*-embedded

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in $X \times \beta Y$ and that for spaces X and Y with non-measurable cardinal and $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$, $\upsilon X \times \upsilon Y$ is a Wallman realcompactification of $X \times Y$ associated with $Z(\upsilon X \times \upsilon Y)_{X \times Y}$. In particular, we show that if X is a P-space and Y is a weakly Lindelöff space, then $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$. Using these, we will show that the following are equivalent :

(1) $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$,

(2) $X \times Y$ is z-embedded in $X \times \upsilon Y$ and $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$,

(3) $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$ and $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$, and

(4) $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$ and $\upsilon X \times \upsilon Y$ is a Wallman realcompactification of $X \times Y$.

For the terminology, we refer to [2, 7].

2. z-closed maps

A subset A of a space X is called *a zero-set in* X if there is a real-valued continuous function f on X such that $A = f^{-1}(0)$. A countable intersection of zero-sets in a space X is a zero-set in X and a finite union of zero-sets in X is a zero-set in X.

Definition 2.1. Let X and Y be spaces. Then the first projection $\pi_X : X \times Y \longrightarrow X$ is called *a z-closed map* if for any zero-set A in $X \times Y$, $\pi_X(A)$ is a closed set in X.

A space X is called *a P-space* if every G_δ-set in X, that is, a countable intersection of open sets in X is open in X. Clearly, a space X is a P-space if and only if every zero-set in X is open in X.

Proposition 2.2.

(1) *Let X be a P-space and Y a Lindelöff space. Then $\pi_X : X \times Y \longrightarrow X$ is a z-closed map.*

(2) *Let X be a space and Y a compact space. Then $\pi_X : X \times Y \longrightarrow X$ is a z-closed map.*

Proof. (1) Let A be a zero-set in $X \times Y$. Suppose that there is an x in $X - \pi_X(A)$. Then $(\{x\} \times Y) \cap A = \emptyset$. For each $y \in Y$, there are zero-sets E_y and F_y in X and Y, resp. such that $x \in int_X(E_y)$, $y \in int_Y(F_y)$ and $(E_y \times F_y) \cap A = \emptyset$. Since Y is a Lindelöff space, there is a sequence (y_n) in Y such that $\cup\{F_{y_n} \mid n \in N\} = Y$. Let $Z = \cap\{E_{y_n} \mid n \in N\}$. Then $(Z \times Y) \cap A = \emptyset$ and $Z \cap \pi_X(A) = \emptyset$. Since X is a P-space and Z is a zero-set in X, Z is an open neighborhood of x in X and hence

$x \notin cl_X(\pi_X(A))$. Since $cl_X(\pi_X(A)) \subseteq \pi_X(A)$, $\pi_X(A)$ is closed in X. Hence π_X is a z-closed map.

Similarly, we have (2). □

A pair (Y, j) or simply Y is called *a compactification(realcompactification, resp.) of a space* X if Y is a compact space(a realcompact space, resp.) and $j : X \longrightarrow Y$ is a dense embedding. The ring of real-valued continuous functions on a space X is denoted by $C(X)$ and $C^*(X)$ denotes the subring of bounded functions of $C(X)$. A subspace S of a space X is called *C-embedded(C^*-embedded, resp.) in* X if every function in $C(S)(C^*(S)$, resp.) extends to a function in $C(X)(C^*(X)$, resp.). Every space X has the unique compactification βX(realcompactification υX, resp.) in which X is densely C^*-embedded(C-embedded, resp.)([2]).

Lemma 2.3. *Let X and Y be spaces and $x \in X$. Then*
(1) *$\{x\} \times Y$ is C-embedded in $X \times \upsilon Y$, and*
(2) *$\{x\} \times Y$ is C^*-embedded in $X \times \beta Y$.*

Proof. (1) Let $f \in C(\{x\} \times Y)$. The map $h : Y \longrightarrow \{x\} \times Y$, defined by $h(y) = (x, y)$, is a homeomorphism. Since $f \circ h : Y \longrightarrow R$ is a continuous map, there is a continuous map $g : \upsilon Y \longrightarrow R$ such that $g\mid_Y = f \circ h$. Then the map $k : X \times \upsilon Y \longrightarrow R$, defined by $k(a, y) = g(y)$, is a continuous map and for any $y \in Y$, $k(x, y) = g(y) = f(h(y)) = f(x, y)$. Hence $k\mid_{X\times Y} = f$ and k is the extension of f to $X \times \upsilon Y$.

Similarly, we have (2). □

Lemma 2.4. ([6]) *Let X and Y be spaces. Then $X \times Y$ is C^*-embedded in $X \times \beta Y$ if and only if $\pi_X : X \times Y \longrightarrow X$ is z-closed.*

A subspace S in a space X is called *z-embedded in X* if for any zero-set Z in S, there is a zero-set A in X such that $Z = A \cap S$.

Recall that a C^*-embedded subspace S of a space X is C-embedded in X if and only if for any zero-set Z in X such that $S \cap Z = \emptyset$, S and Z are completely separated in X, that is, there is a real-valued continuous map f on X such that $S \subseteq f^{-1}(0)$ and $Z \subseteq f^{-1}(1)$([2]).

Proposition 2.5. *Let X and Y be spaces. Then $\pi_X : X \times Y \longrightarrow X$ is a z-closed map if and only if $X \times Y$ is z-embedded in $X \times \upsilon Y$ and $X \times \upsilon Y$ is C^*-embedded in $X \times \beta Y$.*

Proof. Suppose that $X \times Y$ is z-embedded in $X \times \upsilon Y$ and $X \times \upsilon Y$ is C^*-embedded in $X \times \beta Y$. Let A be a zero-set in $X \times Y$ and $x \in X - \pi_X(A)$. Then $\pi_X^{-1}(x) \cap A = \emptyset$. Since $X \times Y$ is z-embedded in $X \times \upsilon Y$, there is a zero-set B in $X \times \upsilon Y$ such that $A = B \cap (X \times Y)$. Then

$(\{x\}\times Y)\cap B=\emptyset$. By Lemma 2.3, $\{x\}\times Y$ is C-embedded in $X\times \upsilon Y$ and hence there are disjoint zero-set C, D in $X\times \upsilon Y$ such that $\{x\}\times Y\subseteq C$ and $B\subseteq D$. Since $X\times \upsilon Y$ is C^*-embedded in $X\times\beta Y$, by Urysohn's extension theorem, $cl_{X\times\beta Y}(C)\cap cl_{X\times\beta Y}(D)=\emptyset$. Since $\{x\}\times\beta Y\subseteq cl_{X\times\beta Y}(C)$ and $cl_{X\times\beta Y}(B)\subseteq cl_{X\times\beta Y}(D)$, $(\{x\}\times\beta Y)\cap cl_{X\times\beta Y}(B)=\emptyset$. Since βY is a compact space, there is a open neighborhood U of x in X such that $(U\times\beta Y)\cap cl_{X\times\beta Y}(B)=\emptyset$ and hence $(U\times Y)\cap A=\emptyset$. Since $U\cap\pi_X(A)=\emptyset$, $x\notin cl_X(\pi_X(A))$ and thus $\pi: X\times Y\longrightarrow X$ is a z-closed map.

The converse is trivial. □

Note that $\pi_X : X\times\upsilon Y\longrightarrow X$ is a z-closed map if and only if $X\times\upsilon Y$ is C^*-embedded in $X\times\beta Y$. Hence we have the following :

Corollary 2.6. *Let X and Y be spaces. Then $\pi_X : X\times Y\longrightarrow X$ is a z-closed map if and only if $X\times Y$ is z-embedded in $X\times\upsilon Y$ and $\pi_X : X\times\upsilon Y\longrightarrow X$ is a z-closed map.*

3. Realcompactification of a product space $X\times Y$

The equality $\upsilon(X\times Y)=\upsilon X\times\upsilon Y$ is to be interpreted to mean that $X\times Y$ is C-embedded in $\upsilon X\times\upsilon Y$. For spaces X and Y, $\upsilon(X\times Y)=\upsilon X\times\upsilon Y$ if and only if $X\times Y$ is C^*-embedded in $\upsilon X\times\upsilon Y$([1]). Let X and Y be spaces with non-measurable cardinal. If $X\times Y$ is C^*-embedded in $X\times\beta Y$, then $\upsilon(X\times Y)=\upsilon X\times\upsilon Y$([6]). A space X is called *a pseudo-compact space* if $C(X)=C^*(X)$, equivalently, $\upsilon X=\beta X$. If $\pi_X : X\times Y\longrightarrow X$ is a z-closed map, then X is a P-space or Y is a pseudo-compact space([6]).

By Proposition 2.2, we have the following proposition:

Proposition 3.1. *Let X and Y be spaces with non-measurable cardinal. Suppose that Y is a pseudo-compact space. Then $\upsilon(X\times\upsilon Y)=\upsilon X\times\upsilon Y$.*

A space X is called *a weakly Lindelöff space* if for any open cover $\mathcal{U}$ of X, there is a countable subfamily $\mathcal{V}$ of $\mathcal{U}$ such that $\cup\{V\mid V\in\mathcal{V}\}$ is dense in X. A space with a dense weakly Lindelöff subspace is also a weakly Lindelöff space.

Proposition 3.2. *Let X and Y be spaces with non-measurable cardinal. Suppose that Y has a dense weakly Lindelöff subspace. Then $\upsilon(X\times\upsilon Y)=\upsilon X\times\upsilon Y$.*

Proof. We will show that $\pi_X : X \times \upsilon Y \longrightarrow X$ is a z-closed map. Take any zero-set Z in $X \times \upsilon Y$ and $x \in X - \pi_X(A)$. Then $(\{x\} \times \upsilon Y) \cap Z = \emptyset$. Hence $(\{x\} \times Y) \cap Z = \emptyset$ and by Lemma 2.3, there are disjoint zero-sets A and B in $X \times \upsilon Y$ such that $\{x\} \times \upsilon Y \subseteq int_{X\times\upsilon Y}(A)$ and $Z \subseteq int_{X\times\upsilon Y}(B)$. For any $y \in \upsilon Y$, there is a zero-sets neighborhood $E_y(F_y$, resp.) of $x(y$, resp.) in $X(\upsilon Y$, resp.) such that $E_y \times F_y \subseteq int_{X\times\upsilon Y}(A)$. Since υY is a weakly Lindelöff space, there is a sequence (y_n) in υY such that $\cup\{F_{y_n} \mid n \in N\}$ is dense in υY. Let $G = \cap\{E_{y_n} \mid n \in N\}$ and $H = \cup\{F_{y_n} \mid n \in N\}$. Since $(G \times H) \cap int_{X\times\upsilon Y}(B) = \emptyset$, $cl_{X\times\upsilon Y}(G \times H) \cap int_{X\times\upsilon Y}(B) = (cl_X(G) \times cl_{\upsilon Y}(H)) \cap int_{X\times\upsilon Y}(B) = (G\times\upsilon Y) \cap int_{X\times\upsilon Y}(B) = \emptyset$. Hence $G \cap \pi_X(Z) = \emptyset$. Since X is a P-space and G is a zero-set in X, $x \notin cl_X(\pi_X(Z))$. So $\pi_X : X \times \upsilon Y \longrightarrow X$ is a z-closed map and thus $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$. □

Let X be a space and A a zero-set in X. Then $cl_{\upsilon X}(A)$ is also a zero-set in υX. Hence for any non-empty zero-set Z in υX, $Z \cap X \neq \emptyset$([7]).

Theorem 3.3. *Let X and Y be spaces with non-measurable cardinal. Suppose that $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$. For any non-empty zero-set Z in $\upsilon X \times \upsilon Y$, $Z \cap (X \times Y) \neq \emptyset$.*

Proof. Let Z be a non-empty zero-set in $\upsilon X \times \upsilon Y$. Since $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$, $Z \cap (X \times \upsilon Y) \neq \emptyset$. Pick $(x, y) \in Z \cap (X \times \upsilon Y)$. Suppose that $Z \cap (X \times Y) = \emptyset$. Then $(\{x\} \times Y) \cap (Z \cap (X \times \upsilon Y)) = \emptyset$. Since $\{x\} \times Y$ is C-embedded in $X \times \upsilon Y$ and $Z \cap (X \times \upsilon Y)$ is a zero-set in $X \times \upsilon Y$, $\{x\} \times Y$ and $Z \cap (X \times \upsilon Y)$ are completely separated in $X \times \upsilon Y$([2]). Hence $(\{x\} \times \upsilon Y) \cap (Z \cap (X \times \upsilon Y)) = \emptyset$. Since $(x, y) \in (\{x\} \times \upsilon Y) \cap (Z \cap (X \times \upsilon Y))$, this is a contradiction. Thus $(X \times Y) \cap Z \neq \emptyset$. □

Definition 3.4. ([8]) Let X be a space and $\mathcal{F}$ a family of closed sets in X. Then $\mathcal{F}$ is called *a separating nest generated intersection ring on* X if

(1) for each closed set H in X and $x \in X - H$, there are A, B in $\mathcal{F}$ such that $x \in A$, $H \subseteq B$ and $A \cap B = \emptyset$,
(2) it is closed under finite unions and countable intersections, and
(3) for any $F \in \mathcal{F}$, there are sequences $(F_n), (G_n)$ in $\mathcal{F}$ such that $F = \cap\{F_n \mid n \in N\}$ and for any $n \in N$, $X - H_{n+1} \subseteq F_{n+1} \subseteq X - H_n \subseteq F_n$.

For any space X, let $Z(X)$ denotes the set of zero-sets in X. Then $Z(X)$ is a separating nest generated intersection ring on X.

Let X be space, $\mathcal{F}$ a separating nest generated intersection ring on X and $(\omega(X, \mathcal{F}), \omega_X)$ the Wallman compactification of X associated with

$\mathcal{F}$([8]). Let $v(X, \mathcal{F}) = \{\alpha \mid \alpha$ is an $\mathcal{F}$-ultrafilter with the countable intersection property$\}$ be the subspace of $\omega(X, \mathcal{F})$ and $v_X : X \longrightarrow v(X, \mathcal{F})$ the corestriction of $\omega_X : X \longrightarrow \omega(X, \mathcal{F})$ with respect to $v(X, \mathcal{F})$. Then $(v(X, \mathcal{F}), v_X)$ is a realcompactification of X(called *the Wallman realcompctification of* X *associated with* $\mathcal{F}$)([8]).

For any $\mathcal{F} \subseteq P(X)$ and $A \subseteq X$, let $\mathcal{F}_A = \{F \cap A \mid F \in \mathcal{F}\}$. For a separating nest generated intersection ring $\mathcal{F}$ on X and $A \subseteq X$, $\mathcal{F}_A$ is a separating nest generated intersection ring on A.

LEMMA 3.5. ([5]) *A realcompactification* Y *of a space* X *is a Wallman realcompactification of* X *if and only if for any non-empty zero-set* Z *in* Y, $Z \cap X \neq \emptyset$. *In case,* $Y = v(Y, Z(Y)_X)$.

By Proposition 2.2 and Proposition 3.2, we have the following corollary.

COROLLARY 3.6. *Let* X *and* Y *be spaces with non-measurable cardinal.*
(1) *If* Y *is a pseudo-compact space, then* $\upsilon X \times \upsilon Y$ *is a Wallman realcompactification of* X *associated with* $Z(\upsilon X \times \upsilon Y)_{X\times Y}$.
(2) *If* X *is a* P*-space and* Y *has a dense weakly Lindelöff subspace, then* $\upsilon X \times \upsilon Y$ *is a Wallman realcompactification of* X *associated with* $Z(\upsilon X \times \upsilon Y)_{X\times Y}$.

Note that for any space X, $\upsilon X = v(X, Z(X))$([2]). Using this, we have the following :

THEOREM 3.7. *Let* X *and* Y *be spaces with non-measurable cardinal. Then the following are equivalent :*
(1) $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$,
(2) $X \times Y$ *is* z*-embedded in* $X \times \upsilon Y$ *and* $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$,
(3) $X \times Y$ *is* z*-embedded in* $\upsilon X \times \upsilon Y$ *and* $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$, *and*
(4) $X \times Y$ *is* z*-embedded in* $\upsilon X \times \upsilon Y$ *and* $\upsilon X \times \upsilon Y$ *is a Wallman realcompactification of* $X \times Y$.

Proof. (1) $\Rightarrow$ (2) Since $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$, $X \times Y$ is C-embedded in $\upsilon X \times \upsilon Y$ and $X \times \upsilon Y$ is C-embedded in $\upsilon X \times \upsilon Y$, because $X \times Y \subseteq X \times \upsilon Y \subseteq \upsilon X \times \upsilon Y$. Hence $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$. Since $X \times Y$ is C-embedded in $\upsilon X \times \upsilon Y$, $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$ and clearly, $X \times Y$ is z-embedded in $X \times \upsilon Y$.

(2)$\Rightarrow$ (3) Since $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$, $Z(X \times \upsilon Y) = Z(\upsilon X \times \upsilon Y)_{X\times \upsilon Y}$ and since $X \times Y$ is z-embedded in $X \times \upsilon Y$, $Z(X \times Y) = Z(X \times \upsilon Y)_{X\times Y}$. Hence $Z(X \times Y) = Z(X \times \upsilon Y)_{X\times Y} = (Z(\upsilon X \times$

$\upsilon Y)_{X\times\upsilon Y})_{X\times Y} = Z(\upsilon X \times \upsilon Y)_{X\times Y}$ and hence $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$

(3) $\Rightarrow$ (4) Since $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$, $\upsilon X \times \upsilon Y$ is a Wallman realcompactification of $X \times Y$, by Theorem 3.3.

(4) $\Rightarrow$ (1) Since $\upsilon X \times \upsilon Y$ is a Wallman realcompactification of $X \times Y$, $\upsilon X \times \upsilon Y = \upsilon(X \times Y, Z(\upsilon X \times \upsilon Y)_{X\times Y})$. Since $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$, $Z(X \times Y) = Z(\upsilon X \times \upsilon Y)_{X\times Y}$. Hence $\upsilon X \times \upsilon Y = \upsilon(X \times Y, Z(X \times Y)) = \upsilon(X \times Y)$ □

Every C^*-embedded subspace S of a space X is z-embedded in X. Hence we have the following corollary :

Corollary 3.8. *Let X and Y be spaces with non-measurable cardinal. Suppose that $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$. Then the following are equivalent :*

(1) *$X \times Y$ is z-embedded in $X \times \beta Y$,*
(2) *$X \times Y$ is z-embedded in $X \times \upsilon Y$,*
(3) *$\upsilon(X \times Y) = \upsilon X \times \upsilon Y$, and*
(4) *$X \times Y$ is C^*-embedded in $X \times \beta Y$.*

Let X and Y be spaces with non-measurable cardinal. Then $\upsilon(X \times \upsilon Y) = \upsilon X \times \upsilon Y$ if Y is a pseudo-compact space. Hence we have the following :

Corollary 3.9. *Let X and Y be spaces with non-measurable cardinal. Suppose that X is not a P-space. Then the followig are equivalent :*

(1) *$\upsilon(X \times Y) = \upsilon X \times \upsilon Y$,*
(2) *Y is a psuedo-compact space and $X \times Y$ is z-embedded in $X \times \upsilon Y$, and*
(3) *Y is a psuedo-compact space and $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$.*

By Proposition 3.2, we have the following :

Corollary 3.10. *Let X and Y be spaces with non-measurable cardinal. Suppose that Y is not a pseudo-compact space with a dense weakly Lindelöff space. Then the followig are equivalent :*

(1) *$\upsilon(X \times Y) = \upsilon X \times \upsilon Y$,*
(2) *X is a P-space and $X \times Y$ is z-embedded in $X \times \upsilon Y$, and*
(3) *X is a P-space and $X \times Y$ is z-embedded in $\upsilon X \times \upsilon Y$.*

A space X is called *a P'-space* if every zero-set in X is a regular closed set in X.

Corollary 3.11. *Let X and Y be spaces with non-measurable cardinal. If $X \times Y$ is a P'-space, then $X \times Y$ is z-embedded in $X \times \upsilon Y$ if and only if $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$.*

Let X and Y be spaces such that $\beta(X \times Y) = \beta X \times \beta Y$. Then $X \times Y$ is a pseudo-compact space([3]) and Y is also a pseudo-compact space. Hence we have the following :

Corollary 3.12. *Let X and Y be spaces with non-measurable cardinal. If $\beta(X \times Y) = \beta X \times \beta Y$, then $\upsilon(X \times Y) = \upsilon X \times \upsilon Y$.*

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Department of Mathematics Education
Dankook University
Yongin-si, Gyeonggi-do 448-701, Republic of Korea
E-mail: kci206@hanmail.net