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STABILITY OF IMPULSIVE CELLULAR NEURAL NETWORKS WITH TIME-VARYING DELAYS†

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Abstract. This paper demonstrates that there is a unique exponentially stable equilibrium state of a class of impulsive cellular neural network with delays. The analysis exploits M-matrix theory and generalized comparison principle to derive some easily verifiable sufficient conditions for the global exponential stability of the equilibrium state. The results extend and improve earlier publications. An example with its simulation is given for illustration of theoretical results.

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1. Introduction

Cellular neural networks(CNNs) are widely used in signal and image processing, associative memories, pattern classification, etc. In particular, delay effect on the stability and other dynamical behaviors of CNNs has been extensively studied in the literature, we refer to [1-6] and the references cited therein. However, besides delay effect, impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly, involving such field as medicine and biology, economics, mechanics, electronics and telecommunications, etc. Many interesting results on impulsive effect have gained, e.g., Refs. [7-14]. Therefore, it is necessary to consider both impulsive effect and delay effect on the stability of CNNs.

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In this paper, we consider the following impulsive CNNs with time-varying delays

$$\begin{cases} x_i'(t) = -c_i x_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) + \sum\limits_{j=1}^{n} b_{ij} g_j(x_j\left(t - \tau_{ij}(t)\right)) + I_i,\ t \geq t_0,\ t \neq t_k, \\ \triangle x_i(t_k) = x_i(t_k) - x_i(t_k^-) = I_k(x_i(t_k^-)),\ i = 1, \cdots, n,\ k \in \mathbb{Z}^+, \end{cases} \tag{1}$$

where n corresponds to the number of neurons, $C = \mathrm{diag}\,(c_1, \cdots, c_n) > 0$, $x_i(t)$ is the state of neuron, $f(x) = \big(f_1(x_1), \cdots, f_n(x_n)\big)^T$ and $g(x) = \big(g_1(x_1), \cdots, g_n(x_n)\big)^T$ are the activation functions of the neurons, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are connection matrices, The time-varying delays $\tau_{ij}(t)(i,j = 1, \cdots, n)$ are bounded functions, i.e. $0 \leq \tau_{ij}(t) \leq \tau$, $I = (I_1, \cdots, I_n)^T$ is the constant input vector, $x_i(t_k^-)$ and $x_i(t_k^+)$ denote the left-hand and right-hand limit at t_k, respectively. $\triangle x_i(t_k)$ is the impulse at moments t_k and $t_1 < t_2 < \cdots$ is a strictly increasing sequences such that

$$\lim_{k\to\infty} t_k = \infty,\ \triangle t_k = t_k - t_{k-1} \geq \theta$$

for $k \in \mathbb{Z}^+$, where the value $\theta > 0$ denotes the minimum time of interval between successive impulses. That is of course consistent with the view that a dynamical system tends to become unstable when subjected to sufficiently frequent impulses[7, 8, 15], in this paper some easily verifiable sufficient conditions on the neural parameters and the impulses is found to guarantee the exponential convergence of the neural states towards the unique equilibrium state. The results obtained by applying M-matrix theory and generalized comparison principle(Lemma 2) enhance the earlier work, both with and without impulses and delays.

In system (1), $I_k(\cdot)$ shows impulsive perturbation of the ith neuron at time t_k. If $I_k(x) \equiv 0$ for all $x \in \mathbb{R}^n, i = 1, \cdots, n, k \in \mathbb{Z}^+$, then the model (1) becomes the continuous non-impulsive DCNNs:

$$x_i'(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} b_{ij} g_j(x_j\left(t - \tau_{ij}(t)\right)) + I_i,\ t \geq t_0. \tag{2}$$

2. Preliminaries

In this section, we shall introduce some basic definitions, assumptions and lemmas. Let $PC(I, \mathbb{R}^n) \overset{\triangle}{=} \big\{\psi : I \to \mathbb{R}^n \big| \psi(t^+) = \psi(t)$ for $t \in I$, $\psi(t^-)$ exists for $t \in (t_0, \infty)$, $\psi(t^-) = \psi(t)$ for all but points $t_k \in (t_0, \infty)\big\}$, where $I \subset \mathbb{R}$ is an interval. Especially, let $PC = PC\big([-\tau, 0], \mathbb{R}^n\big)$. For $x \in \mathbb{R}^n, \phi \in PC$, $\|x\|$ denote a vector norm defined by $\|x\| = \max_{1\leq i\leq n}\{|x_i|\}$, $\|\phi\| = \max_{1\leq i\leq n} \sup_{-\tau\leq s\leq 0} |\phi_i(s)|$, and write $\overline{x}(t) = \sup_{t-\tau\leq s\leq t} x(s)$. For matrix $A = (a_{ij})_{n\times n}$, $|A|$ denotes absolute value matrix given by $|A| = (|a_{ij}|)_{n\times n}(i, j = 1, \cdots, n)$.

Definition 2.1. For any given $t_0 \in \mathbb{R}$, $\phi \in PC$, a function $x(t) \in PC([t_0 - \tau, +\infty), \mathbb{R}^n)$ is called a solution of (1) with the initial condition

$$x(t_0 + s) = \phi(s), s \in [-\tau, 0],$$

if $x(t)$ satisfies (1) for $t \geq t_0$. Especially, a point x^* in $\mathbb{R}^n$ is called an equilibrium point of (1), if $x(t) = x^*$ is a solution of (1).

Definition 2.2. The equilibrium point $x^* = (x_1^*, \cdots, x_n^*)^T$ of (1) is said to be globally exponentially stable, if there exist constants $\beta \geq 1$ and $\lambda > 0$ such that

$$\|x(t) - x^*\| \leq \beta \big\|\phi - x^*\| e^{-\lambda(t-t_0)}, t \geq t_0.$$

Throughout this paper, we consider the activation functions of the neurons satisfying the following assumption:

(H1) $f_i(\cdot)$ and $g_i(\cdot)$ are Lipschitz continuous, i.e., there exist constants $M_i > 0,\ N_i > 0$ such that

$$|f_i(x) - f_i(y)| \leq M_i|x - y|,\ |g_i(x) - g_i(y)| \leq N_i|x - y|,$$

for any $x, y \in \mathbb{R}, i = 1, \cdots, n$. Let $M = \operatorname{diag}(M_1, \cdots, M_n)$, $N = \operatorname{diag}(N_1, \cdots, N_n)$.

Definition 2.3 ([16])**.** A real matrix $D = (d_{ij})_{n\times n}$ is said to be an M-matrix, if $d_{ij} \leq 0 (i \neq j)$, and all leading principal minors of D are positive.

Lemma 2.1 ([16, 17])**.** *If $d_{ij} \leq 0 (i \neq j)$, D is an M-matrix if and only if there exists a positive vector $\xi > 0$ such that $A\xi > 0$ or $A^T\xi > 0$.*

Lemma 2.2 ([18])**.** *Let $x_i, y_i \in C\big([t_0 - \tau, +\infty), \mathbb{R}\big)$ satisfy the following conditions:*

i) $x_i(t) < y_i(t), i = 1, \cdots, n,\ t \in [t_0 - \tau, t_0]$.

ii) $x_i'(t) \leq f_i(t; x_1(t), \cdots, x_n(t); \overline{x}_1(t), \cdots, \overline{x}_n(t))$,
$y_i'(t) > f_i(t; y_1(t), \cdots, y_n(t); \overline{y}_1(t), \cdots, \overline{y}_n(t)), t \geq t_0, i = 1, \cdots, n$.

iii) $f_i \in C\big([t_0, +\infty) \times \mathbb{R}^{n\times n}, \mathbb{R}\big)$ *is a quasi-monotonic nondecreasing function with respect to x_i, that is, when $x_i = y_i,\ x_j \leq y_j (j \neq i, j = 1, \cdots, n), \overline{x}_i \leq \overline{y}_i (i = 1, \cdots, n)$, the following inequality:*

$$f_i(t; x_1(t), \cdots, x_n(t); \overline{x}_1(t), \cdots, \overline{x}_n(t)) \leq f_i(t; y_1(t), \cdots, y_n(t); \overline{y}_1(t), \cdots, \overline{y}_n(t))$$

hold for $i = 1, \cdots, n$, then

$$x_i(t) < y_i(t), i = 1, \cdots, n, t \geq t_0. \tag{3}$$

Lemma 2.3 ([5])**.** *Under assumption* (H1)*, the system* (2) *has a unique equilibrium point x^*, if $D \triangleq C - \big(|A|M + |B|N\big)$ is an M-matrix.*

3. Main Results

Let x^* be an equilibrium point of impulsive system (1) and $x(t)$ be any solution of (1). For analytical convergence, let

$$y_i(t) = x_i(t) - x_i^*,\ J_k(y_i(t_k^-)) = y_i(t_k^-) + I_k(y_i(t_k^-) + x_i^*)),$$

$$F_j(y_j(t)) = f_j(y_j(t)+x_j^*) - f_j(x_j^*),\ G_j(y_j(t-\tau_{ij}(t))) = g_j(y_j(t-\tau_{ij}(t))+x_j^*) - g_j(x_j^*)),$$

Then Eq. (1) can be reduced to the following system

$$\begin{cases} y_i'(t) = -c_i y_i(t) + \sum\limits_{j=1}^{n} a_{ij} F_j\left(y_j(t)\right) + \sum\limits_{j=1}^{n} b_{ij} G_j(y_j\left(t - \tau_{ij}(t)\right)),\ t \geq t_0,\ t \neq t_k, \\ y_i(t_k) = J_k(y_i(t_k^-)),\ i = 1, \cdots, n,\ k \in \mathbb{Z}^+. \end{cases} \tag{4}$$

where the functions F_j and G_j inherit the properties of f_j and g_j, namely

$$F_j(0) = G_j(0) = 0,\ |F_j(u)| \leq M_j|u|,\ |G_j(u)| \leq N_j|u|,$$

and the function $J_k(\cdot)$ are assumed to be continuous. Let us consider $J_k(\cdot)$ of the form

$$y_i(t_k) = J_k(y_i(t_k^-)) = d_k y_i(t_k^-),\ k \in \mathbb{Z}^+, \tag{5}$$

where $d_k \neq 0$ for $k \in \mathbb{Z}^+$. It is clear that the stability of the zero solution of system (4) is equivalent to the stability of the equilibrium point x^* of system (1). Therefore, we may mainly discuss the stability of the zero solution of system (4). Other authors have considered impulses of the form (5) in stability investigations of impulsive neural networks. One category [13,14,19-26] is restricted to impulsive jumps with small magnitudes $0 < |d_k| < 1$ independent of the inter-impulse intervals $\triangle t_k = t_k - t_{k-1}$. Another category [11, 12, 27, 28] includes some linkage between the magnitudes $|d_k|$ and intervals $\triangle t_k$, and this is the case here.

Theorem 3.1. *Under the assumptions of Lemma 2.3, impulsive DCNNs* (1) *has a unique equilibrium point, which is globally exponentially stable and exponential convergence rate equals* $\lambda - \eta$, *if the magnitudes* $|d_k|$ *satisfying*

(H2) $0 < |d_k| \leq e^{\eta\theta}$, *where the number* $\theta > 0$ *defines* $\triangle t_k \geq \theta$, $0 < \eta < \lambda$, $\lambda > 0$ *is the solution the following inequality:*

$$\big((\lambda E - C) + |A|M + |B|Ne^{\lambda\tau}\big)\xi < 0,\ \xi \in \mathbb{R}^n \text{ is a positive constant vector.} \tag{6}$$

Proof. Under the assumptions of Lemma 2.3, the non-impulsive system (2) has a unique equilibrium point x^*. In view of $J_k(0) = 0 (i = 1, \cdots, n, k \in \mathbb{Z}^+)$, then x^* is the unique equilibrium point of the impulsive system (1). Due to D being an M-matrix, using Lemma 2.1, there exist $\xi_i > 0 (i = 1, \cdots, n)$, such that

$$-c_i\xi_i + \sum_{j=1}^{n} \xi_j\left(|a_{ij}|M_j + |b_{ij}|N_j\right) < 0, (i = 1, \cdots, n).$$

By using continuity, there exists a sufficiently small constant $\lambda > 0$, such that

$$(\lambda - c_i)\xi_i + \sum_{j=1}^{n} \xi_j \left(|a_{ij}|M_j + |b_{ij}|N_j e^{\lambda\tau}\right) < 0, (i = 1, \cdots, n). \tag{7}$$

that is, the inequality (6) has at least one positive solution λ. In view of $\xi_i > 0 (i = 1, \cdots, n)$, there exists a constant $\mu > 1$ such that $\mu\xi_i > 1 (i = 1, \cdots, n)$. For the initial conditions: $y(t_0 + s) = \phi(s), s \in [-\tau, 0]$, where $\phi \in PC$ and $t_0 \in \mathbb{R}$, it is clear that

$$|y_i(t)| < \mu\xi_i\phi_\varepsilon e^{-\lambda(t-t_0)}, t \in [t_0 - \tau, t_0], \tag{8}$$

where $\phi_\varepsilon = \|\phi\| + \varepsilon$.

First, we prove that

$$|y_i(t)| < \mu\xi_i\phi_\varepsilon e^{-\lambda(t-t_0)}, t \geq t_0, i = 1, \cdots, n, t \neq t_k. \tag{9}$$

Let $w_i(t) = \mu\xi_i\phi_\varepsilon e^{-\lambda(t-t_0)}$. From (8), so $|y_i(t)| < w_i(t)$ for all $t \in [t_0 - \tau, t_0]$, and from (7), we can deduce that

$$\begin{aligned} w_i'(t) =& - \lambda\mu\xi_i\phi_\varepsilon e^{-\lambda(t-t_0)} \\ >& \left[-c_i\xi_i + \sum_{j=1}^{n} \xi_j \left(|a_{ij}|M_j + |b_{ij}|N_j e^{\lambda\tau}\right)\right] \phi_\varepsilon e^{-\lambda(t-t_0)} \\ =& - c_i\xi_i\mu\phi_\varepsilon e^{-\lambda(t-t_0)} + \sum_{j=1}^{n} \xi_j|a_{ij}|M_j\mu\phi_\varepsilon e^{-\lambda(t-t_0)} \\ &+ \sum_{j=1}^{n} \xi_j|b_{ij}|N_j\mu\phi_\varepsilon e^{-\lambda(t-\tau-t_0)} \\ \geq& - c_i w_i(t) + \sum_{j=1}^{n} |a_{ij}|M_j w_j(t) + \sum_{j=1}^{n} |b_{ij}|N_j \overline{w}_j(t), \end{aligned} \tag{10}$$

where $t \geq t_0,\ i = 1, \cdots, n,\ t \neq t_k$. On the other hand, calculating the upper right derivative $D^+|y_i(t)|$ along the solution of system (4), we have

$$\begin{aligned} D^+|y_i(t)| =& \operatorname{sgn}(y_i(t))y_i'(t) \\ =& \operatorname{sgn}(y_i(t)) \left[-c_i y_i(t) + \sum_{j=1}^{n} \xi_j a_{ij} F_j(y_j(t)) + + \sum_{j=1}^{n} b_{ij} G(y_j(t - \tau_{ij}(t)))\right] \\ \leq& - c_i|y_i(t)| + \sum_{j=1}^{n} |a_{ij}|M_j|y_j(t)| + \sum_{j=1}^{n} |b_{ij}|N_j|\overline{y}_j(t)|, \end{aligned} \tag{11}$$

hold for $t \geq t_0,\ i = 1, \cdots, n,\ t \neq t_k$. Obviously, the right function of Eq. (11) is quasi-monotonic nondecreasing with respect to $|y_i|$. Therefore, from (10) and

(11), by Lemma 2.2, Eq. (9) holds. Letting $\varepsilon \to 0$, $\widetilde{\xi} = \mu(\xi_1, \cdots, \xi_n)^T$, then we obtain

$$|y(t)| \le \widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)}, t \ge t_0, i = 1, \cdots, n, t \ne t_k. \tag{12}$$

Without loss of generality, we assume that $t_0 \le t_1$. Obviously, $|y(t)| \le \eta_0\widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)}$ for all $t \in [t_0, t_1)$, where $\eta_0 = 1$. Next, we prove that under assumption (H2) the following inequality

$$|y(t)| \le \eta_0\eta_1 \cdots \eta_{k-1}\widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)} \tag{13}$$

holds for all $t \in [t_{k-1}, t_k)$, where $\eta_k = e^{\eta(t_k - t_{k-1})}, k \in \mathbb{Z}^+$.

Suppose that for all $m = 1, \cdots, k$, the inequalities

$$|y(t)| \le \eta_0\eta_1 \cdots \eta_{m-1}\widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)}, t \in [t_{m-1}, t_m). \tag{14}$$

hold. Noticing that $0 < |d_k| \le e^{\eta\theta} \le \eta_k$, it follows from (5) and (14) that

$$|y(t_k)| \le e^{\eta\theta}|y_i(t_k^-)| \le \eta_0\eta_1 \cdots \eta_{k-1}\eta_k\widetilde{\xi}\|\phi\|e^{-\lambda(t_k-t_0)}. \tag{15}$$

This, together with (14), lead to

$$|y(t)| \le \eta_0\eta_1 \cdots \eta_{k-1}\eta_k\widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)} < \eta_0\eta_1 \cdots \eta_{k-1}\eta_k\widetilde{\xi}\phi_\varepsilon e^{-\lambda(t-t_0)},\ t \in [t_k-\tau, t_k].$$

By using the same method with (9), and letting $\varepsilon \to 0$, we have

$$|y(t)| \le \eta_0\eta_1 \cdots \eta_k\widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)}, t \in [t_k, t_{k+1}). \tag{16}$$

In view of (14) and (16), by induction, the inequality (13) holds. Therefore, we can use (13) to conclude that

$$\begin{aligned}|y(t)| \le& e^{\eta(t_1-t_0)} \cdots e^{\eta(t_{k-1}-t_{k-2})}\widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)} \\ \le& e^{\eta(t-t_0)}\widetilde{\xi}\|\phi\|e^{-\lambda(t-t_0)} \\ =& \widetilde{\xi}\|\phi\|e^{-(\lambda-\eta)(t-t_0)},\ t \in [t_{k-1}, t_k),\ k \in \mathbb{Z}^+.\end{aligned}$$

So,

$$\|x(t) - x^*\| \le \widetilde{\xi}\|\phi - x^*\|e^{-(\lambda-\eta)(t-t_0)},\ t \ge t_0.$$

The proof is complete. □

Remark 3.1. In the paper [12], Mohamad limited $|d_k|$ as $0 < |d_k| < e^{\eta\theta}$, but he only considered the Hopfield-type neural networks with impulses without delays. In this paper, we study the global exponential stability of the equilibrium point on impulsive CNNs with time-varying delays under the assumption of $0 < |d_k| < e^{\eta\theta}$. Here, the constant $\theta > 0$ determines the size of the impulse, $\eta > 0$ defines how large the impulse magnitudes can be, and the number $\lambda-\eta > 0$ determines the convergence rate of the network. Furthermore, the methods we use in this paper are quite different from other references. The following example is given to illustrate the usefulness of the results in this paper.

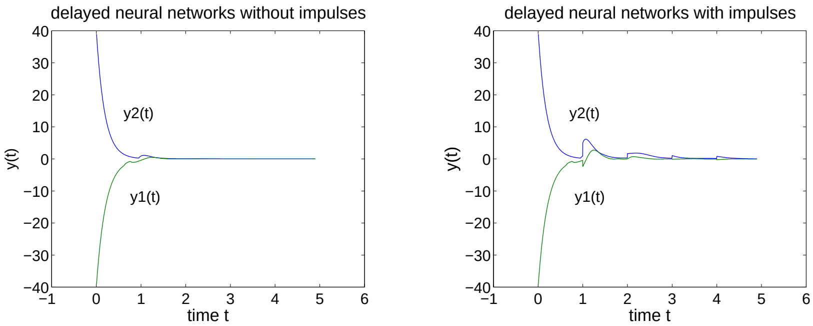

Fig.1. Exponential convergence of neural network (17). The impulsive jumps are characterized by $d_k = 5.55$ at times $t_k = 1, 2, \cdots$ and $\triangle t_k = 1$.

4. Illustrative Examples

Consider the impulsive DCNNs given by

$$\begin{cases} y_1'(t) = -6y_1(t) + 0.5y_1(t) - 0.1y_2(t) + 0.5|y_1(t-\tau_{11}(t))|, \\ y_2'(t) = -5y_2(t) + 0.2y_1(t) + 0.4y_2(t) - 0.4|y_1(t-\tau_{21}(t))| + 0.3|y_1(t-\tau_{22}(t))|, \end{cases} \tag{17}$$

for $t > 0$, $t \neq t_k = 1, 2, \cdots$, where $\tau_{ij}(t) = |\sin((i+j)t)| \leq 1 = \tau$ for $i, j = 1, 2$. The impulsive jumps are characterized by $y_i(t_k^+) = d_k y_i(t_k^-)$ for $t = t_k = 1, 2, \cdots$. The parameters of conditions are as follows

$$C = \begin{pmatrix} 6 & 0 \\ 0 & 5 \end{pmatrix},\ A = \begin{pmatrix} 0.5 & -0.1 \\ 0.2 & 0.4 \end{pmatrix},\ B = \begin{pmatrix} 0.5 & 0 \\ -0.4 & 0.3 \end{pmatrix},\ M = N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$D = C - |A|M - |B|N = \begin{pmatrix} 5 & -0.1 \\ -0.6 & 4.3 \end{pmatrix}.$$

we easily observe D is an M-matrix. Let $\xi = (1, 10)^T$ and $\lambda = 1.716$ which satisfies the inequality $(\lambda E - C + |A|M + |B|Ne^{\lambda\tau})\xi < 0$. Let us pick the number $\eta = 1.715 < \lambda$ in order to illustrate the usefulness of the condition (H2), in which case $\triangle t_k = \theta = 1$. The magnitudes of the impulsive jumps then satisfy $|d_k| \leq e^{\eta\theta} \leq 5.55$. By Theorem 3.1, the equilibrium state $y^* = 0$ of the network (17) is unique and globally exponentially stable. The exponential convergence dynamics of the network (17) with and without impulses are shown in Fig. 1.

Consider again the impulsive network (17) for $t > 0$, $t \neq t_k = 2, 4, \cdots$ subjected to impulsive jumps characterized by $y_i(t_k^+) = d_k y_i(t_k^-)$ for $t_k = 2, 4, \cdots$. The inter-impulse intervals are $\triangle t_k = \theta = 2$. On choosing the same number $\eta = 1.2$, the magnitudes of the impulsive jumps satisfy $|d_k| \leq e^{\eta\theta} \leq 30.93$, and as before the equilibrium state $y^* = 0$ of the network (17) is unique and globally exponentially stable. The exponential convergence dynamics of the network (17) with and without impulses are shown in Fig. 2. The severe restriction $0 < |d_k| < 1$ of the impulse magnitudes considered in [13, 14, 19-26] is merely a special case of the criterion $0 < |d_k| < e^{\eta\theta}$ obtained in this paper.

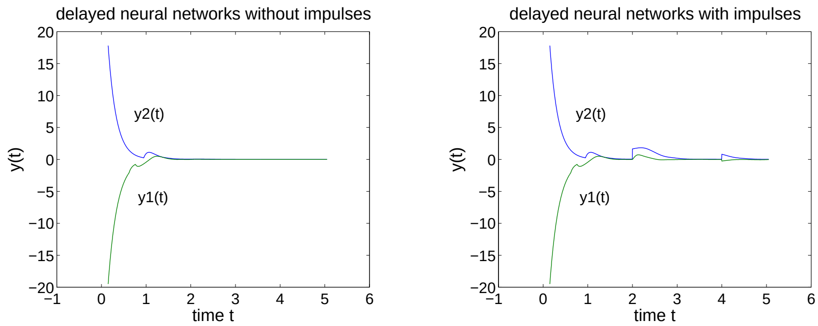

Fig.2. Exponential convergence of neural network (17). The impulsive jumps are characterized by $d_k = 30.93$ at times $t_k = 2, 4, \cdots$ and $\triangle t_k = 2$.

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