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MODELING AND ANALYSIS FOR OPPORTUNISTIC SPECTRUM ACCESS†

YUTAE LEE* AND DONG BO SIM

ABSTRACT. We present an analytic model of an unslotted opportunistic spectrum access (OSA) network and evaluate its performance such as interruption probability, service completion time, and throughput of secondary users. Numerical examples are given to show the performance of secondary users in cognitive networks. The developed modeling and analysis method can be used to evaluate the performance of various OSA networks.

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1. Introduction

The exponential growth in wireless services has recently led to an increasing demand for more bandwidth resources and hence a shortage of them[8]. This expected shortage problem in spectrum supply is reported to be due not to spectrum scarcity, but to the inefficient and static nature of current spectrum management policy[9]. Recent extensive measurements of spectrum efficiency indicate that a large portion of the licensed spectrum lies unused in space and time. As a solution for inefficient spectrum usage, the so-called opportunistic spectrum access (OSA) has been intensively studied, in which secondary users not having a license for spectrum usage is allowed to opportunistically occupy an idle spectrum owned by licensee named primary users in a manner that limits the level of interference perceived by the primary users[3, 6].

The performance of OSA networks has been analyzed mathematically in a few works[1, 2, 4, 5, 6, 10]. Lee[2] presented an analytic model to evaluate the performance of a slotted OSA network. Tang[4] presented an analytic model of

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OSA under unreliable spectrum sensing, assuming that the channel occupancy times of primary and secondary packets are both exponentially distributed.

In this paper, assuming a general channel occupancy times for primary and secondary packets, we develop an analytical model to evaluate the performance of an unslotted OSA network. The developed modeling and analysis can be used to evaluate the performance of various OSA networks.

The organization of the paper is as follows. In Section 2, the network model is described in detail. The interrupted probability is derived in Section 3. The service completion time and the throughput of secondary users are derived in Section 4. Numerical examples are presented in Section 5 and conclusions are given in Section 6.

2. Network model

We consider a primary network with a single channel. A channel is at on-state if it is used by primary users, and at off-state otherwise. The channel occupancy is modelled as an alternating renewal process consisting of on- and off-periods. We assume that the length of on-period T_{on} follows a general distribution with mean $E[T_{on}]$, and the length of off-period T_{off} follows an exponential distribution with rate α. Note that $E[T_{off}] = 1/\alpha$.

We consider N secondary users seeking spectrum opportunities in the channel. We assume that each secondary user has single transceiver and so the secondary user cannot sense the channel during it is transmitting. We do not assume time synchronization between primary users and secondary users.

The OSA scheme considered in this paper is described as follows: Each secondary user has to wait for a random backoff interval before sensing the channel. If a secondary user senses the channel to be occupied, then the secondary user will wait for another backoff interval to sense it again. If a secondary user senses the channel to be unoccupied, the secondary user transmits a packet. When a primary user returns to the channel before a secondary user finishes its transmission, the secondary user stops transmission. In this case, the interrupted packet will be retransmitted if the channel is sensed to be unoccupied after a random backoff interval. Otherwise, the secondary user will wait for another backoff interval to retransmit the interrupted packet.

Let V_i be the random variable representing the backoff interval of the ith secondary user, where the random variables $\{V_i, 1 \leq i \leq N\}$ are i.i.d. with exponential distribution of rate v. The packet transmission time B of secondary users is generally distributed with Laplace Stieljes Transform (LST) $B^*(s)$.

3. Interruption probability

In our analysis, although existing spectrum sensing technologies cannot achieve faultless spectrum detection and undetected errors degrade system performance, we assume perfect spectrum sensing. We also assume that secondary users operate under saturation conditions, where each secondary user always has at least

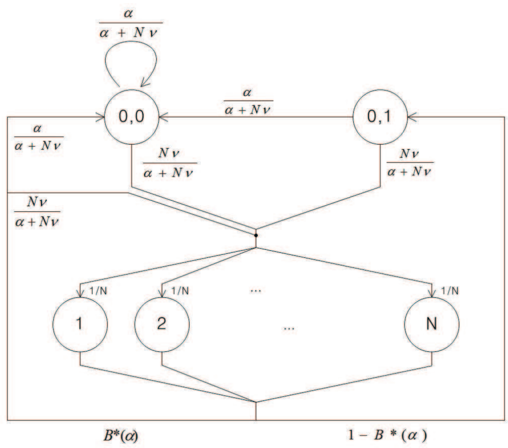

FIGURE 1. state transition diagram.

one packet awaiting transmission. Since the saturation condition means that the system has reached its maximum traffic handling capacity, the saturation performance is a fundamental performance defined as the limit reached by the system performance as the offered load increases.

To analyze the performance of our network system, we first observe the state of the system at the time epochs at which a primary or secondary user accesses the channel. Let $W(t_k)$ indicate the user who accesses the channel at the kth embedded point t_k. The value $W(t_k) = i$, $1 \le i \le N$, means that the ith secondary user accesses the channel, and $W(t_k) = 0$ means that a primary user accesses the channel. If a secondary packet is in transmission when a primary user returns to the channel, then the transmission of the packet is interrupted. Thus, a primary user may access an idle channel or interrupt a transmission of a secondary packet to access the channel. Thus, when $W(t_k) = 0$, we use another variable $S(t_k)$, which indicates the channel state immediately before the primary user accesses the channel. The value $S(t_k) = 0$ means that the channel is idle at t_k, and $S(t_k) = 1$ means that a transmission of a secondary user is interrupted. Then our system can be described by state space $\{(0,0), (0,1), 1, 2, \cdots, N\}$ at each embedded point, where state $(0, j)$, $j = 0, 1$, represents $W(t_k) = 0$ and $S(t_k) = j$, and state i, $1 \le i \le N$, represents $W(t_k) = i$. Considering these embedded points, our system can be modelled as a discrete-time Markov chain with state space $\{(0,0), (0,1), 1, 2, \cdots, N\}$. The state transition diagram of the Markov chain describing the state of the system is presented in Fig. 1.

Let

$$p_{0,j} \equiv \lim_{k\to\infty} P\{W(t_k) = 0, S(t_k) = j\}, \quad (1)$$

$$p_i \equiv \lim_{k\to\infty} P\{W(t_k) = i\} \quad (2)$$

be the stationary probability of the described Markov chain. The stationary probabilities satisfy the following balance equations:

$$p_{0,1} = [1 - B^*(\alpha)] \sum_{i=1}^{N} p_i, \tag{3}$$

$$p_i = \left(p_{0,0} + p_{0,1} + B^*(\alpha) \sum_{i=1}^{N} p_i \right) \frac{Nv}{\alpha + Nv} \frac{1}{N}, \quad 1 \le i \le N. \tag{4}$$

Then, from the balance equations and normalization condition

$$p_{0,0} + p_{0,1} + \sum_{i=1}^{N} p_i = 1, \tag{5}$$

we get

$$p_{0,0} = \frac{\alpha}{\alpha + Nv + Nv\,[1 - B^*(\alpha)]}, \tag{6}$$

$$p_{0,1} = \frac{Nv\,[1 - B^*(\alpha)]}{\alpha + Nv + Nv\,[1 - B^*(\alpha)]}, \tag{7}$$

$$p_i = \frac{v}{\alpha + Nv + Nv\,[1 - B^*(\alpha)]}, \quad 1 \le i \le N. \tag{8}$$

Define P_I to be the interruption probability that a primary user interrupts a transmission of secondary packet when the primary user accesses the channel. The probability P_I is given by

$$P_I = \frac{p_{0,1}}{p_{0,0} + p_{0,1}} = \frac{Nv\,[1 - B^*(\alpha)]}{\alpha + Nv\,[1 - B^*(\alpha)]}. \tag{9}$$

4. Service completion time and throughput

We introduce service completion time of secondary packets that is the time period needed for the packet to be successfully transmitted after it is positioned in the head-of-line (HOL) position of the buffer for the first time. Under saturation condition, each secondary user has another packet after successful packet transmission, and service completion times of secondary packets always start with idle channel. The completion time, which is one of important performance measures when we consider the quality of service of secondary users, depends on which user first accesses the idle channel. Without loss of generality, we consider service completion time for the first secondary user. If we define C as the service completion time for the 1st secondary user, then

$$\begin{aligned} C = {} & 1_{\left\{V_1 < (V_2, \cdots, V_N, T_{off})^-\right\}} \left[V_1 + 1_{\{B \le T_{off}\}} B + 1_{\{B > T_{off}\}} \left(T_{off} + T_{on} + C'\right)\right] \\ & + 1_{\left\{T_{off} < (V_1, \cdots, V_N)^-\right\}} \left[T_{off} + T_{on} + C'\right] \\ & + 1_{\left\{(V_2, \cdots, V_N)^- < (V_1, T_{off})^-\right\}} \Big[(V_2, \cdots, V_N)^- \\ & \qquad + 1_{\{B \le T_{off}\}} \left(B + C'\right) + 1_{\{B > T_{off}\}} \left(T_{off} + T_{on} + C'\right) \Big], \end{aligned} \tag{10}$$

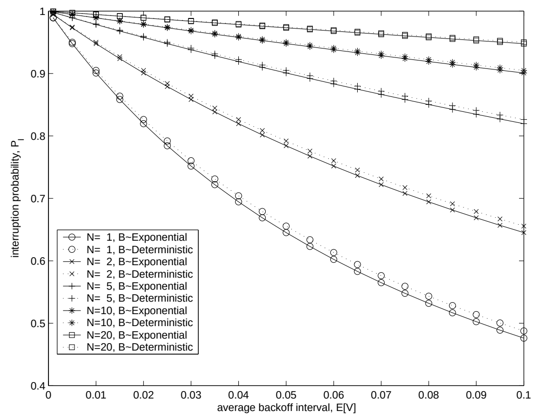

FIGURE 2. interruption probability.

where 1_A is an indicator random variable, C' is a random variable with the same distribution as C, and $(v_1, \cdots, v_n)^-$ denotes the minimum of $v_1, \cdots, v_n$. Then, we get

$$E[C] = \left[\frac{\alpha}{vB^*(\alpha)} + N\frac{1 - B^*(\alpha)}{B^*(\alpha)}\right]\left[\frac{1}{\alpha} + E[T_{on}]\right]. \tag{11}$$

Another important performance measure of OSA network is the saturation throughput of secondary users. The saturation throughput U of secondary users is defined as the long-term proportion of time that secondary users transmit packets without interruption by primary users. The saturation throughput U of secondary users is obtained by

$$U = N\frac{E[B]}{E[C]}. \tag{12}$$

5. Numerical examples

In this section we present numerical examples for the performance measures such as interruption probability, average service completion time, and throughput of secondary users. The on-period T_{on} and the off-period T_{off} are assumed to be an exponential distribution with mean 0.5 and 1.0, respectively, which lead to the channel available probability 2/3. Here, we test the cases B being exponentially distributed and deterministic. We set $E[B] = 0.1$ and $N = 1, 2, 5, 10, 20$. We vary the average backoff period $E[V] = 1/\alpha$ from 0.001 to 0.1. Note that all parameters are given in dimensionless units and can be modified to reflect other situations.

Fig.2 and Fig.3 shows the interruption probability of primary users and the average service completion time of secondary users for various values of N as a function of average backoff interval $E[V]$. We see that an increasing $E[V]$

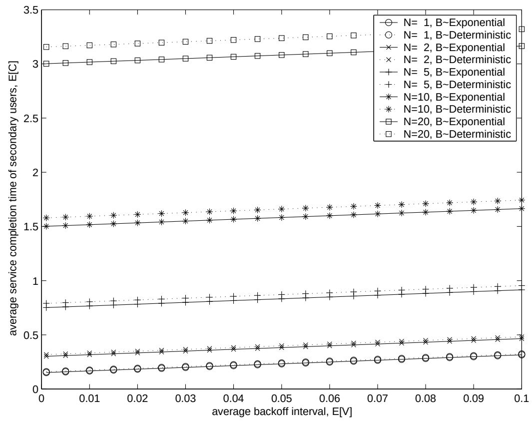

FIGURE 3. average service completion time.

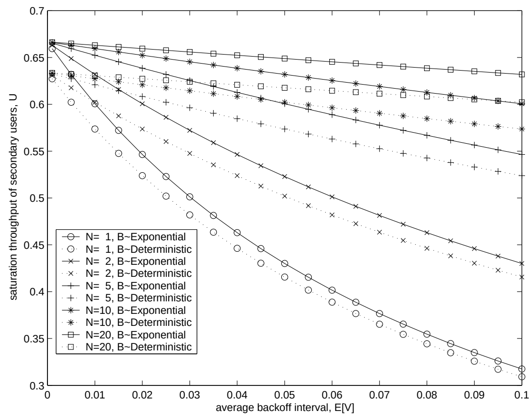

FIGURE 4. saturation throughput.

can significantly reduce the interruption probability at a very small increase of average service completion time. Both the interruption probability and the average service completion time increase as the number N of secondary users increases.

Fig.4 plots the throughput of secondary users as the average backoff interval $E[V]$ increases. It is seen from this case that a decreasing $E[V]$ or an increasing N can increase throughput.

Fig.2, Fig.3, and Fig.4 show that the secondary packets with exponential transmission time achieves better performance than those with deterministic transmission time.

6. Conclusion

In this paper, we have developed an analytic model to evaluate performance of an unslotted OSA network. We have derived performance measures such as interruption probability, service completion time, and saturation throughput of secondary users. Numerical examples are given to show the performance of secondary users in cognitive networks. The developed modeling and analysis can be used to evaluate performance of various OSA schemes.

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Yutae Lee received B.Sc., M.Sc., and Ph.D from KAIST. Since 2001 he has been at Dongeui University. His research interests include queueing theory and performance analysis of telecommunication networks.

Department of Information and Communication Engineering, Dongeui University, Busan 614-714, Korea.
e-mail: ylee@deu.ac.kr

Dong Bo Sim received B.Sc. from Dongeui University. He is currently working toward the M.S. degree in opportunistic spectrum access networks with the Department of Information and Communication Engineering, Dongeui University.

Department of Information and Communication Engineering, Dongeui University, Busan 614-714, Korea.
e-mail: miyeuni84@deu.ac.kr