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# APPROXIMATE SOLUTIONS TO MHD SQUEEZING FLUID FLOW

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Abstract. In this paper, a steady axisymmetric MHD flow of two dimensional incompressible fluids is studied under the influence of a uniform transverse magnetic field. The governing equations are reduced to nonlinear boundary value problem by applying the integribility conditions. Optimal Homotopy Asymptotic Method (OHAM) is applied to obtain solution of reduced fourth order nonlinear boundary value problem. For comparison, the same problem is also solved by Variational Iteration Method (VIM).

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## 1. Introduction

Squeezing flows have many applications in food industry, especially in chemical engineering [1, 2, 3, 4]. Some practical examples of squeezing flow include polymer processing, compression and injection molding. Grimm [5] studied numerically, the thin Newtonian liquids films being squeezed between two plates. Squeezing flow coupled with magnetic field is widely applied to bearing with liquid-metal lubrication [2, 6, 7, 8]. Nonlinear differential equations can be solved analytically by various perturbation techniques. These techniques are very simple in calculating the solution, but the limitations of these methods are based on the assumption of small parameter. The researchers were looking for some new techniques which are independent of the small parameter. An excellent review of these methods is given by He [9] in his paper. In the last decade, the idea of homotopy was combined with perturbation. The fundamental work was done by (Liao [10]) and (JH. He [11, 12]. This paper applies the so-called Optimal Homotopy Asymptotic Method (Marinca et al. [14, 15, 16]. In a series of papers by (Marinca et al. [17, 18]), Islam et al. [19, 20] and (S. Iqbal et al. [21])

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have not only applied this method successfully for some important problems in science and technology, but they have also shown its effectiveness, generalization and reliability. In this paper, we use Optimal Homotopy Asymptotic Method (OHAM) to study the squeezing MHD fluid flow between two infinite planar plates slowly approaching each other. The problem is studied in the influence of inertial terms and magnetic field.

## 2. Basic equations and problem formulation

Consider a squeezing flow of an incompressible Newtonian fluid in the presence of a magnetic field of a constant density $\rho$ and viscosity $\mu$ , squeezed between two large planar parallel plates, separated by a small distance $2H$ and the plates approaching each other with a low constant velocity $V$, as illustrated in figure 1 and the flow can be assumed to quasi-steady [1, 3, 28]

The Navier-Stokes equations [3, 4] governing such flow in the presence of magnetic field, when inertial terms are retained in the flow, are:

$$\nabla V.\mathbf{u} = 0, \tag{1}$$

$$\rho\left[\frac{\partial \mathbf{u}^{\cdot}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u}\right] = \nabla.\mathbf{T} \ + \mathbf{J} \ \times \mathbf{B} + \rho\mathbf{f}, \tag{2}$$

where $u$ is the velocity vector, $\nabla$ denotes the material time derivative, $T$ is the Cauchy stress tensor, $T = -pI + \mu A_1$, where $A_1 = \nabla u + (u)^T$, $J$ is the electric current density, $B$ is the total magnetic field and $B = B_0 + b$, where $B_0$ represents the imposed magnetic field and $b$ denotes the induced magnetic field. In the absence of displacement currents, the modified Ohm's law and Maxwell's equations ([22] and the references therein) are

$$J = \sigma[E + u \times B], \tag{3}$$

$$divB = 0,\ \nabla \times B = \mu_m J,\ curlE = \frac{\partial B}{\partial t}, \tag{4}$$

in which $\sigma$ is the electrical conductivity, $E$ the electric field and $\mu_m$ the magnetic permeability.

The following assumptions are made in order to lead our discussion:

1. The density $\rho$, magnetic permeability $\mu_m$ and electric field conductivity $\sigma$, are assumed to be constant throughout the flow field region.
2. The electrical conductivity $\sigma$ of the fluid considers being finite.
3. Total magnetic field $B$ is perpendicular to the velocity field $V$ and the induced magnetic field $b$ is negligible compared with the applied magnetic field $B_0$ so that the magnetic Reynolds number is small ([22] and the references therein).
4. We assume a situation where no energy is added or extracted from the fluid by the electric field, which implies that there is no electric field present in the fluid flow region.

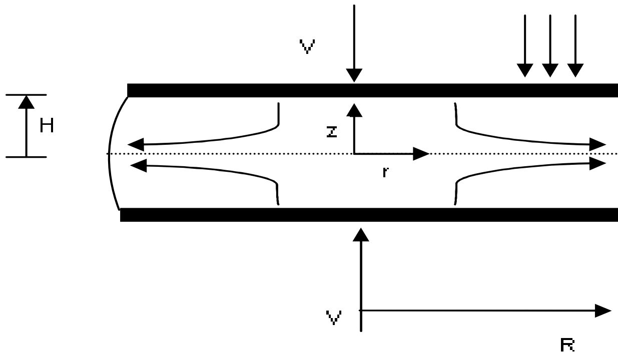


FIGURE 1. A steady squeezing axisymmetric fluid flow between two parallel plates.

Under these assumptions, the magnetohydrodynamic force involved in Eq. (2) can be put into the form,

$$J \times B = -\sigma B_0^2 u. \tag{5}$$

We consider an incompressible Newtonian fluid, squeezed between two large planar, parallel smooth plates which is separated by a small distance $2H$ and moving towards each other with velocity $V$. We assume that the plates are non-conducting and the magnetic field is applied along the z-axis. For small values of the velocity $V$, as shown in the Figure 1, the gap distance $2H$ between the plates changes slowly with time t, so that it can be taken as constant, the flow is steady [2, 28]. An axisymmetric flow in cylindrical coordinates $r, \theta, z$ with z-axis perpendicular to plates and $z = \pm H$ at the plates. Since we have axial symmetry, so $u$ is represented by $u = (u_r(r,z), 0, u_z(r,z))$. When body forces are negligible, Navier-Stokes Eqs. (1-2) in cylindrical coordinates where there is no tangential velocity ($u_\theta = 0$), are:

$$\rho\left(u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2}\right) + \sigma B_0^2 u_r, \tag{6}$$

$$\rho\left(u_z\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial r} + \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2}\right), \tag{7}$$

where $p$ is the pressure, and equation of continuity is:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{\partial u_z}{\partial z} = 0. \tag{8}$$

The boundary conditions require

$$\begin{aligned} &u_r = 0,\ u_z = -V\ at\ z = H \\ &\tfrac{\partial u_r}{\partial z} = 0,\ u_z = 0\ at\ z = 0. \end{aligned} \tag{9}$$

Introducing the axisymmetric Stokes stream function $\psi$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial z},\ u_z = -\frac{1}{r}\frac{\partial\psi}{\partial r} \tag{10}$$

The continuity equation is satisfied using Eq. (11), substituting Eqs. (3-5) and Eq. (11) in Eq. (7-8) we obtain

$$-\frac{\rho}{r^2}\frac{\partial\psi}{\partial r}E^2\psi = -\frac{\partial p}{\partial r} + \frac{\mu}{r}\frac{\partial E^2\psi}{\partial z} - \frac{\sigma B_0^2}{r}\frac{\partial\psi}{\partial z} \tag{11}$$

$$-\frac{\rho}{r^2}\frac{\partial\psi}{\partial z}E^2\psi = -\frac{\partial p}{\partial z} + \frac{\mu}{r}\frac{\partial E^2\psi}{\partial r} \tag{12}$$

Eliminating the pressure from Eqs. (11) and (12) by integribility condition we get the compatibility equation

$$-\rho\left[\frac{\partial\left(\psi, \frac{E^2\psi}{r^2}\right)}{\partial(r,z)}\right] = \frac{\mu}{r}E^2\psi - \frac{\sigma B_0^2}{r}\frac{\partial^2\psi}{\partial z^2} \tag{13}$$

where $E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$. The stream function can be expressed as [1, 3, 33-34]:

$$\psi(r,z) = r^2 F(z), \tag{14}$$

In view of Eq. (14), the compatibility Eq. (13) and the boundary conditions (9) take the form:

$$F^{(iv)}(z) - \frac{\sigma B_0^2}{r}F^{''}(z) + 2\frac{\rho}{\mu}F(z)F^{'''}(z) = 0, \tag{15}$$

subject to

$$\begin{gathered} F(0) = 0,\ F^{''}(0) = 0 \\ F(H) = \tfrac{V}{2},\ F^{'}(H) = 0. \end{gathered} \tag{16}$$

Introducing the following non-dimensional parameters $F^* = F/V/2, z^* = z/H$, $R_e = \rho H/\mu/V,\ m = B_0 H\sqrt{\sigma/\mu}$. For simplicity omitting the $*$ the boundary value problem (14, 15a, 15b) becomes

$$F^{(iv)}(z) - m^2 F^{''}(z) + R_e F(z)F^{'''}(z) = 0, \tag{17}$$

with the boundary conditions

$$\begin{gathered} F(0) = 0,\ F^{''}(0) = 0 \\ F(1) = 1,\ F^{'}(1) = 0. \end{gathered} \tag{18}$$

where $R_e$ is the Reynolds number and $m$ is Hartmann number. This problem has been solved by OHAM and for comparison it has been solved by VIM and numerically by RK-4 using mathematica.

## 3. Basic idea of OHAM

We apply OHAM to the following differential equation:

$$L(F(z)) + g(z) + NF(z) = 0, \; B\big(F, \frac{dF}{dz}\big) = 0 \tag{19}$$

$$F(0) = F^{''}(0) = F^{'}(1) = 0, \; F(1) = 1, \tag{20}$$

where $L$ is a linear operator, $z$ denotes independent variable, $F(z)$ is an unknown function, $g(z)$ is a known function, $N$ is a nonlinear operator and $B$ is a boundary operator.

According to OHAM we construct a homotopy $\phi(z,p) : R \times [0,1] \to R$ which satisfies

$$\begin{aligned} &(1-p)[L(\phi(z,p) + g(z))] = H(p)[L(\phi(z,p) + g(z) + N(\phi(z,p)))], \\ &B\Bigg(\phi(z,p), \frac{\partial \phi(z,p)}{\partial z}\Bigg) = 0 \end{aligned} \tag{21}$$

where $z$ and $p \in [0,1]$ is an embedding parameter, $H(p)$ is a nonzero auxiliary function for $p \neq 0$, $H(0) = 0$ and $\phi(z,p)$ is an unknown function. Obviously, when $p = 0$ and $p = 1$ it holds that $\phi(z,0) = F_0(z)$ and $\phi(z,1) = F(z)$, respectively. Thus, $p$ as varies from 0 to 1, the solution $\phi(z,p)$ approaches from $F_0(z)$ to $F(z)$, where $F_0(z)$ is obtained from Eq (23) for $p = 0$ :

$$L(F_0(z)) + g(z) = 0, \; B\big(F_0, \frac{dF_0}{dz} = 0\big). \tag{22}$$

Next, we choose optimal $H(p)$ identification of the unknown parameter in the trial function in the form

$$H(p) = pC_1 + p^2C_2 + p^3C_3 + ...... \tag{23}$$

where $C_1, C_2, C_3, ...$ are constants.

To get an approximate solution, we expand $\phi(z,p,C_i)$ in Taylor's series about $p$ in the following manner,

$$\phi(z,p,C_i) = F_0(z) + \Sigma_{k=1}^{\infty} F_k(z, C_1, C_2, C_3, ......C_k)p^k. \tag{24}$$

Substituting Eq. (21) into Eq. (18) and equating the coefficient of like powers of $p$, we obtain the following linear equations.

Zeroth order problem is given by Eq. (19) and the first and second order problems are given by Eqs. (22-23) respectively:

$$L(F_1(z)) + g(z) = C_1N_0(F_0(z)), \; B\big(F_1, \frac{dF_1}{dz}\big) = 0. \tag{25}$$

$$\begin{aligned} &L(F_2(z)) - L(F_1(z)) = C_1N_0(F_0(z)) + \\ &C_1\Bigg(L(F_1(z)) + N_1(F_0(z), F_1(z))\Bigg), \; B\big(F_2, \tfrac{dF_2}{dz}\big) = 0. \end{aligned} \tag{26}$$

The general governing equations for $F_k(z)$ are given by:

$$L(F_k(z)) - L(F_{k-1}(z)) = C_1 N_0(F_0(z)) + \Sigma_{i=1}^{k-1} C_i \Big( L(F_{k-i}(z)) + N_{k-i}(F_0(z), F_1(z), ....., F_{k-1}(z)) \Big), \quad k = 2,3,4 \; B\big(F_k, \tfrac{dF_k}{dz}\big) = 0. \tag{27}$$

where $N_m(F_0(z), F_1(z), ....., F_{k-1}(z))$ is the coefficient of $p^m$ in the expansion of $N(\phi(z,p))$ about the embedding parameter

$$N(\phi(z,p,C_i)) = C_1 N_0(F_0(z)) + \Sigma_{m=1}^{\infty} N_m \Big( F_0, F_1, F_2, ....., F_m), \tag{28}$$

It has been observed that the convergence of the series (21) depends upon the auxiliary constants $C_1, C_2, C_3, ...$ If it is convergent at $p = 1$, one has

$$\mathfrak{F}(z,p,C_i) = F_0(z) + \Sigma_{i=1}^{\infty} F_i(z, C_1, C_2, C_3, ......C_i). \tag{29}$$

Substituting Eq. (26) into Eq. (15) it results the following expression for residual:

$$R(z, C_1, ......C_m) = L(\mathfrak{F}(z, C_1, C_2, ......C_m)) + g(z) + N(\mathfrak{F}(z, C_1, C_2, ......C_m)). \tag{30}$$

If $R = 0$, then $\mathfrak{F}$ will be the exact solution. Generally it doesn't happen, especially in nonlinear problems.

There are many methods like Method of Least Squares, Galerkin's Method, Ritz Method, and Collocation Method to find the optimal values of $C_1, C_2, C_3, ....$ We apply the Method [14, 15, 16, 17, 18] as under: If $k_i \in (a,b)$ for $i = 1, 2, ....m$ and substituting $k_i$ into Eq. (27), we obtain the equation

$$R(k_1, C_i) = R(k_2, C_i) = .... = R(k_m, C_i) = 0, \; for \; i = 1, 2, ...m, \tag{31}$$

with these known constants, the approximate solution (of order) is well-determined.

## 4. Application of OHAM

In this section, we apply OHAM to the following nonlinear problem:

$$F^{(iv)}(z) - m^2 F^{''}(z) + R_e F(z) F^{'''}(z) = 0, \tag{32}$$

$$F(0) = F^{''}(0) = F^{'}(1) = 0, \; F(1) = 1, \tag{33}$$

According to Eq. (1) we have:

$$L(F(z)) = F^{(iv)}(z) - m^2 F^{''}(z), \tag{34}$$

$$g(z) = 0, \tag{35}$$

$$N(F(z)) = R_e F(z) F^{'''}(z). \tag{36}$$

Using Eq. (18), we construct a family of equations for the given problem (28-29):

$$(1-p)L(\phi(z,p)) = H(p)[L(\phi(z,p)) + N(\phi(z,p)))], \tag{37}$$

$$B\left( \phi(z,p), \frac{\partial \phi(z,p)}{\partial z} \right) = 0. \tag{38}$$

$$(1-p)\bigg(\phi^{(iv)}(z,p)-m^2\phi^{''}(z,p)\bigg)=H(p)\bigg(\phi^{(iv)}(z,p)-m^2\phi^{''}(z,p) + R_e\phi(z,p)(\phi^{'''}(z,p))\bigg), \tag{39}$$

$$B\bigg(\phi(z,p),\frac{\partial\phi(z,p)}{\partial z}\bigg)=0. \tag{40}$$

Expanding $\phi(z,p)v$ in a Taylor series with respect to $p$, we obtain:

$$\phi(z,p,C_i)=F_0(z)+\Sigma_{k\geq 1}F_k(z,C_i)p^k, i=1,2,..... \tag{41}$$

Using Eqs. (28, 29, 35 and 37) we get the following cases:

**Zeroth-order problem:**

$$F_0^{(iv)}(z)-m^2F_0^{''}(z)=0,\ F_0(0)=F_0^{''}(0)=F_0^{'}(1)=0,\ F_0(1)=1, \tag{42}$$

with solution

$$F_0=\frac{mz\ cosh(m)-sinh(mz)}{m\ cosh(m)-sinh(m)} \tag{43}$$

is the initial guess which satisfies the boundary conditions (16a,16b), in literature this is known as Newtonian solution in the absence of inertial terms in the equations of motion.

**First order problem:**

$$F_1^{(iv)}(z,C_1)-m^2F_1^{''}(z,C_1)-F_0^{(iv)}(z)+m^2F_0^{''}(z) = C_1F_0^{(iv)}(z)-m^2C_1F_1^{''}(z)+R_eC_1F_0(z)F_0^{'''}(z), \tag{44}$$

$$F_1(0)=F_1(1)=F_1^{'}(1)=F_1^{''}(0)=0. \tag{45}$$

The solution of (40) and (41) is

$$\begin{aligned}F(z,C_1)=\ &192mR_ez(cosh(m))^2C_1-128mR_ez^3(cosh(m))^2C_1\\&+2mR_ezcosh(2m)C_1-2mR_ez^3cosh(2m)C_1\\&-64mR_ezcosh(m)cosh(mz)C_1-384R_ezcosh(m)sinh(m)C_1\\&-32m^2R_ezcosh(m)sinh(m)C_1+128mR_ez^3cosh(m)sinh(m)C_1\\&+28m^2R_ez^3cosh(m)sinh(m)C_1-3R_ezsinh(2m)C_1\\&+R_ez^3sinh(2m)C_1+256R_ecosh(m)sinh(mz)C_1\\&+2R_esinh(2mz))C_1)/(64m(m(cosh(m))-(sinh(m)))^2C_1).\end{aligned} \tag{46}$$

**Second-order problem:**

$$\begin{aligned}&F_2^{(iv)}(z,C_1,C_2)-m^2F_2^{''}(z,C_1,C_2)-F_1^{(iv)}(z,C_1)+m^2F_1^{''}(z,C_1\\&=C_1F_1^{(iv)}(z,C_1)-m^2C_1F_1^{''}(z,C_1)+C_2F_0^{(iv)}(z)-m^2C_2F_0^{''}(z)\\&R_eC_2F_0(z)F_0^{'''}(z)+R_eC_1F_1(z,C_1)F_0^{'''}(z)+R_eC_1F_0(z)F_1^{'''}(z,C_1),\end{aligned} \tag{47}$$

$$F_2(0)=F_2(1)=F_2^{'}(1)=F_2^{''}(0)=0. \tag{48}$$

**Note:** On demand we will provide the solution of $F_2$. For the second order approximation, adding $F_0(z)$, $F_1(z, C_1)$, and $F_2(z, C_1, C_2)$, we obtain $\mathfrak{F}$, where

$$\mathfrak{F} = F_0(z) + F_1(z, C_1) + F_2(z, C_1, C_2). \tag{49}$$

Substituting Eq. (45) in Eq. (16a), we obtain the residual as:

$$R_e(z, C_1, C_2) = \mathfrak{F}^{(iv)}(z) - m^2\mathfrak{F}(z) + R_e\mathfrak{F}(z)\mathfrak{F}(z). \tag{50}$$

We obtain the expression for the residual $R$, using Mathematica 7.

**Determination of Constants** $(C_i, i = 1, 2)$

Here for constants $C_1$ and $C_2$, we use the Method [15, 16]:

$$R(k_1, C_1) = R(k_2, C_2) = 0, \tag{51}$$

where $k_i \in (0, 1)$ for $i = 1, 2$ with $C_1 = -0.7939381382939315$ and $C_2 = 0.021809256981632913$. Using these constants in Eq. (45), we get the second order OHAM approximation, whose results are given in Table1, Table 2 and Figures (2-7), at different magnetic fields and Reynolds numbers.

## 5. He's variational iteration method

In this section we apply variational iteration method [23, 24, 25, 26] to the boundary value problem given in (16a-16b). More recently Herisanu and Marinca an improvement of this method was recently proposed in [27], where they suggested an optimal variational iteration algorithm.

In [25, 26], correction functional for Eq. (17) can be written as

$$F_{n+1}(z) = F_n(z) + \int_0^z \lambda(\xi)(LF_n(\xi) + N\tilde{F}_n(\xi) - g(\xi))d\xi, \tag{52}$$

where $L$ and $N$ are linear and nonlinear operators, respectively, $g(\xi)$ is the source homogeneous term and $\lambda$ is a general Lagrange's multiplier which can be identified optimally via the variational theory, and $\tilde{F}_n$ as a restricted variation which means $\delta\tilde{F}_n = 0$. Applying VIM to the given problem (16a-16b), we obtain the correction functional as

$$F_{n+1}(z) = F_n(z) + \int_0^z \lambda(\xi)\left(F_n^{(iv)}(\xi) - m^2\mathfrak{F}_n^{''}(\xi) + R_e\mathfrak{F}_n(\xi)\mathfrak{F}_n^{''}(\xi)\right)d\xi. \tag{53}$$

Taking variation of both sides with respect to independent variable $F_n$ and after some manipulations applying integration by parts four times we obtain

$$\delta F_{n+1} = (1 - \lambda^{'''})\delta F_n + \lambda^{''}\delta F_n^{'} + \lambda^{'}\delta F_n^{''} + \lambda\delta F_n^{'''} + \int_0^z \lambda^{(iv)}\delta F_n d\xi. \tag{54}$$

Applying the conditions $\delta F_{n+1} = 0$, this leads to the following conditions

$$(1 - \lambda^{'''}) \mid_{\xi=z}= 0,\ \lambda^{''} \mid_{\xi=z}= 0,\ \lambda^{'} \mid_{\xi=z}= 0,\ \lambda \mid_{\xi=z}= 0,\ \lambda^{(iv)} \mid_{\xi=z}= 0, \tag{55}$$

which gives

$$\lambda(\xi) = \frac{(\xi - z)^3}{3!} \tag{56}$$

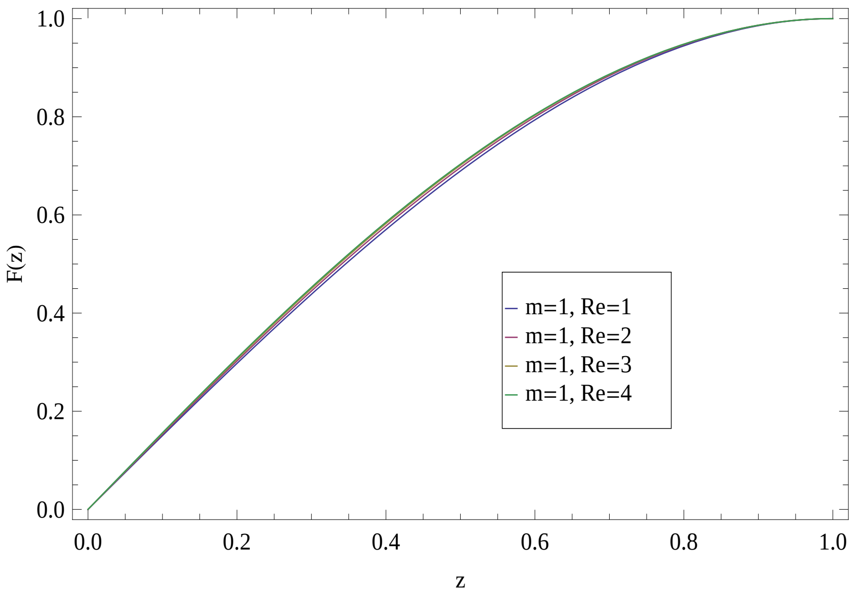


FIGURE 2. Comparison of squeezing flow for a fixed magnetic field effect $m = 1$ and increasing Reynolds numbers $R_e = 1, 2, 3, 4$.

Substituting this value of Lagrange's multiplier into the functional (48), we obtain the iteration formula

$$F_{n+1}(z) = F_n(z) + \int_0^z \frac{(\xi - z)^3}{3!} \left( F_n^{(iv)}(\xi) - m^2 F_n^{''}(\xi) + R_e F_n(\xi) F_n^{'''}(\xi) \right) d\xi. \quad (57)$$

First we have apply the above functional for . , and then for better accuracy VIM has been combined with Pade approximation. The results of VIM Pade are plotted as shown in Fig. 7, which are still not comparable with second order OHAM. The other major drawback in VIM for higher values of is the occurrence of tedious and lengthy expressions and their calculations.

## 6. Results and discussions

Comparisons of results have been made through different Reynolds numbers $R_e$ and magnetic field effect $m$. Fig. 2 shows comparisons of $F(z)$ for a fixed magnetic field $m = 1$ with increasing Reynolds numbers $R_e = 1, 2, 3, 4$. It is observed that increasing Reynolds number $R_e$, slightly effect the OHAM results obtain for the squeezing flow. Fig. 3 shows comparisons of $F(z)$ for a fixed magnetic field $m$ with increasing Reynolds numbers $R_e = 1, 4, 10$. It is observed that much increase in Reynolds numbers affect the results. This is possible as the flow being viscous which hold good for low Reynolds numbers.

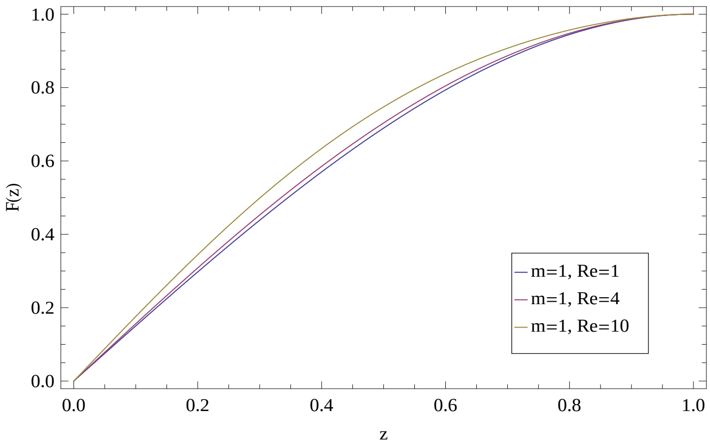


Figure 3. Comparison of squeezing flow for a fixed magnetic field effect $m = 1$ and increasing Reynolds numbers $R_e = 1, 4, 10$.

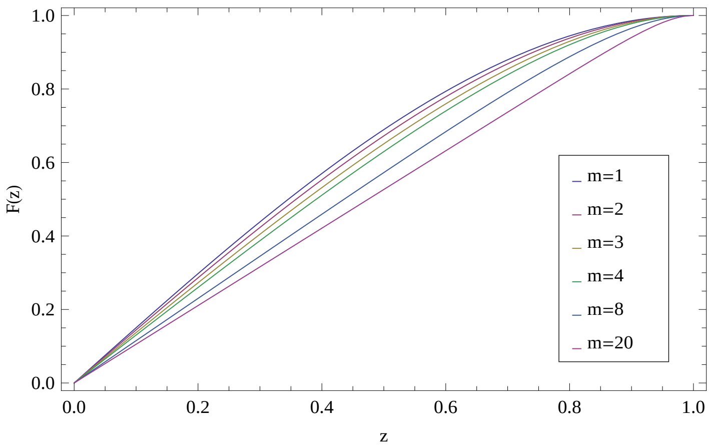


Figure 4. TComparison of squeezing flow for a fixed Reynolds number $R_e = 1$ and increasing magnetic field effect $m = 1, 2, 3, 4, 8, 20$.

Fig. 4 shows comparisons of $F(z)$ for a fixed Reynolds number with increasing magnetic field effect $m = 1, 2, 3, 4, 8, 20$. It is observed that fluid flow is affected with the application of increasing magnetic field effect $m$.

Fig. 5 also shows comparisons of $F(z)$ for a fixed Reynolds number $R_e = 1$ with a sharp increase in the magnetic field effect for $m = 1, 2, 3, 4, 8, 20$ for a clear view. It is observed that fluid flow is much affected with sharp increase in the magnetic field effect $m$.

Table 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.1 | 0.150265 | 0.15293 | 0.154955 | 0.156218 | 0.175911 | 0.139679 |
| 0.2 | 0.297424 | 0.302387 | 0.306107 | 0.308363 | 0.344336 | 0.27576 |
| 0.3 | 0.438387 | 0.444965 | 0.449782 | 0.452557 | 0.498671 | 0.40527 |
| 0.4 | 0.570093 | 0.57739 | 0.582554 | 0.585287 | 0.633941 | 0.526298 |
| 0.5 | 0.68952 | 0.696569 | 0.701324 | 0.703518 | 0.747277 | 0.638084 |
| 0.6 | 0.793695 | 0.799621 | 0.803366 | 0.804726 | 0.838004 | 0.740576 |
| 0.7 | 0.879695 | 0.883885 | 0.886307 | 0.886838 | 0.907244 | 0.83335 |
| 0.8 | 0.944641 | 0.946898 | 0.948052 | 0.948051 | 0.956954 | 0.913787 |
| 0.9 | 0.985687 | 0.98635 | 0.986634 | 0.986529 | 0.988387 | 0.974489 |
| 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

Table 2

| x | m=1 | m=3 | m=4 | m=8 | m=20 |
|---|---|---|---|---|---|
| 0. | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.1 | 0.150265 | 0.13709 | 0.130403 | 0.11507 | 0.105312 |
| 0.2 | 0.297424 | 0.272583 | 0.259843 | 0.230068 | 0.210625 |
| 0.3 | 0.438387 | 0.404759 | 0.387214 | 0.344866 | 0.315938 |
| 0.4 | 0.570093 | 0.531649 | 0.511107 | 0.459205 | 0.421249 |
| 0.5 | 0.68952 | 0.650894 | 0.629618 | 0.572545 | 0.526551 |
| 0.6 | 0.793695 | 0.759591 | 0.740103 | 0.683769 | 0.631824 |
| 0.7 | 0.879695 | 0.854106 | 0.838843 | 0.790543 | 0.736971 |
| 0.8 | 0.944641 | 0.929845 | 0.920578 | 0.887936 | 0.841352 |
| 0.9 | 0.985687 | 0.980966 | 0.977843 | 0.965381 | 0.94035 |
| 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

Fig. 6 shows comparisons of second order OHAM solutions with fifth order VIM Pade. VIM Pade is a kind of modified VIM, in which a Pade approximant is combined with typical VIM, which produced good results at the extended domain, but it is clear from the Fig. 7, that second order OHAM solution is much better than the fifth iterative VIM Pade solution. VIM Pade even fail to satisfy the boundary conditions at and this is not the case with OHAM.

Table 1 and Table 2 are constructed to see the behavior of OHAM results at discrete points of the desired domain by varying Reynolds number $R_e$ and magnetic field effect $m$. A special rhythm of the results and the satisfaction of the boundary conditions give guarantee that OHAM capture the exact behavior of squeezing flow.

In Table 3 and Table 4 OHAM solution is compared with the numerical results calculated by Range-Kutta Method (RK-4), for various Reynolds numbers and different magnetic field effect. It can be seen from the tables (3-4) that OHAM results are matching the numerical results.

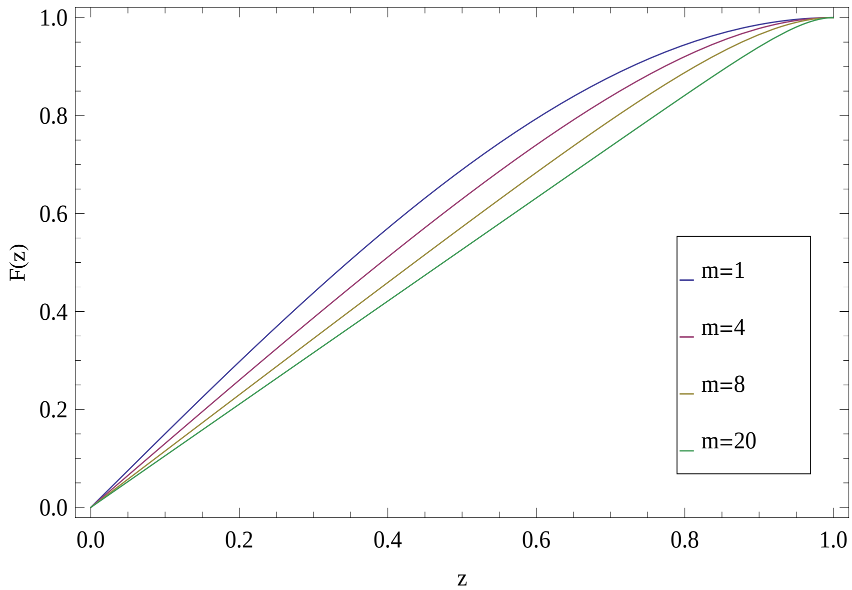


Figure 5. Comparison of squeezing flow for a fixed Reynolds number $R_e = 1$ and increasing magnetic field effect $m = 1, 4, 8, 20$.

Table 3. m=1

| x | Numerical $R_e$=1 | OHAM $R_e$=1 | Numerical $R_e$=4 | OHAM $R_e$=4 | Numerical $R_e$=10 | OHAM $R_e$=10 |
|---|---|---|---|---|---|---|
| 0. | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.1 | 0.150294 | 0.150265 | 0.158104 | 0.156218 | 0.167616 | 0.175911 |
| 0.2 | 0.297481 | 0.297424 | 0.311962 | 0.308363 | 0.329031 | 0.344336 |
| 0.3 | 0.438467 | 0.438387 | 0.457539 | 0.452557 | 0.478907 | 0.498671 |
| 0.4 | 0.570189 | 0.570093 | 0.591193 | 0.585287 | 0.613252 | 0.633941 |
| 0.5 | 0.689624 | 0.68952 | 0.709771 | 0.703518 | 0.729428 | 0.747277 |
| 0.6 | 0.793796 | 0.793695 | 0.810642 | 0.804726 | 0.825843 | 0.838004 |
| 0.7 | 0.879779 | 0.879695 | 0.891666 | 0.886838 | 0.901576 | 0.907244 |
| 0.8 | 0.944696 | 0.944641 | 0.95112 | 0.948051 | 0.901576 | 0.956954 |
| 0.9 | 0.985707 | 0.985687 | 0.987612 | 0.986529 | 0.988978 | 0.988387 |
| 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

## 7. Conclusion

In this paper, a squeezed axisymmetric fluid flow between two parallel plates under a transverse magnetic field is analyzed. We applied a new powerful analytic technique, OHAM for the reduced nonlinear boundary value problem. For

TABLE 4. $R_e$=1

| x | Numerical m=3 | OHAM m=3 | Numerical m=8 | OHAM m=8 | Numerical m=20 | OHAM m=20 |
|---|---|---|---|---|---|---|
| 0. | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.1 | 0.137044 | 0.13709 | 0.114976 | 0.11507 | 0.105391 | 0.105312 |
| 0.2 | 0.272494 | 0.272583 | 0.229882 | 0.230068 | 0.210782 | 0.210625 |
| 0.3 | 0.404637 | 0.404759 | 0.344604 | 0.344866 | 0.316173 | 0.315938 |
| 0.4 | 0.531508 | 0.531649 | 0.458904 | 0.459205 | 0.421563 | 0.421249 |
| 0.5 | 0.650756 | 0.650894 | 0.572276 | 0.572545 | 0.526952 | 0.526551 |
| 0.6 | 0.759478 | 0.759591 | 0.683628 | 0.683769 | 0.632324 | 0.631824 |
| 0.7 | 0.854035 | 0.854106 | 0.790607 | 0.790543 | 0.737586 | 0.736971 |
| 0.8 | 0.929817 | 0.929845 | 0.888173 | 0.887936 | 0.842051 | 0.841352 |
| 0.9 | 0.980963 | 0.980966 | 0.965578 | 0.965381 | 0.940861 | 0.94035 |
| 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

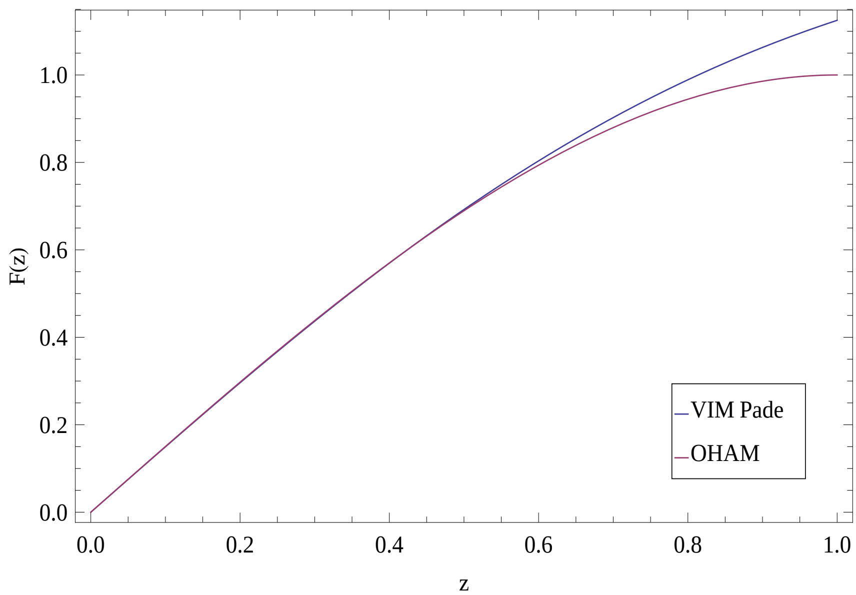


FIGURE 6. : Comparison between OHAM and VIM with Pade approximation of squeezing flow for a fixed Reynolds number $R_e = 1$ and magnetic field effect $m = 1$.

comparison, the same problem is also solved by VIM and numerical method (RK-4). Furthermore, this method provides us a convenient way to control the convergence and we can easily adjust the desired convergence regions. This approach is simple in applicability, as it does not require discretization like other

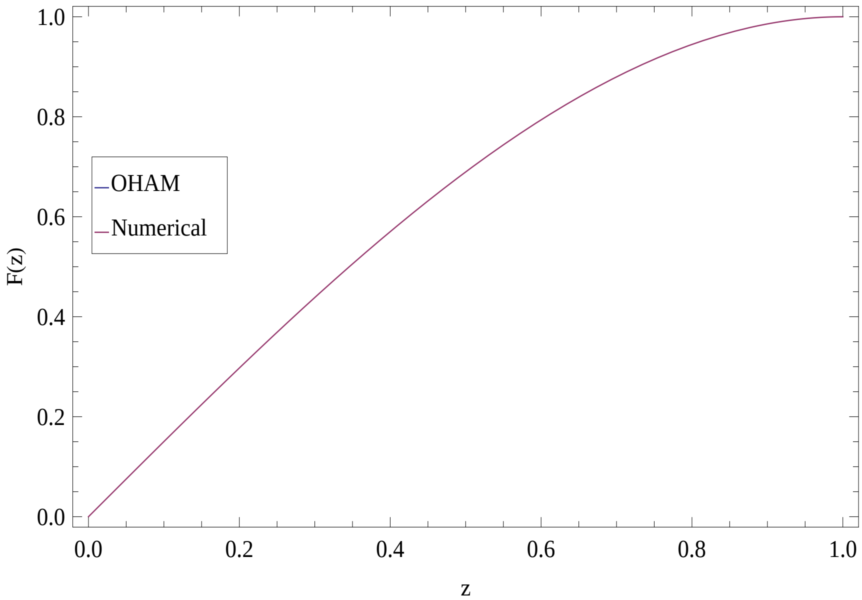


Figure 7. Comparison between OHAM and Numerical solution of squeezing flow for a fixed Reynolds number $R_e = 1$ and magnetic field effect $m = 1$.

numerical and approximate methods. Moreover, this technique is fast converging to the exact solution and requires less computational work. This confirms our belief that the efficiency of the OHAM gives it much wider applicability. Mathematica software is used for symbolic derivations of some of the equations.

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