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STABLY PERIODIC SHADOWING AND DOMINATED SPLITTING

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Abstract. Let f be a diffeomorphism of a closed n-dimensional smooth manifold. In this paper, we introduce the notion of C^1-stably periodic shadowing property for a closed f-invariant set, and prove that for a transitive set Λ, if f has the C^1-stably periodic shadowing property on Λ, then Λ admits a dominated splitting.

1. Introduction

It has been a main subject in differentiable dynamical systems during last decades to understand the influence of a robust dynamic property on the behavior of the tangent map of the system. The best known result of this type might be the (now verified) stability conjecture of Palis and Smale, which states that structural stability implies Axiom A and the strong transversality. In the context of the stability conjecture, we study the notion of the dominated splittingwhich is more general than that of uniform hyperbolicity. The notions have turned out to be important in the study of differentialbe dynamical systems([1, 2, 6, 7, 9, 8, 12]).

In this paper we are concerned with the so-called *stably periodic shadowing property*, and show that if a transitive set is C^1-stably periodic shadowable, then it admits a dominated splitting.

Let M be a closed C^∞manifold, and denote by d the distance on M induced from a Riemannian metric $\|\cdot\|$ on the tangent bundle TM.

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Denote by Diff(M) the space of diffeomorphisms of M endowed with the C^1-topology. Let $f \in \text{Diff}(M)$, and let Λ be a closed f-invariant set. Denote by $f|_\Lambda$ the restriction of f to a subset Λ of M.

For δ >0, a sequence of points $\{x_i\}_{i=a}^{b} \subset M$ $(-\infty \le a < b \le \infty)$ is called a *δ-pseudo- orbit* of $f \in \text{Diff}(M)$ if $d(f(x_i), x_{i+1}) < \delta$ for all $a \le i \le b-1$.

We say that $f|_\Lambda$ has the *shadowing property* if for every $\epsilon > 0$, there is a $\delta > 0$ such that for any δ-pseudo-orbits of f contained in Λ can be ϵ-shadowed, but a shadowing point $y \in M$ is not necessarily contained in Λ. Now we introduce the notion of the periodic shadowing property which studied in [11]. We say that f has the *periodic shadowing property* if for every $\epsilon > 0$, there is a $\delta > 0$ such that if $\xi = \{x_i\}_{i\in\mathbb{Z}}$ is a periodic δ-pseudo-orbit, then there exists a periodic point $p \in M$ such that $d(f^i(p), x_{i+1}) < \epsilon$, $i \in \mathbb{Z}$. We say that Λ is *locally maximal* if there is a neighborhood U of Λ such that $\bigcap_{n\in\mathbb{Z}} f^n(U) = \Lambda$. We say that Λ admits a *dominated splitting* if the tangent bundle $T_\Lambda M$ has a continuous Df-invariant splitting $E \oplus F$ and there exist constants $C > 0$ and $0 < \lambda < 1$ such that

$$\|D_x f^n|_{E(x)}\| \cdot \|D_x f^{-n}|_{F(f^n(x))}\| \le C\lambda^n,$$

for all $x \in \Lambda$ and $n \ge 0$.

DEFINITION 1.1. Let $\Lambda \subset M$ be a closed f-invariant set. We say that f has the *C^1-stably periodic shadowing property* on Λ if there exist a C^1-neighborhood $\mathcal{U}(f)$ of f and a compact neighborhood U of Λ such that :

(a) $\Lambda = \Lambda_f(U) = \bigcap_{n\in\mathbb{Z}} f^n(U)$(locally maximal).
(b) for any $g \in \mathcal{U}(f)$, $g|_{\Lambda_g(U)}$ has the periodic shadowing property, where $\Lambda_g(U) = \bigcap_{n\in\mathbb{Z}} g^n(U)$ is called the continuation of $\Lambda(U)$.

DEFINITION 1.2. Let $\Lambda \subset M$ be a closed f-invariant set. A splitting $T_\Lambda M = E \oplus F$ is called a *l-dominated splitting* for a positive integer l and dimension $i \in \{1, 2, \ldots, n-1\}$ if E and F are Df-invariant and

$$\|Df^l|_{E(x)}\|/m(Df^l|_{F(x)}) \le \frac{1}{2},$$

for all $x \in \Lambda$, where $m(A) = \inf\{\|Av\| : \|v\| = 1\}$ denotes the mininorm of a linear map A.

We say that Λ is *hyperbolic* if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E^s \oplus E^u$ and there exists constants $C > 0$ and $0 < \lambda < 1$ such that

$$\|D_x f^n|_{E_x^s}\| \le C\lambda^n \text{ and } \|D_x f^{-n}|_{E_x^u}\| \le C\lambda^n$$

for all $x \in \Lambda$ and $n \geq 0$. It is well-known that if Λ is hyperbolic, then it admits a dominated splitting. Now we are in position to state the main theorem of our paper.

Theorem Let Λ be a transitive set. If f has the C^1-stably periodic shadowing property on Λ, then Λ admits a dominated splitting.

2. Some known results

Let M as before, and let $f \in \mathrm{Diff}(M)$. To prove the results, we will use the following of the well known Franks' Lemma.

LEMMA 2.1. [3] *Let $\mathcal{U}(f)$ be any given C^1-neighborhood of f. Then there exists $\epsilon > 0$ and a C^1-neighborhood of $\mathcal{U}_0(f) \subset \mathcal{U}(f)$ of f such that for given $g \in \mathcal{U}_0(f)$, a finite set $\{x_1, x_2, \ldots, x_N\}$, a neighborhood U of $\{x_1, x_2, \ldots, x_N\}$, and linear maps $L_i : T_{x_i}M \to T_{g(x_i)}M$ satisfying $\|L_i - D_{x_i}g\| \leq \epsilon$ for all $1 \leq i \leq N$, there exists $\widehat{g} \in \mathcal{U}(f)$ such that $\widehat{g}(x) = g(x)$ if $x \in \{x_1, x_2, \ldots, x_N\} \cup (M \backslash U)$ and $D_{x_i}\widehat{g} = L_i$ for all $1 \leq i \leq N$.*

We recall the R. Mãné's result in [10] for the uniformly hyperbolic family of periodic sequences of linear maps on $\mathbb{R}^n$. Let $GL(n)$ be the group of linear isomorphisms of $\mathbb{R}^n$. We say that a sequence $\xi : \mathbb{Z} \to GL(n)$ is *periodic* if there is $k > 0$ such that $\xi_{j+k} = \xi_j$ for $j \in \mathbb{Z}$. In the case, the finite subset $\mathcal{A} = \{\xi_i : 0 \leq i \leq k-1\} \subset GL(n)$ is called *periodic family with period k*. For a periodic family $\mathcal{A} = \{\xi_i : 0 \leq i \leq n-1\}$, we denote

$$\mathcal{C}_{\mathcal{A}} = \xi_{n-1} \circ \xi_{n-2} \circ \ldots \circ \xi_0.$$

DEFINITION 2.2. [5] We say that the periodic family $\mathcal{A} = \{\xi : 0 \leq i \leq n-1\}$ admits a l-dominated splitting and dimension $i \in \{1, 2, \ldots, n-1\}$ if there is a splitting $\mathbb{R}^n = E \oplus F$ satisfying

(1) E and F are $\mathcal{C}_{\mathcal{A}}$-invariant; i.e., $\mathcal{C}_{\mathcal{A}}(E) = E$ and $\mathcal{C}_{\mathcal{A}}(F) = F$.
(2) For any $k = 0, 1, 2, \ldots,$

$$\frac{\|\mathcal{A}^l|_{E_k}\|}{m(\mathcal{A}^l|_{F_k})},$$

where

$$E_k = \xi_{k-1} \circ \xi_{k-2} \circ \ldots \circ \xi_0(E),$$
$$F_k = \xi_{k-1} \circ \xi_{k-2} \circ \ldots \circ \xi_0(F),$$

and $\mathcal{A}^l = \xi_{k+l-i}$ for $i = 0, \ldots, k-1$.

We will use the following two perturbation theorems which can be found in [2].

THEOREM 2.3. *Given any* $\epsilon > 0$ *and* $C > 0$*, there is* $n(\epsilon, C) > 0$ *which satisfies the following property; Given any periodic family* $\mathcal{A} = \{A_0, A_1, \ldots, A_{k-1}\}$ *which satisfies the period* $n_0 \geq n_1(\epsilon, C)$ *and*

$$\max\{\|A_i\|, \|A_i^{-1}\|\} \leq C$$

for $i = 0, 1, \ldots, k-1$*, one can find periodic family* $\mathcal{B} = \{B_0, B_1, \ldots, B_{k-1}\}$ *such that*

$$\max\{\|B_i - A_i\|, \|B_i^{-1} - A_i^{-1}\|\} < \epsilon$$

for $i = 0, 1, \ldots, k-1$, $\det(\mathcal{C}_{\mathcal{A}}) = \det(\mathcal{C}_{\mathcal{B}})$ *and the eigenvalues of* $\mathcal{C}_{\mathcal{B}}$ *are all real, with multiplicity* 1 *and difference moduli.*

THEOREM 2.4. *Given any* $\epsilon > 0$ *and* $C > 0$*, there is* $n(\epsilon, C) > 0$ *and* $l(\epsilon, C) > 0$ *which satisfies the following property; Given any periodic family* $\mathcal{A} = \{A_0, A_1, \ldots, A_{k-1}\}$ *which satisfies the period* $n_0 \geq n_1(\epsilon, C)$ *and* $\max\{\|A_i\|, \|A_i^{-1}\|\} \leq C$ *for* $i = 0, 1, \ldots, k-1$*, if* $\mathcal{A}$ *do not admits any* $l(\epsilon, C)$*-dominated splitting, then one can find periodic family* $\mathcal{B} = \{B_0, B_1, \ldots, B_{k-1}\}$ *such that* $\max\{\|B_i - A_i\|, \|B_i^{-1} - A_i^{-1}\|\} < \epsilon$ *for* $i = 0, 1, \ldots, k-1$, $\det(\mathcal{C}_{\mathcal{A}}) = \det(\mathcal{C}_{\mathcal{B}})$ *and the eigenvalues of* $\mathcal{C}_{\mathcal{B}}$ *are all real and the same modulus.*

To prove Main Theorem, we need another result about uniformly contracting family which can be found in [10]. Let $\mathcal{A} = \{\xi_i : 0 \leq i \leq k-1\} \subset GL(n)$ be a periodic family. We say the sequence $\mathcal{A}$ is *uniformly contracting family* if there is a constant $\delta > 0$ such that every δ-perturbation of $\mathcal{A}$ is sink; i.e., for any $\mathcal{B} = \{\zeta_i : 0 \leq i \leq k-1\}$ with $\|\zeta_i - \xi_i\| < \delta$, all eigenvalues of $\mathcal{C}_{\mathcal{B}}$ have moduli less than 1. Similarly, we can define the notion of *uniformly expanding periodic family.*

THEOREM 2.5. [10] *For any* $\delta > 0$ *and* $K > 0$*, there are constants* $C > 0$, $0 < \lambda < 1$ *and positive integer* m *such that if* $\mathcal{A} = \{A_0, A_1, \ldots, A_{n-1}\}$ *is a uniformly contracting periodic family which satisfies*

$$\max_{i=0,1,\ldots,n-1}\{\|A_i\|, \|A_i^{-1}\|\} < K$$

for $n > m$*, then*

$$\prod_{j=0}^{k-1} \| \prod_{i=0}^{m-1} A_{i+mj} \| \leq C\lambda^k,$$

where $k = [\frac{n}{m}]$.

3. Proof of Theorem

Let M be as before and let $f \in \mathrm{Diff}(M)$. From now on, we introduce the notion of pre-sinks and pre-sources for periodic points of diffomorphisms of M, and prove that for every C^1-stably periodic shadowing diffeomorphism f on a transitive set, any g C^1-nearby f has neither pre-sink nor pre-source with the orbit staying in a neighborhood of a transitive set. Let $f \in$ Diff(M) and p be a periodic point of f. We say $p \in P(f)$ is *pre-sink(resp. pre-source)* if $D_p f^{\pi(p)}$ has a multiplicity one eigenvalue equal to 1 and -1 and the other eigenvalues have norm less than 1(resp. bigger than 1). In the next Lemma, let Λ be a closed f-invariant set, U be a compact neighborhood of Λ, and $\mathcal{U}(f)$ be a C^1-neighborhood of f.

LEMMA 3.1. *Let Λ be a transitive set of f. Suppose f has the C^1-stably periodic shadowing property on Λ with respect to $\mathcal{U}(f)$ and U. Then for any $g \in \mathcal{U}(f)$, g has neither pre-sink nor pre-source with the orbit staying in U.*

Proof. Assume that there is g C^1-nearby f such that g has a pre-sink p with $O_g(p) \subset U$. choose $\epsilon_1 > 0$ with $B_{\epsilon_1}(p) \subset U$ and g_1 C^1-nearby g such that for 0$\leq i \leq \pi(p)$-2, if $x \in B_{\epsilon_1}(g^i(p))$, then

$$g_1(x) = \exp_{g^{i+1}(p)} \circ D_{g^i(p)} g \circ \exp^{-1}_{g^i(p)}(x),$$

and if $x \in B_{\epsilon_1}(g^{\pi(p)-1}(p))$, then

$$g_1(x) = \exp_p \circ D_{g^{\pi(p)-1}(p)} g \circ \exp^{-1}_{g^{\pi(p)-1}(p)}(x).$$

Then $g_1^{\pi(p)}(p) = g^{\pi(p)}(p) = p$. Let $p_i = f^i(p)$ and $A_i = D_{p_i} g$. For $\delta > 0$, let us construct a sequence $y_i \in T_pM, i \in \mathbb{Z}$, as follow :

$$\begin{aligned}
&y_0 = O_p, \\
&y_{i+1} = Ay_i + \frac{\delta}{2}, \quad \text{if} \quad 0 \leq i \leq K-1, \\
&y_{i+1} = Ay_i - \frac{\delta}{2}, \quad \text{if} \quad K \leq i \leq 2K, \\
&y_{i-1} = Ay_i - \frac{\delta}{2}, \quad \text{if} \quad -K+1 \leq i \leq 0, \quad \text{and} \\
&y_{i-1} = Ay_i + \frac{\delta}{2}, \quad \text{if} \quad -2K \leq i \leq -K.
\end{aligned}$$

Then $y_{i+4K} = y_i$, for $i \in \mathbb{Z}$. Therefore $\{y_i\}_{i\in\mathbb{Z}}$ is a 4K-periodic δ-pseudo orbit. Put $y_k = \frac{k\delta}{2}$. Then

$$\xi = \Big\{ -\frac{2K\delta}{2}, -\frac{2(K-1)\delta}{2}, \dots, O_p, \frac{\delta}{2}, \dots, \frac{2K\delta}{2} \Big\}$$

is a 4K-periodic δ-pseudo orbit. Let $x_i = \exp_p(y_i)$. Then $\{x_i\}_{i\in\mathbb{Z}}$ is a 4K-periodic δ-pseudo orbit of g in M, where $x_0 = p = \exp_p(O_p)$. By our assumption, p is a pre-sink of g, $D_pg^{\pi(p)}$ has a multiplicity one eigenvalue such that $|\lambda| = 1$, and the other eigenvalues of $D_pg^{\pi(p)}$ are with moduli less than 1. Denote that E^c_p is the eigenspace corresponding to λ, and E^s_p is the eigenspace corresponding to the eigenvaulues with moduli less than 1. Then $T_pM = E^c_p \oplus E^s_p$. If $\lambda \in \mathbb{R}$, then $\dim E^c_p = 1$, and if $\lambda \in \mathbb{C}$ then $\dim E^c_p = 2$.

At first, we consider $\lambda \in \mathbb{R}$. For simplicity, we may assume that $\lambda = 1$ and $g^{\pi(p)}(p) = g(p) = p$. Then there is a small arc $\mathcal{I}_p \subset B_{\epsilon_1}(p) \bigcap \exp_p(E^c_p(\epsilon_1))$ centered at p such that $g_1^{\pi(p)}(\mathcal{I}_p) = g(\mathcal{I}_p) = \mathcal{I}_p$ with $\mathrm{diam}(\mathcal{I}_p) = \mathrm{K}\delta = \mu$. Here $E^c_p(\epsilon_1)$ is the ε_1-ball in E^c_p centered at the origin O_p. Take $\epsilon = \frac{\mu}{8}$. Since f has the C^1-stably periodic shadowing property on Λ, g has the shadowing property on $\Lambda_g(U) = \bigcap_{n\in\mathbb{Z}} g^n(U)$. Let us construct a sequence $y_i \in T_pM$, $i \in \mathbb{Z}$, as follow :

$$\begin{aligned}
&y_0 = O_p, \\
&y_{i+1} = y_i + \frac{\delta}{2}, \quad \text{if} \quad 0 \le i \le K-1, \\
&y_{i-1} = y_i - \frac{\delta}{2}, \quad \text{if} \quad -K+1 \le i \le O_p, \\
&y_{i+1} = y_i - \frac{\delta}{2}, \quad \text{if} \quad K \le i \le 2K \quad \text{and} \\
&y_{i-1} = y_i + \frac{\delta}{2}, \quad \text{if} \quad -2K \le i \le -K.
\end{aligned}$$

Then $y_{i+4K} = y_i$, for $i \in \mathbb{Z}$. Therefore $\{y_i\}_{i\in\mathbb{Z}}$ is a 4K-periodic δ-pseudo orbit in $\exp^{-1}(\mathcal{I}_p)$. Let $x_i = \exp_p(y_i)$. Then $\{x_i\}_{i\in\mathbb{Z}}$ is a 4K-periodic δ-pseudo orbit of g on $\mathcal{I}_p$, where $x_0 = p = \exp_p(O_p)$. Thus we can take a periodic shadowing point, say z, in $M \setminus \mathcal{I}_p$ or $\mathcal{I}_p$.

Suppose z belongs to $\mathcal{I}_p$ with $d(z, x_0(= p)) < \frac{\mu}{4}$. Since g is an identity map on $\mathcal{I}_p$, we can find $n \in \mathbb{Z}$ such that

$$d(g^n(z), x_n) = d(z, \exp_p(y_0 + \frac{n\delta}{2})) > \frac{\mu}{4} > \epsilon.$$

That is, g dose not have the periodic shadowing property. This implies that the shadowing point z belongs to $M \setminus \mathcal{I}_p$. Since p is a pre-sink, we know that for $z \in M \setminus \mathcal{I}_p$, $g^n(z) \to \mathcal{I}_p$ as $n \to \infty$. Then we easy to see that g dose not have the periodic shadowing property. Consequently, if f has the C^1-stably periodic shadowing property on Λ, then for any g C^1-nearby f, g has neither pre-sink nor pre-source.

Next we consider the case $\lambda \in \mathbb{C}$. For simplicity, we may assume that $g^{\pi(p)}(p) = g(p) = p$. Then there exist ϵ_0 and g_1 C^1-nearby g such that

$$g_1(x) = \exp_{g(p)} \circ D_p g \circ \exp^{-1}(x)$$

if $x \in B_{\epsilon_0}(p)$. Then $g_1(p) = g(p) = p$. With a small modification of the map $D_p g_1$, we may suppose that there is $m > 0$ (the minimal number) such that $D_p g_1^m(v) = v$ for any $v \in E_p^c(\epsilon_0) \cap B_{\epsilon_0}(p)$. Hence we can get a small disk $\mathcal{D}_p \subset exp_p(E_p^c(\epsilon_0)) \cap B_{\epsilon_0}(p)$ such that $g_1^m(\mathcal{D}_p) = \mathcal{D}_p$.

Hence we only treat the case when the pseudo orbits are in $M \setminus \mathcal{D}_p$. The other case can be proved as in the above case. To prove this, we assume $g_1^m(\mathcal{D}_p) = \widetilde{g}(\mathcal{D}_p) = \mathcal{D}_p$. By applying Proposition 3.1 in [4], we can choose $h \in \mathrm{Diff(M)}$ and $\epsilon_1 > 0$ such that

- $d_1(h, \widetilde{g}) < \delta$,
- $h(p) = \widetilde{g}(p) = p$,
- $h(x) \neq \widetilde{g}(p)$ for $x \in B_{\epsilon_1}(p)$ and
- $h(x) = \widetilde{g}(x)$ for $x \notin B_{\epsilon_1}(p)$.

Since p is pre-sink, for any point $y \in M \setminus \mathcal{D}_p$, we have

$$h^k(y) \to z \in \mathcal{D}_p \quad \text{as} \quad k \to \infty, \text{and} \quad h^k(y) \to \infty \quad \text{as} \quad k \to -\infty.$$

Thus we get a contradiction by the periodic shadowing property for g_1. The contradiction shows that if $f \in \mathrm{Diff(M)}$ has the C^1-stably periodic shadowing property on Λ then it does not have the pre-sinks. The case for pre-source can be proved in a similar way. □

We say Λ is a nontrivial transitive set if Λ is not just one orbit.

Lemma 3.2. [12, Corollary 2.7.1] *Let Λ be a nontrivial transitive set. Then there are a sequence $\{g_n\}_{n\in\mathbb{N}}$ of diffeomorphisms and a sequence $\{P_n\}$ of periodic orbits of g_n with period $\pi(P_n) \to \infty$ such that $g_n \to f$ in the C^1-topology and $P_n \to_H \Lambda$ as $n \to \infty$, where $\to_H$ is the Hausdorff limit, and $\pi(P_n)$ is the period of P_n.*

From Lemma 3.4, we can choose $p_n \in P_n$ such that

$$\mathcal{A}_n = \{D_{p_n} g_n, D_{g_n(p_n)} g_n, \ldots, D_{g_n^{\pi(p_n)-1}} g_n\}$$

is a family of periodic linear maps.

LEMMA 3.3. [8, Lemma 3.3] *Let* Λ *and* P_n *be as in Lemma 3.2, and let* $\mathcal{A}_n$ *be given as in the above. Then for any* $\epsilon > 0$ *there exists* $n_0(\epsilon) > 0$ *such that for any* $n > n_0(\epsilon)$, $\mathcal{A}_n$ *is neither* ϵ*-uniformly contraction for* ϵ*-uniformly expanding.*

LEMMA 3.4. [8, Lemma 3.4] *Let* Λ *and* P_n *be as in Lemma 3.2. Then for any* $\epsilon > 0$ *there are* $n(\epsilon)$ *and* $l(\epsilon) > 0$ *such that for any* $n > n(\epsilon)$ *if* P_n *does not admit a* $l(\epsilon)$*-dominated splitting, then there exists* g C^1 *nearby* f *which preserves the orbit of* P_n *and* P_n *is a pre-sink or pre-source for* g.

From the above facts, we get the following propositions.

PROPOSITION 3.5. *Let* Λ *and* P_n *be as in Lemma 3.2. If* f *has the* C^1*-stably periodic shadowing property on* Λ*, then there are constants* N *and* $l > 0$ *such that* P_n *admits a* l*-dominated splitting for any* $n > N$.

PROPOSITION 3.6. [1] *Let* $\{g_n\}$ *be a sequence in* $\mathrm{Diff}(M)$ *which converges to* f*, and let* Λ_{g_n}*-invariant set for each* $n \in \mathbb{N}$. *Suppose* $\lim \Lambda_{g_n} = \Lambda$. *If each* Λ_{g_n} *admits a* l*-dominated splitting for* g_n*, then* Λ *admits a* l*-dominated splitting for* f.

End of the Proof of Theorem. Let Λ be a transitive set. If f has the C^1-stably periodic shadowing property on Λ. Then for any g C^1 nearby f, g has neither pre-sink nor pre-source with the orbit staying in U. Therefore Λ admits a dominated splitting, by the result of Bonatti, Diáz and Pujals [1].

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