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INELASTIC FLOWS OF CURVES ACCORDING TO EQUIFORM IN GALILEAN SPACE

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Abstract. In this paper, we derive a set of the partial differential equations that characterize an inelastic flow of a curve in a 3-dimensional Galilean space. Also, we give necessary and sufficient condition for an inelastic flow.

1. Introduction

The flow of a curve is called to be inelastic if the arc-length of a curve is preserved. Inelastic curve flows have growing importance in many applications such engineering, computer vision, structural mechanics and computer animation ([2], [7]). Physically, inelastic curve flows give rise to motion which no strain energy is induced. The swinging motion of a cord of fixed length can be described by inelastic curve flows. There exist such motions in many physical applications.

G. S. Chirikjian and J. W. Burdick([1]) studied applications of inelastic curve flows. M. Gage and R. S. Hamilton([5]) and M. A. Grayson([6]) investigated shrinking of closed plane curves to a circle via the heat equation. Also, D. Y. Kwon and F. C. Park([9], [10]) derived the evolution equation for an inelastic plane and space curve. Recently, N. Gurbuz([7]) examined inelastic flows of space-like, time-like and null curve in a 3-dimensional Minkowski space.

A Galilean space may be considered as the limit case of a pseudo-Euclidean space in which the isotropic cone degenerates to a plane. The limit transition corresponds to the limit transition from the special theory of relatively to classical mechanics ([12]). On the study of a Galilean space, B. Divjak and M. Sipus([3]) investigated the properties of helical surfaces, ruled screw surfaces and rotation surface in 3-dimensional

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Galilean space G_3. Also, A. O. Ogrenmis([11]) studied the Frenet formula and Mannheim curve of AW(k)-type in pseudo-Galilean space.

In this paper, we derive the evolution equations for inelastic flows of a curve in G_3. Furthermore, we give necessary and sufficient condition for inelastic flows.

2. Preliminaries

The Galilean space G_3 is a Cayley-Klein space equipped with the projective metric of signature $(0, 0, +, +)$. The absolute figure of the Galilean space consist of an ordered triple $\{w, f, I\}$, where w is the ideal (absolute) plane, f is the line (absolute line) in w and I is the fixed elliptic involution of points of f.

In the non-homogeneous coordinates the similarity group H_8 has the form

$$\begin{aligned} \bar{x} &= a_{11} + a_{12}x \\ \bar{y} &= a_{21} + a_{22}x + a_{23}y\cos\theta + a_{23}z\sin\theta \\ \bar{z} &= a_{31} + a_{32}x - a_{23}y\sin\theta + a_{23}z\cos\theta \end{aligned} \tag{2.1}$$

where a_{ij} and θ are real numbers ([4]). In what follows the real numbers a_{12} and a_{23} will play the special role. In particular, for $a_{12} = a_{23} = 1$, (2.1) defines the group $B_6 \subset H_8$ of isometries of the Galilean space G_3.

The Galilean scalar product can be written as

$$\langle \mathbf{x}, \mathbf{y} \rangle = \begin{cases} x_1x_2, & \text{if } \ x_1 \neq 0 \quad \text{or} \quad x_2 \neq 0 \\ y_1y_2 + z_1z_2, & \text{if } \ x_1 = 0 \quad \text{and} \quad x_2 = 0, \end{cases}$$

where $\mathbf{x} = (x_1, y_1, z_1)$ and $\mathbf{y} = (x_2, y_2, z_2)$. It leaves invariant the Galilean norm of the vector $\mathbf{x}$ defined by

$$||\mathbf{x}|| = \begin{cases} x_1, & \text{if} \quad x_1 \neq 0 \\ \sqrt{y_1^2 + z_1^2}, & \text{if} \quad x_1 = 0. \end{cases}$$

A curve $\alpha : I \subset \mathbb{R} \to G_3$ of the class C^∞ in the Galilean space G_3 is defined by the parametrization

$$\alpha(s) = (s, y(s), z(s)),$$

where s is a Galilean invariant arc-length of α. Then the curvature $\kappa(s)$ and the torsion $\tau(s)$ are given by

$$\kappa(s) = \sqrt{\ddot{y}(s)^2 + \ddot{z}(s)^2}, \quad \tau(s) = \frac{\det(\dot{\alpha}(s), \ddot{\alpha}(s), \dddot{\alpha}(s))}{\kappa^2(s)},$$

respectively.
On the other hand, the Frenet vectors of $\alpha(s)$ in G_3 are defined by

$$\begin{aligned}\mathbf{t}(s) &= \dot{\alpha}(s) = (1, \dot{y}(s), \dot{z}(s)),\\ \mathbf{n}(s) &= \frac{1}{\kappa(s)}\ddot{\alpha}(s) = \frac{1}{\kappa(s)}(0, \ddot{y}(s), \ddot{z}(s)),\\ \mathbf{b}(s) &= \frac{1}{\kappa(s)}(0, -\ddot{z}(s), \ddot{y}(s)).\end{aligned}$$

The vectors $\mathbf{t}, \mathbf{n}, \mathbf{b}$ are called the vector of tangent, principal normal and binormal of α, respectively. For their derivatives the following Frenet formula satisfies([cf. 4])

$$\begin{aligned}\dot{\mathbf{t}}(s) &= \kappa(s)\mathbf{n}(s),\\ \dot{\mathbf{n}}(s) &= \tau(s)\mathbf{b}(s),\\ \dot{\mathbf{b}}(s) &= -\tau(s)\mathbf{n}(s).\end{aligned} \tag{2.2}$$

3. Frenet formulas in equiform geometry in G_3

Let $\alpha : I \to G_3$ be a curve in the Galilean space G_3. We define the equiform parameter of α by

$$\sigma := \int \frac{1}{\rho} ds = \int \kappa ds,$$

where $\rho = \frac{1}{\kappa}$ is the radius of curvature of the curve α. Then, we have

$$\frac{ds}{d\sigma} = \rho. \tag{3.1}$$

Let h be a homothety with the center in the origin and the coefficient λ. If we put $\tilde{\alpha} = h(\alpha)$, then it follows

$$\tilde{s} = \lambda s \quad \text{and} \quad \tilde{\rho} = \lambda\rho,$$

where $\tilde{s}$ is the arc-length parameter of $\tilde{\alpha}$ and $\tilde{\rho}$ the radius of curvature of this curve. Therefore, σ is an equiform invariant parameter of α ([4]).

From now on, we define the Frenet formula of the curve α with respect to the equiform invariant parameter σ in G_3.

The vector

$$\mathrm{T} = \frac{d\alpha}{d\sigma}$$

is called a tangent vector of the curve α. From (2.2) and (3.1) we get

$$\mathrm{T} = \frac{d\alpha}{ds} \cdot \frac{ds}{d\sigma} = \rho \cdot \frac{d\alpha}{ds} = \rho \cdot \mathbf{t}. \tag{3.2}$$

We define the principal normal vector and the binormal vector by

$$\mathrm{N} = \rho \cdot \mathbf{n}, \quad \mathrm{B} = \rho \cdot \mathbf{b}. \tag{3.3}$$

Then, we easily show that $\{\mathrm{T}, \mathrm{N}, \mathrm{B}\}$ are an equiform invariant orthonormal frame of the curve α.
On the other hand, the derivations of these vectors with respect to σ are given by

$$\begin{aligned} \mathrm{T}' =& \frac{d\mathrm{T}}{d\sigma} = \dot{\rho}\mathrm{T} + \mathrm{N}, \\ \mathrm{N}' =& \frac{d\mathrm{N}}{d\sigma} = \dot{\rho}\mathrm{N} + \rho\tau\mathrm{B}, \\ \mathrm{B}' =& \frac{d\mathrm{B}}{d\sigma} = \rho\tau\mathrm{N} + \dot{\rho}\mathrm{B}. \end{aligned}$$

Definition 3.1. The function $\mathcal{K} : I \to \mathbb{R}$ defined by

$$\mathcal{K} = \dot{\rho}$$

is called the equiform curvature of the curve α.

Definition 3.2. The function $\mathcal{T} : I \to \mathbb{R}$ defined by

$$\mathcal{T} = \rho\tau = \frac{\tau}{\kappa}$$

is called the equiform torsion of the curve α.

Thus, the formula analogous to the Frenet formula in the equiform geometry of the Galilean space have the following form

$$\begin{aligned} \mathrm{T}' &= \mathcal{K} \cdot \mathrm{T} + \mathrm{N}, \\ \mathrm{N}' &= \mathcal{K} \cdot \mathrm{N} + \mathcal{T} \cdot \mathrm{N}, \\ \mathrm{B}' &= \mathcal{T} \cdot \mathrm{N} + \mathcal{K} \cdot \mathrm{B}. \end{aligned} \tag{3.4}$$

The equiform parameter $\sigma = \int \kappa(s)ds$ for closed curves is called the total curvature, and it plays an important role in global differential geometry of Euclidean space. Also, the function $\frac{\tau}{\kappa}$ has been already known as a conical curvature and it also has interesting geometric interpretation.

4. Inelastic flows of curves according to equiform in G_3

We assume that $F : [0, l] \times [0, w] \to G_3$ is a one parameter family of smooth curve in the Galilean space G_3, where l is the arc-length of initial curve. Let u be the curve parametrization variable, $0 \leq u \leq l$. We put $v = ||\frac{\partial F}{\partial u}||$, from which the arc-length of F is defined by $s(u) = \int_0^u v du$.

Also, the operator $\frac{\partial}{\partial s}$ is given in terms of u by $\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u}$ and the arc-length parameter is given by $ds = vdu$.

On the equiform invariant orthonormal frame $\{\mathrm{T}, \mathrm{N}, \mathrm{B}\}$ of a curve α in G_3, any flow of F can be given by

$$\frac{\partial F}{\partial t} = f\mathrm{T} + g\mathrm{N} + h\mathrm{B}, \tag{4.1}$$

where f, g, h are the tangential, principal normal, binormal speeds of the curve in G_3, respectively. We put $s(u,t) = \int_0^u vdu$, it is called the arc-length variation of F. From this, the requirement that the curve not be subject to any elongation or compression can be expressed by the condition

$$\frac{\partial}{\partial t}s(u,t) = \int_0^u \frac{\partial v}{\partial t}du = 0, \tag{4.2}$$

for all $u \in [0, l]$.

Definition 4.1. A curve evolution $F(u,t)$ and its flow $\frac{\partial F}{\partial t}$ in G_3 are said to be inelastic if

$$\frac{\partial}{\partial t}\left|\left|\frac{\partial F}{\partial u}\right|\right| = 0.$$

Theorem 4.2. *(Necessary and Sufficient Conditions for an Inelastic Flow) Let $\frac{\partial F}{\partial t} = f\mathrm{T} + g\mathrm{N} + h\mathrm{B}$ be a flow of F in G_3. The flow is inelastic if and only if*

$$\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u} + vf\mathcal{K}. \tag{4.3}$$

Proof. From the definition of F, we have

$$v^2 = \left\langle \frac{\partial F}{\partial u}, \frac{\partial F}{\partial u} \right\rangle. \tag{4.4}$$

Since u and t are independent coordinates, $\frac{\partial}{\partial u}$ and $\frac{\partial}{\partial t}$ commute. So, by differentiating of (4.4) we have

$$\begin{aligned}
2v\frac{\partial v}{\partial t} &= \frac{\partial}{\partial t}\left\langle \frac{\partial F}{\partial u}, \frac{\partial F}{\partial u}\right\rangle \\
&= 2\left\langle \frac{\partial F}{\partial u}, \frac{\partial}{\partial u}(\frac{\partial F}{\partial t})\right\rangle \\
&= 2\left\langle \frac{\partial F}{\partial u}, \frac{\partial}{\partial u}(f\mathrm{T} + g\mathrm{N} + h\mathrm{B})\right\rangle \\
&= 2v\left\langle \mathrm{T}, (\frac{\partial f}{\partial u} + vf\mathcal{K})\mathrm{T} + (vf + \frac{\partial g}{\partial u} + vg\mathcal{K} + vh\mathcal{T})\mathrm{N} \right. \\
&\qquad\qquad \left. +(vg\mathcal{T} + \frac{\partial h}{\partial u} + vh\mathcal{K})\mathrm{B}\right\rangle \\
&= 2v\left(\frac{\partial f}{\partial u} + vf\mathcal{K}\right).
\end{aligned}$$

□

Lemma 4.3. *The flow* $\frac{\partial H}{\partial t} = f\mathrm{T} + g\mathrm{N} + h\mathrm{B}$ *of the curve* F *is inelastic if and only if*

$$\frac{\partial f}{\partial s} = -f\mathcal{K}.$$

Proof. From Theorem 4.2,

$$\begin{aligned}
\frac{\partial}{\partial t}s(u,t) &= \int_0^u \frac{\partial v}{\partial t}du \\
&= \int_0^u (\frac{\partial f}{\partial u} + vf\mathcal{K})du \\
&= 0
\end{aligned}$$

for all $u \in [0, l]$. It follows that $\frac{\partial f}{\partial u} = -vf\mathcal{K}$ or $\frac{\partial f}{\partial s} = -f\mathcal{K}$. The argument can be reversed to show sufficient, completing the proof. □

Lemma 4.4.

$$\begin{aligned}
\frac{\partial \mathrm{T}}{\partial t} &= \left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}\right)\mathrm{N} + \left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K}\right)\mathrm{B}, \\
\frac{\partial \mathrm{N}}{\partial t} &= -\left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}\right)\mathrm{T} + \Psi\mathrm{B}, \\
\frac{\partial \mathrm{B}}{\partial t} &= -\left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K}\right)\mathrm{T} - \Psi\mathrm{N}, \quad \Psi = \left\langle \frac{\partial \mathrm{N}}{\partial t}, \mathrm{B}\right\rangle.
\end{aligned}$$

Proof. Using the Frenet formula and Lemma 4.3, we calculate

$$\begin{aligned}
\frac{\partial \mathrm{T}}{\partial t} &= \frac{\partial}{\partial t}\frac{\partial F}{\partial s} = \frac{\partial}{\partial s}\frac{\partial F}{\partial t} = \frac{\partial}{\partial s}(f\mathrm{T} + g\mathrm{N} + f\mathrm{B}) \\
&= \frac{\partial f}{\partial s}\mathrm{T} + \frac{\partial g}{\partial s}\mathrm{N} + \frac{\partial h}{\partial s}\mathrm{B} + f\mathcal{K}\mathrm{T} + f\mathrm{N} + g\mathcal{K}\mathrm{N} + g\mathcal{T}\mathrm{B} + h\mathcal{T}\mathrm{N} + h\mathcal{K}\mathrm{B} \\
&= \left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}\right)\mathrm{N} + \left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K}\right)\mathrm{B}.
\end{aligned}$$

Differentiating Frenet frame with respect to t, we obtain:

$$\begin{aligned}
0 = \frac{\partial}{\partial t}\langle \mathrm{T}, \mathrm{N}\rangle &= \left\langle \frac{\partial \mathrm{T}}{\partial t}, \mathrm{N}\right\rangle + \left\langle \mathrm{T}, \frac{\partial \mathrm{N}}{\partial t}\right\rangle \\
&= \left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}\right) + \left\langle \mathrm{T}, \frac{\partial \mathrm{N}}{\partial t}\right\rangle.
\end{aligned}$$

$$0 = \frac{\partial}{\partial t}\langle \mathrm{T}, \mathrm{B}\rangle = \left\langle \frac{\partial \mathrm{T}}{\partial t}, \mathrm{B}\right\rangle + \left\langle \mathrm{T}, \frac{\partial \mathrm{B}}{\partial t}\right\rangle = \left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K}\right) + \left\langle \mathrm{T}, \frac{\partial \mathrm{B}}{\partial t}\right\rangle.$$

$$0 = \frac{\partial}{\partial t}\langle \mathrm{N}, \mathrm{B}\rangle = \left\langle \frac{\partial \mathrm{N}}{\partial t}, \mathrm{B}\right\rangle + \left\langle \mathrm{N}, \frac{\partial \mathrm{B}}{\partial t}\right\rangle = \Psi + \left\langle \mathrm{N}, \frac{\partial \mathrm{B}}{\partial t}\right\rangle.$$

From the above equations, we obtain $\frac{\partial \mathrm{N}}{\partial t} = -\left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}\right)\mathrm{T} + \Psi\mathrm{B}$ and $\frac{\partial \mathrm{B}}{\partial t} = -\left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K}\right)\mathrm{T} - \Psi\mathrm{N}$. □

The following theorem states the conditions on the curvature and the torsion for the curve flow $F(u,t)$ to be inelastic.

THEOREM 4.5. *(Equations for Inelastic Evolution) If the curve flow* $\frac{\partial F}{\partial t} = f\mathrm{T} + g\mathrm{N} + h\mathrm{B}$ *is inelastic, then the following system of partial differential equations holds:*

$$\begin{aligned}
\frac{\partial \mathcal{K}}{\partial t} &= \frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}. \\
\frac{\partial \mathcal{T}}{\partial t} &= -\left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K}\right). \\
\Psi &= \frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathcal{T}) + \frac{\partial}{\partial s}(h\mathcal{K}) + \mathcal{T}\left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}\right).
\end{aligned}$$

Proof. Noting that $\frac{\partial}{\partial s}\frac{\partial \mathrm{T}}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \mathrm{T}}{\partial s}$,

$$\frac{\partial}{\partial s}\frac{\partial \mathrm{T}}{\partial t} = \frac{\partial}{\partial s}\left((\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T})\mathrm{N} + (\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K})\mathrm{B}\right)$$

$$
\begin{aligned}
&= \left(\frac{\partial^2 g}{\partial s^2} + \frac{\partial f}{\partial s} + \frac{\partial}{\partial s}(g\mathcal{K}) + \frac{\partial}{\partial s}(h\mathcal{T}) \right) \mathrm{N} \\
&\quad + \left(\frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathcal{T}) + \frac{\partial}{\partial s}(h\mathcal{K}) \right) \mathrm{B} \\
&\quad + \left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T} \right) (\mathcal{K}\mathrm{N} + \mathcal{T}\mathrm{B}) \\
&\quad + \left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K} \right) (\mathcal{T}\mathrm{N} + \mathcal{K}\mathrm{B}).
\end{aligned}
$$

By using Lemma 4.4, we have the following equation:

$$
\begin{aligned}
\frac{\partial}{\partial t}\frac{\partial \mathrm{T}}{\partial s} &= \frac{\partial}{\partial t}(\mathcal{K}\mathrm{T} + \mathrm{N}) \\
&= \frac{\partial \mathcal{K}}{\partial t}\mathrm{T} + \mathcal{K}\left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T} \right)\mathrm{N} + \mathcal{K}\left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K} \right)\mathrm{B} \\
&\quad - \left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T} \right)\mathrm{T} + \Psi\mathrm{B}.
\end{aligned}
$$

By combining the above two equations, we have

$$
\frac{\partial \mathcal{K}}{\partial t} = \frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T}
$$

and

$$
\Psi = \frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathcal{T}) + \frac{\partial}{\partial s}(h\mathcal{K}) + \mathcal{T}\left(\frac{\partial g}{\partial s} + f + g\mathcal{K} + h\mathcal{T} \right).
$$

Since $\frac{\partial}{\partial s}\frac{\partial \mathrm{B}}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \mathrm{B}}{\partial s}$, we have from Lemma 4.4

$$
\begin{aligned}
\frac{\partial}{\partial s}\frac{\partial \mathrm{B}}{\partial t} &= \frac{\partial}{\partial s}\left(-(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K})\mathrm{T} - \Psi\mathrm{N} \right) \\
&= -\left(\frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathcal{T}) + \frac{\partial}{\partial s}(h\mathcal{K}) \right)\mathrm{T} \\
&\quad - \left(\frac{\partial h}{\partial s} + g\mathcal{T} + h\mathcal{K} \right)(\mathcal{K}\mathrm{T} + \mathrm{N}) \\
&\quad - \frac{\partial \Psi}{\partial s}\mathrm{N} - \Psi(\mathcal{K}\mathrm{N} + \mathcal{T}\mathrm{B}).
\end{aligned}
$$

By using Frenet formula, we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\partial \mathrm{B}}{\partial s} &= \frac{\partial}{\partial t}(\mathcal{T}\mathrm{N}+\mathcal{K}\mathrm{B})\\ &= \frac{\partial \mathcal{T}}{\partial t}\mathrm{N}+\frac{\partial \mathcal{K}}{\partial t}\mathrm{B}-\mathcal{T}\left(\frac{\partial g}{\partial s}+f+g\mathcal{K}+h\mathcal{T}\right)\mathrm{T}\\ &\quad +\mathcal{T}\Psi\mathrm{B}-\mathcal{K}\left(\frac{\partial h}{\partial s}+g\mathcal{T}+h\mathcal{K}\right)\mathrm{T}-\mathcal{K}\Psi\mathrm{N}.\end{aligned}$$

Thus we have $\frac{\partial \mathcal{T}}{\partial t}=-\left(\frac{\partial h}{\partial s}+g\mathcal{T}+h\mathcal{K}\right).$ □

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